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Optimal Hedge Ratio for Brent Oil Market; Baysian Approach

Mohsen Mehrara[a], Monire Hamldar[b]

Faculty of Economics, University of Tehran, Kargar-e-shomali, Po Box 14166-6445, Tehran, Iran

[a,b]E-mail address: mmehrara@ut.ac.ir , hamldar@ut.ac.ir

ABSTRACT

This paper examines the optimal hedging ratio (OHR) for the Brent Crude Oil Futures using daily data over the period 1990/17/8-2014/11/3. To gain OHR, it is employed a Vector Autoregressive (VAR) and Vector Error Correction (VEC) and Baysian Vector Autoregressive (BVAR) models. At last, the efficiency of these calculated OHR are compared through Edrington's index.

Keywords: Optimal Hedging Ratio; OHR; VAR; VECM; BVAR; Edrington's efficiency index

1. INTRODUCTION

Price risk management, using hedging tools like futures and options and their effectiveness, is an active area of research. Hedging decisions based on futures contracts have to deal with finding optimal hedge ratio and hedging effectiveness. Role of hedging using multiple risky assets and using futures market for minimizing the risk of spot market fluctuation has been extensively researched.

Traditionally, hedging is envisaged using a hedge ratio of '-1', i.e., taking a position in futures contract which is equal in magnitude and opposite in sign to the position in spot market. If the movement of changes in spot prices and futures prices is exactly the same, then such a strategy eliminates the price risk. Such a perfect correlation between spot and futures prices is rarely observed in markets and hence a need was felt for a better strategy. Johnson (1960) proposed 'minimum variance hedge ratio (MVHR)', which factored in less than perfect relationship between spot and futures prices. Risk was defined as the variance of returns on a two-asset hedged position.

Following Doan, Litterman and Sims (1984), the Bayesian approach to the estimation of vector autoregressive (VARs) is employed. The forecasting models have traditionally been formulated as simultaneous equation structural models. However, for a variety of reasons structural models have proved unreliable for forecasting. The VAR models have also been criticized insofar as they lack strong theoretical justification over and above the use of theory as a guide in deciding which variables to include in the analysis. Doan, Litterman and Sims (1984) in an attempt to improve the forecasting performance of unrestricted VARs suggested that they could be estimated using Bayesian techniques which take account of any prior information which may be available to the modeler. It is this Bayesian approach to parameter estimation in vector autoregressive which is employed in this study.

2. METHODOLOGY AND EMPIRICAL RESULTS

In this section we employ the VAR, VEC and BVAR to calculate the optimal hedge ratio(OHR). The financial variables used in the model are spot and futures price of Brent Oil. The data series are obtained from Energy Information Agency (EIA). The data are daily from 1990/17/8-2014/11/3.

2. 1. Hedge Ratio and Hedging Effectiveness

The optimal hedge ratio is defined as the ratio of the size of position taken in the futures market to the size of the cash position which minimizes the total risk of portfolio. The return on an unhedged and a hedged portfolio can be written as:

$$R_U = S_{t+1} - S_t$$

$$R_H = (S_{t+1} - S_t) - H(F_{t+1} - F_t)$$

Variances of an unhedged and a hedged portfolio are:

$$Var(U) = \sigma_S^2$$

$$Var(H) = \sigma_S^2 + H^2\sigma_F^2 - 2H\sigma_{S,F}$$

where, S_t and F_t are natural logarithm of spot and futures prices, H is the hedge ratio, R_H and R_U are return from unhedged and hedged portfolio, σ_S and σ_F are standard deviation of the spot and futures return and $\sigma_{S,F}$ is the covariance.

Hedging effectiveness is defined as the ratio of the variance of the unhedged position minus variance of hedge position over the variance of unhedged position.

$$Edrington's\ efficiency\ index = \frac{Var(U)-Var(H)}{Var(U)} \quad (1)$$

2. 2. Models for Calculating Hedging Effectiveness and Hedge Ratio

Several models have been used to calculate hedge ratio and hedging effectiveness such as Vector Autoregressive regression (VAR) model, Vector Error Correction model (VEC), Baysian Vector Autoregressive regression (BVAR) model.

The VAR models eliminates problems of autocorrelation but it does not consider the possibility of long term integration. The advantage of the Bayesian approach to statistics is that provides a general method for combining a modeler’s beliefs with the evidence contained in the data.

2. 3. The VAR Model

The bivariate VAR Model is preferred over the simple OLS estimation because it eliminates problems of autocorrelation between errors and treat futures prices as endogenous variable. The VAR model is represented as

$$R_{st} = \alpha_s + \sum_{i=1}^{k} \beta_{si} R_{st-i} + \sum_{j=1}^{l} \gamma_{fj} R_{ft-j} + \varepsilon_{st} \quad (2)$$
$$R_{ft} = \alpha_f + \sum_{i=1}^{k} \beta_{fi} R_{ft-i} + \sum_{j=1}^{l} \gamma_{sj} R_{st-j} + \varepsilon_{ft} \quad (3)$$

The error terms in the equations, ε_{St}, and ε_{Ft} are independently identically distributed (IID) random vector. The minimum variance hedge ratio are calculated as

$$\text{h} = \frac{\sigma_{sf}}{\sigma_{ff}} \quad (4)$$
$$\sigma_{sf} = cov(\varepsilon_{st}, \varepsilon_{ft})$$
$$\sigma_{ff} = var(\varepsilon_{ft})$$

The VAR model does not consider the possibility of long term integration between spot and futures returns.

2. 4. The Error Correction Model

VAR model does not consider the possibility that the endogenous variables could be co-integrated in the long term. If two prices are co-integrated in long run then Vector Error Correction model is more appropriate which accounts for long-run co-integration between spot and futures prices (Lien & Luo, 1994; Lien, 1996). If the futures and spot series are co-integrated of the order one, then the Vector error correction model of the series is given as:

$$R_{st} = \alpha_s + \beta_s S_{t-1} + \gamma_f F_{t-1} + \sum_{i=1}^{k} \beta_{si} R_{st-i} + \sum_{j=1}^{l} \gamma_{fj} R_{ft-j} + \varepsilon_{st} \quad (5)$$
$$R_{ft} = \alpha_f + \beta_f F_{t-1} + \gamma_s S_{t-1} + \sum_{i=1}^{k} \beta_{fi} R_{ft-i} + \sum_{j=1}^{l} \gamma_{sj} R_{st-j} + \varepsilon_{ft} \quad (6)$$

where, S_t and F_t are natural logarithm of spot and futures prices. The assumptions about the error terms are same as for VAR model. The minimum variance hedge ratio and hedging effectiveness are estimated by following similar approach as in case of VAR model.

2. 5. The BVAR Model

A Bayesian approach to vector autoregressive has in particular been put forward by Doan, Litterman and Sims (1984). The original Litterman or Minnesota prior was based on the idea that each series is best described as a random walk around an unknown deterministic component. Consider the *n* variable vector autoregressive of order *p*, VAR(*p*), given by (7)

$$\text{yt} = \Gamma 1 \text{ yt-1} + ... + \Gamma \text{py t-p} + \mu + \varepsilon \quad (7)$$

where yt is an (*n* x 1) vector of non-stationary time series, m is an (*n* x 1) vector of constants coefficients and et is an *n* x 1 vector of error terms. Γ_1 through $\Gamma_p$represent (*n* x *n*) matrices of parameters to be estimated. Hence the prior distribution is centered around the random walk specification for variable *n* given by (8) below.

$$\text{Y(n,t)} = \mu\text{(n)} + \text{y(n,t-1)} + \varepsilon\text{(n,t)} \quad (8)$$

As described in Litterman (1986), the standard error on the coefficient estimate for lag l of variable j in equation i is given by a standard deviation of the coefficient on lag l of variable j in equation i given by a standard deviation function of the form S(i, j, l) given by equation (9) below.

$$S(i, j, l)= \frac{[\gamma.g(l).f(i,j)]s_i}{s_j} \qquad (9)$$

where

$$f(i, j) = 1 \text{ if } i = j \quad \text{and} \quad w_{ij} \quad \text{otherwise}$$

The "hyperparameter" γ and functions g(l) and f(i, j) determine the tightness or weight attaching to the prior in (8) above. Given the functional specifications of g(l) and f(i, j), γ can simply be interpreted as the standard deviation on the first own lag. It is also often termed the "overall tightness" of the prior. The function *g(l)* determines the tightness on lag one relative to lag l. The tightness around the prior mean is normally assumed to increase with increasing lag length. This is achieved by allowing *g(l)* decay harmonically with decay factor d, i.e. $g(l) = l^{-d}$. The tightness of the prior on variable j relative to variable i in the equation for variable i is determined by the function $f(i, j)$; this can be the same across all equations in which case w_{ij} is equal to a constant (w) and the prior is said to be *symmetric*.

Alternatively, the tightness of the prior for variable j relative to variable i (in the equation for variable i) can vary depending upon the particular equation and/or variable in question (this is known as a *general* prior). However, the flexibility inherent in the specification of a general prior may not always be desirable. On the one hand, as argued by Doan (1990), it simply transfers the problem of over-parameterization to one of having to estimate or search over too many hyperparameters. However, in a situation where the analyst has strong prior views that one of the variables is exogenous, the general prior may improve forecasting performance. In particular, the equations for exogenous variables may best be specified as univariate autoregressive with no feedback from the other variables in the system. This can be achieved by setting very low values for the off-diagonal elements in f(i, j) which correspond to that particular variable.

Finally, the multiplicative ratio s_i/s_j in equation (9) reflects the fact that in general the prior cannot be completely specified without reference to the data. In particular it corrects for differences in the scale used in the measurement of each variables included in the system. For example, how tight a standard deviation of 0.5 is on the lags of prices in an equation for the interest rate will depend on whether the price index is based to equal unity or 100 in the base period.

Litterman (1986) argues that the scale of the response of one variable to another is "a function of the relative size of *unexpected* movements in the two variables rather than the relative sizes of their overall standard errors". Hence, he suggests scaling the standard error on the prior by the ratio of the standard deviations of the residuals (s_i) from a univariate autoregressive for variable i to the standard deviation of the residuals (s_j) from a univariate autoregressive for variable j (both with p lags).

2. 6. The VAR Estimates

To calculate the hedge ratio and hedging effectiveness, system of equations is solved. We used covariance and variance errors from the equation [2, 3] to calculate hedge ratio and hedging effectiveness (equation [1]) of futures contracts. The covariance and variance errors and OHR equations are given in Table 1 and hedge effectiveness is presented in Table 2.

Table 1

	Cov (ε_s, ε_f)	Var (ε_s)	Var (ε_f)	OHR
Future1	0.000366	0.000453	0.000502	0.729

Table2

	Var (U)	Var (H)	Hedging Effectivenss, E
Future1	0.000453	0.000186	0.5894

2. 7. The VECM Estimates

Using the same approach as in case of VAR model, errors are estimated and hedging effectiveness and hedge ratio are calculated for VECM model. Results of the equation [5, 6] and OHR are presented in Table 3. Table 4 illustrates the estimates hedging effectiveness of futures contracts.

Table 3

	Cov (ε_s, ε_f)	Var (ε_s)	Var (ε_f)	OHR
Future1	0.000454	0.000526	0.000609	0.7454

Table 4

	Var (U)	Var (H)	Hedging Effectivenss, E
Future1	0.000526	0.00005188	0.9013

2. 8. The BVAR Estimates

Errors are estimated through Baysian Vector Autoregressive (BVAR) model and hedging effectiveness and hedge ratio are calculated. OHR are presented in Table 5. Table 6 illustrates the estimate of hedging effectiveness of futures contracts.

Table 5

	Cov ($\boldsymbol{\varepsilon_s}$, $\boldsymbol{\varepsilon_f}$)	Var ($\boldsymbol{\varepsilon_s}$)	Var ($\boldsymbol{\varepsilon_f}$)	OHR
Future1	0.000368	0.000468	0.000506	0.7173

Table 6

	Cov (U)	Var (H)	Hedging Effectiveness, E
Future1	0.000468	0.000207	0.5576

3. CONCLUSION

To hedging risk, it is important to evaluate the hedging effectiveness of derivatives. In the present paper, we report hedge ratios of Brent Oil futures through three alternative modeling frameworks: VAR model, VECM model and BVAR model. We compare the hedging effectiveness of the contacts using these models, *ex post* (in-sample) and *ex ante* (out-of-sample) introduced by Edrington. The results show the VEC model is more effective than the other models used in this paper.

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