Simulation of isotropic acoustic metamaterials

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ABSTRACT The model use to study electromagnetic metamaterials in transverse electric mode was modified to study the pressure distribution in an acoustic metamaterial in a two dimensional geometry. An electromagnetic wave of 30GHz in transverse magnetic mode at normal incidence propagating through a two dimensional isotropic semi infinite double negative metamaterial slab of 640×830 cells embedded in free space with low loss damping frequency 10^8 s^{-1} was studied by finite difference time dependent method with Yee's algorithm with an explicit leapfrog scheme. The multiple cycle m - n - m pulses generating beams were used as sources. The simulations for refractive indices $n = -1$ and $n = -6$ at different time steps are presented. For $n = -1$, the sub-wavelength imaging is apparent, diverging the waves from source into two sources, one inside and the other outside the slab. The reverse propagation is much more significant for $n = -6$ showing that the group velocity is much larger inside the metamaterial by the closely packed wave front, making the continuation of the wave pattern much more significant through the metamaterial into the normal space.

INTRODUCTION

Metamaterials are artificially fabricated materials designed to control, direct and manipulate light as well as sound waves in fluids and solids. These assemblies fashioned from replacing the molecules of conventional materials with artificial atoms on a scale much less than the relevant wavelength, derive their properties from their exactly-designed structures. The core concept of metamaterial is the negative index of refraction for particular wavelengths, electromagnetic or sound waves propagating in reverse. The first metamaterials were developed for electromagnetic waves. Acoustic wave, a longitudinal wave have common physical concepts and in the two dimensional case, when there is only one polarization mode, the electromagnetic wave has scalar wave formulation and the optical and acoustic metamaterials share many similar implementation approaches as well. The controlling of the various forms of sound waves is mostly accomplished through the bulk modulus β , mass density ρ , and chirality. In negative index materials, the density and bulk modulus are analogies of the electromagnetic parameters, permittivity and permeability. Also materials have mass and intrinsic degrees of stiffness. Together these form a resonant system, and may be excited by appropriate sonic frequencies for pulses at audio frequencies.

An experimental acoustic metamaterial based on the transmission line model, as well as unit cells for isotropic and anisotropic metamaterials were proposed by Zang [1]. To simulate two dimensional electromagnetic materials in transverse electric mode, finite difference time domain (FDTD) method is widely used [2]-[4]. A dispersive FDTD scheme or a leap frog scheme has also been used to determine the values of electric and magnetic fields as well as induced electric current and induced magnetic current values inside the metamaterials [5], [6]. The notations used in these dispersive schemes are adopted from Taflov and Hagness [7]. The existence of an analogy between the acoustic metamaterial parameters and electromagnetic metamaterial parameters in transverse magnetic mode, first proposed by Cummer and Schurig [8] was used by Torrent and Dehesa [9].

In this work, the model used by Ziolkowski [4] to study electromagnetic metamaterials using transverse electric mode was modified to transverse magnetic mode to study the pressure

distribution of an acoustic wave in metamaterials. The simulation were performed for electromagnetic wave of 30GHz in transverse magnetic mode at normal incidence through a two dimensional isotropic semi infinite double negative metamaterial slab of 640×830 cells embedded in free space with low loss damping frequency 10^8 s^{-1} by finite difference time dependent method with Yee's algorithm which calculates the electric and magnetic field in an explicit leapfrog scheme. For the simulations the mesh size used was $100 \mu\text{m}$ with time steps of 22.39 ps . The analogy between acoustic and electromagnetic metamaterials is used to illustrate the behaviour of magnitude of z component of electric field distribution in electromagnetic metamaterial or magnitude of pressure distribution of acoustic metamaterial for refraction indices $n \approx -1$ and $n \approx -6$.

2. DRUDE METAMATERIAL MODEL

The pressure and the particle velocity are the parameters used to describe an acoustic wave which is a longitudinal wave. In the two dimensional Cartesian coordinate with z invariance, the time harmonic acoustic equations for fluid with anisotropic or isotropic medium are:

$$\begin{aligned}\frac{\partial P}{\partial x} &= -j\omega\rho_x u_x, \\ \frac{\partial P}{\partial y} &= -j\omega\rho_y u_y, \\ \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} &= -j\omega\beta P\end{aligned}\tag{1}$$

where u_x and u_y are particle velocities in x and y directions, P is acoustic scalar pressure, ρ_x and ρ_y are the dynamic density along x and y directions and β is the dynamic compressibility. These lead to the pressure equation:

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + k^2 P = 0 \quad (k = \pm\omega\sqrt{\beta\rho})\tag{2}$$

In electromagnetism, both electric and magnetic fields are transverse waves. However the two wave systems have the common physical concepts such as wave vector, wave impedance and power flow. The manner in which the permittivity and permeability behave in electromagnetism is closely analogous to the behaviour of compressibility and density in an acoustic system. Moreover in a two dimensional case, when there is only one polarization mode the acoustic waves in fluid and the Maxwell equations for electromagnetic wave have identical form under certain variable exchange. An analogy has been proposed between acoustic and electromagnetic metamaterials in Cartesian coordinates [1]. For the same z invariant Cartesian coordinates, the Maxwell's equations for the transverse magnetic mode are:

$$\begin{aligned}\frac{\partial E_z}{\partial x} &= -j\omega\mu_y H_y, \\ \frac{\partial E_z}{\partial y} &= j\omega\mu_x H_x, \\ \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} &= -j\omega\epsilon_z E_z\end{aligned}\tag{3}$$

where E is the electric field, H is the magnetic field intensity, μ and ε are the magnetic permeability and electric permittivity along respective directions. Observing the two sets of equations given in equation 1 for acoustic waves and in equation 3 for electromagnetic waves, it can be seen that these two sets are equivalent under the following variable exchange.

$$P \leftrightarrow -E_z, \quad u_x \leftrightarrow -H_y, \quad u_y \leftrightarrow H_x, \quad \rho_x \leftrightarrow \mu_y, \quad \rho_y \leftrightarrow \mu_x, \quad \varepsilon_z \leftrightarrow \beta \quad (4)$$

The boundary conditions in electromagnetism are that the normal component of electric field E_z and tangential component of magnetic field H are continuous while at a fluid interface, the normal component of particle velocity u_y and pressure P are continuous. Therefore the boundary conditions are preserved under this variable exchange. For normal incidence of acoustic waves, solids obey the same equations developed for fluids with the modification that the speed of sound in the solid is the bulk sound depending on the bulk modulus and the shear modulus. For normal incidence, the boundary between two fluids is considered to be at $x=0$ as show in figure 1. A plane wave travelling in positive x -direction in a medium 1 $P_i = P_{ia}e^{j(\omega t - k_1 t)}$ strikes the boundary generating a reflected wave $P_r = P_{ra}e^{j(\omega t - k_1 t)}$ and a transmitted wave $P_t = P_{ta}e^{j(\omega t - k_2 t)}$. The relevant particle velocities are $u_i = u_{ia}e^{j(\omega t - k_1 t)}$, $u_r = u_{ra}e^{j(\omega t + k_1 t)}$ and $u_t = u_{ta}e^{j(\omega t - k_2 t)}$ respectively. The boundary conditions require that at $x = 0$:

$$\begin{aligned} P_{ia} &= P_{ra} + P_{ta} \\ u_{ia} &= u_{ra} + u_{ta} \end{aligned} \quad (5)$$

The reflection and transmission coefficients are given by:

$$\begin{aligned} r_p &= -r_u = \frac{\rho_2 c_2 - \rho_1 c_1}{\rho_2 c_2 + \rho_1 c_1} \\ t_p &= \frac{2\rho_2 c_2}{\rho_2 c_2 + \rho_1 c_1}; \quad t_u = \frac{2\rho_1 c_1}{\rho_2 c_2 + \rho_1 c_1} \end{aligned} \quad (6)$$

The reflection coefficient of intensity r_I and transmission coefficient of intensity t_I are:

$$r_I = \left(\frac{\rho_2 c_2 - \rho_1 c_1}{\rho_2 c_2 + \rho_1 c_1} \right)^2; \quad t_I = 1 - r_I = \frac{4\rho_1 c_1 \rho_2 c_2}{(\rho_2 c_2 + \rho_1 c_1)^2} \quad (7)$$

where c_1 and c_2 are the velocities of sound in the two mediums [1].

The Drude metamaterial model is the most popular and widely used model in electromagnetic metamaterial simulations. The Lossy Drude polarization and magnetization models are used in the frequency domain in Drude metamaterial model and describe the electric permittivity and magnetic permeability as:

$$\begin{aligned}\varepsilon(\omega) &= \varepsilon_0 \left(1 - \frac{\omega_{pe}^2}{\omega(\omega - j\Gamma_e)} \right) \\ \mu(\omega) &= \mu_0 \left(1 - \frac{\omega_{pm}^2}{\omega(\omega - j\Gamma_m)} \right)\end{aligned}\quad (8)$$

where ω_{pe} and ω_{pm} are the electric and magnetic plasma frequencies while Γ_e and Γ_m are the electric and magnetic damping (collision) frequencies respectively. The Governing equations in Drude metamaterial model for the electromagnetic metamaterials are,

$$\begin{aligned}\mathbf{D} &= \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon_0 \varepsilon_r \mathbf{E} \\ \mathbf{B} &= \mu_0 \mathbf{H} + \mathbf{M} = \mu_0 \mu_r \mathbf{H}\end{aligned}\quad (9)$$

where ε_0 and ε_r are electric permittivity of free space and relative permittivity of the medium and μ_0 and μ_r are magnetic permeability of free space and relative permeability of the medium. Magnetic permeability and electric permittivity are only dependent on the frequency. $\mathbf{M}$ is the magnetization and $\mathbf{P}$ is the electric polarization. If a time harmonic variation of $e^{j\omega t}$ is used, then the corresponding time domain equations for the polarization and magnetization are:

$$\begin{aligned}\frac{\partial^2 \mathbf{P}}{\partial t^2} + \Gamma_e \frac{\partial \mathbf{P}}{\partial t} &= \varepsilon_0 \omega_{pe}^2 \mathbf{E} \\ \frac{\partial^2 \mathbf{M}}{\partial t^2} + \Gamma_m \frac{\partial \mathbf{M}}{\partial t} &= \mu_0 \omega_{pm}^2 \mathbf{H}\end{aligned}\quad (10)$$

If the induced electric and magnetic currents respectively, are $\mathbf{J} = \partial \mathbf{P} / \partial t$ and $\mathbf{K} = \partial \mathbf{M} / \partial t$, the governing equations for modelling wave propagation in electromagnetic metamaterials are finally derived as:

$$\begin{aligned}\frac{1}{\varepsilon_0 \omega_{pe}^2} \frac{\partial \mathbf{J}}{\partial t} + \frac{\Gamma_e}{\varepsilon_0 \omega_{pe}^2} \mathbf{J} &= \mathbf{E} \\ \frac{1}{\mu_0 \omega_{pm}^2} \frac{\partial \mathbf{K}}{\partial t} + \frac{\Gamma_m}{\mu_0 \omega_{pm}^2} \mathbf{K} &= \mathbf{H}\end{aligned}\quad (11)$$

For simplicity, it is assumed that the boundary of the domain Ω is a perfect conductor, leading to $\hat{n} \times \mathbf{E} = 0$ on $\partial\Omega$ where $\hat{n}$ is the unit outward normal to $\partial\Omega$. Moreover, it is assumed that the initial conditions are $\mathbf{A}(\mathbf{x}, 0) = \mathbf{A}_0(\mathbf{x})$ ($\mathbf{A} = \mathbf{E}, \mathbf{H}, \mathbf{J}, \mathbf{K}$). For the transverse magnetic mode, the field components E_z , H_x and H_y are:

$$\begin{aligned}E_z &= \frac{1}{\varepsilon_0 \omega_{pe}^2} \left(\frac{\partial J_z}{\partial t} + \Gamma_e J_z \right) \\ H_x &= \frac{1}{\mu_0 \omega_{pm}^2} \left(\frac{\partial K_x}{\partial t} + \Gamma_m K_x \right) \\ H_y &= \frac{1}{\mu_0 \omega_{pm}^2} \left(\frac{\partial K_y}{\partial t} + \Gamma_m K_y \right)\end{aligned}\quad (12)$$

3. YEE'S ALGORITHM

The finite-difference time domain (FDTD) method is very popular in modelling electromagnetic fields. Yee's algorithm is based on the time-dependent Maxwell's curl equations and it couples the equations in order to solve for multiple field components simultaneously. The spatially staggered grid simplifies the contours involved in the curl equations, maintaining the continuity of the tangential components of the electric and magnetic fields, as well as simplifying the implementation of the boundary conditions. Yee's algorithm uses a fully explicit leapfrog scheme in time that also involved second-order central differences, so the field components are staggered temporally. This means the electric field is calculated before or after, but not simultaneously with the magnetic field thereby defining the leapfrog scheme. Any three dimensional vector field component can be written as $F(i\Delta x, j\Delta y, k\Delta z, n\Delta t) = F_{i,j,k}^n$ where $\Delta x, \Delta y$ and Δz are the mesh sizes in the x, y or z direction respectively, and Δt is the time increment. So, the second order finite difference for the first partial derivative of F in the x direction evaluated at time $t_n = n\Delta t$ is

$$\frac{\partial F}{\partial x}(i\Delta x, j\Delta y, k\Delta z, n\Delta t) = \frac{F_{(i+1/2),j,k}^n - F_{(i-1/2),j,k}^n}{\Delta x} \quad (13)$$

The Yee algorithm is a conditionally stable algorithm, and there exists a maximum allowable time step $\Delta t_{\max}$ in order for the algorithm to remain stable.

$$\Delta t_{\max} \leq \frac{1}{c\sqrt{(1/\Delta x)^2 + (1/\Delta y)^2 + (1/\Delta z)^2}} \Rightarrow c\Delta t_{\max} \sqrt{(1/\Delta x)^2 + (1/\Delta y)^2 + (1/\Delta z)^2} \leq 1 \quad (14)$$

where c is the speed of light in vacuum. In one dimension, the Courant number (S) is defined as, $S = c\Delta t / \Delta x$ and $S = 1$. For $\Delta x = \Delta y = \Delta z$, in order for the algorithm to remain stable $S = 1/\sqrt{2}$ in two dimensional case and $S = 1/\sqrt{3}$ in three dimensional case.

Simulating wave propagation from scattering or waveguides requires an unbounded domain or a domain large enough so that waves do not reflect of the domain boundaries back into the computational domain and interfere with the wave propagation being analyzed. As it is impossible to have an unbounded domain in scientific computing, the absorbing boundary conditions (ABCs) have emerged. This is an additional domain surrounding the computational domain. It is connected to the computational domain boundary, but the fields are computed separately in this external domain. Here all tangential properties are preserved, and the fields are continuous across the boundary. It is made so that it absorbs waves that come in contact with that region. The external region is designed as a lossy material, so that

it does not really absorb the wave but severely dampen the wave as it enters that region by removing its power so quickly that there's nothing left to reflect back off the outer boundary [7]. The Yee lattice used in FDTD is shown in figure 2. Equation 12 in the form given in equation 13 read:

$$\begin{aligned}
 E_z|_{i,j+1}^{n+1} &= \frac{1}{\epsilon_0 \omega_{pe}^2} \frac{\partial J_z}{\partial t} \Big|_{i,j+1}^{n+1} + \frac{\Gamma_e}{\epsilon_0 \omega_{pe}^2} J_z|_{i,j+1}^{n+1} \\
 H_x|_{i,j+\frac{1}{2}}^{n+\frac{1}{2}} &= \frac{1}{\mu_0 \omega_{pm}^2} \frac{\partial K_x}{\partial t} \Big|_{i,j+\frac{1}{2}}^{n+\frac{1}{2}} + \frac{\Gamma_m}{\mu_0 \omega_{pm}^2} K_x|_{i,j+\frac{1}{2}}^{n+\frac{1}{2}} \\
 H_y|_{i-\frac{1}{2},j+1}^{n+\frac{1}{2}} &= \frac{1}{\mu_0 \omega_{pm}^2} \frac{\partial K_y}{\partial t} \Big|_{i-\frac{1}{2},j+1}^{n+\frac{1}{2}} + \frac{\Gamma_m}{\mu_0 \omega_{pm}^2} K_y|_{i-\frac{1}{2},j+1}^{n+\frac{1}{2}}
 \end{aligned} \tag{15}$$

Yee's algorithm utilizes central differencing in time for the $\mathbf{E}$ and $\mathbf{H}$ components and then solves them by using a leapfrog scheme. All of the $\mathbf{E}$ components in the modelled space inside the metamaterial are computed and stored in memory by using the previously computed values of $\mathbf{E}$ and the newly updates $\mathbf{H}$ field data. At the next step $\mathbf{H}$ is recomputed based on the previously obtained $\mathbf{H}$ and the newly obtained $\mathbf{E}$. This process is continued until the time-stepping is terminated. The relevant components of the FDTD scheme used inside the metamaterial was obtained from the equations given by [7]:

$$\begin{aligned}
 E_z|_{i-\frac{1}{2},j+\frac{1}{2}}^{n+\frac{1}{2}} &= C_a(m) E_z|_{i-\frac{1}{2},j+\frac{1}{2}}^n + C_b(m) \left(H_y|_{i,j+\frac{1}{2}}^n - H_y|_{i-1,j+\frac{1}{2}}^n + H_x|_{i-\frac{1}{2},j}^n - H_x|_{i-\frac{1}{2},j+1}^n - J_x|_{i-\frac{1}{2},j+\frac{1}{2}}^n * \Delta x \right) \\
 H_x|_{i-\frac{1}{2},j+1}^{n+1} &= D_a(m) H_x|_{i-\frac{1}{2},j+1}^n + D_b(m) \left(E_z|_{i-\frac{1}{2},j+\frac{1}{2}}^{n+\frac{1}{2}} - E_z|_{i-\frac{1}{2},j+\frac{3}{2}}^{n+\frac{1}{2}} - K_y|_{i,j+\frac{1}{2}}^{n+\frac{1}{2}} * \Delta x \right) \\
 H_y|_{i,j+\frac{1}{2}}^{n+1} &= D_a(m) H_y|_{i,j+\frac{1}{2}}^n + D_b(m) \left(E_z|_{i+\frac{1}{2},j+\frac{1}{2}}^{n+\frac{1}{2}} - E_z|_{i-\frac{1}{2},j+\frac{1}{2}}^{n+\frac{1}{2}} - K_x|_{i,j+\frac{1}{2}}^{n+\frac{1}{2}} * \Delta x \right)
 \end{aligned} \tag{16}$$

the coefficients are given by

$$\begin{aligned}
 C_a &= \frac{1 - (\sigma \Delta t / 2 \epsilon_r \epsilon_0)}{1 + (\sigma \Delta t / 2 \epsilon_r \epsilon_0)}; \quad C_b = \frac{\Delta t / \epsilon_r \epsilon_0 \Delta x}{1 + (\sigma \Delta t / 2 \epsilon_r \epsilon_0)}; \\
 D_a &= \frac{1 - (\sigma^* \Delta t / 2 \mu_r \mu_0)}{1 + (\sigma^* \Delta t / 2 \mu_r \mu_0)}; \quad D_b = \frac{\Delta t / 2 \mu_r \mu_0 \Delta x}{1 + (\sigma^* \Delta t / 2 \mu_r \mu_0)}
 \end{aligned} \tag{17}$$

where σ and σ^* are the electric conductivity and the equivalent magnetic loss respectively. $\Delta x = \Delta y = \Delta z$ is the cell size or mesh size, Δt is the time step and m standing for the media is 2 for the two dimensional simulation, so that C_a , C_b , D_a and D_b are arrays of two elements. This dispersive scheme was used to calculate the parameters within the metamaterial.

4. TRANSVERSE MAGNETIC MODES IN METAMATERIALS

The simulations done by Ziolkowski [4] for transverse electric mode in y direction was modified to transverse magnetic (TMz) mode. For normal mode the reflection coefficient $R=0$ and transmission coefficient $T=1$. The general index of refraction of the medium (index i) through the wave is propagating is $n_i = \sqrt{\epsilon_r \mu_r}$. A semi infinite double negative (DNG) medium in which both

the relative permittivity and permeability are negative i.e., $\varepsilon_r < 0$, $\mu_r < 0$ at a certain frequency embedded in free space was considered. Then $n_i = -\sqrt{\varepsilon_r \mu_r}$, the negative sign is taken to satisfy the causality. For the simulations, lossy Drude polarization and magnetization models given in equation 4 were used. A matched slab was defined in x-y plane such that the transmitted refractive index n_{trans} and incident refractive index n_{inc} were equal ($n_{trans} = n_{inc}$). Also in the slab the electric frequency ω_{pe} and magnetic plasma frequency ω_{pm} were considered equal ($\omega_{pe} = \omega_{pm} = \omega_p$) as well as the electric damping frequency Γ_e and magnetic damping frequency Γ_m as equal ($\Gamma_e = \Gamma_m = \Gamma$). The simulations were performed for two indices of refraction, $n \approx -1$ and $n \approx -6$. In all cases, damping frequency was taken as $\Gamma = 10^8 \text{ s}^{-1}$, the electromagnetic or acoustic frequency is $f_0 = 30 \text{ GHz}$, the mesh size $\Delta x = \lambda_0 / 100 = 100 \mu\text{m}$, time step $\Delta t = 22.39 \text{ ps}$ and the simulation domain size was 640×830 cells. The multiple cycle m - n - m pulses were used in both cases to generate the sources that are smooth thus producing minimal noise. These simulations were done in time domain and the pulses are given by the expression:

$$f(t) = \begin{cases} 0; & t < 0 \\ g_{on}(t) \sin(\omega t); & 0 < t < mT_p \\ \sin(\omega t); & mT_p < t < (m+n)T_p \\ g_{off}(t) \sin(\omega t); & (m+n)T_p < t < (m+n+m)T_p \\ 0; & (m+n+m)T_p < t \end{cases} \quad (18)$$

where

$$x_{on}(t) = \frac{t}{mT_p}, \quad x_{off}(t) = \frac{t - (m+n)T_p}{mT_p}$$

and the continuous functions are:

$$\begin{aligned} g_{on} &= 10x_{on}^3(t) - 15x_{on}^4(t) + 6x_{on}^5(t) \\ g_{off} &= 1 - [10x_{off}^3(t) - 15x_{off}^4(t) + 6x_{off}^5(t)] \end{aligned} \quad (19)$$

Consider the case when $n \approx -1$, then $\omega_p = 2\pi\sqrt{2}f_0 \approx 2.6657 \times 10^{11} \text{ rad s}^{-1}$. So that $\Gamma = 3.75 \times 10^{-4} \omega_p$. Similarly when $n \approx -6$, then $\omega_p = 2\pi\sqrt{7}f_0 \approx 4.9871 \times 10^{11} \text{ rad s}^{-1}$ and $\Gamma = 2.01 \times 10^{-4} \omega_p$. The Gaussian beam varies spatially as $\exp(-x^2 / \omega_0^2)$ and the waist ω_0 was set to 50. The single slab has a depth of $2\lambda_0 = 200$ cells and width of $6\lambda_0 = 600$ cells. The source was placed 40 cells from the bottom and 200 cells away from the slab. The numerical simulations were done using Matlab.

For the refractive index $n \approx -1$, number of snapshots for equal time intervals were captured for the simulations for z-component of electric field E_z . These at the instances of 500, 1500 and 5000 time steps are presented in figure 3, 4 and 5 respectively. The highest values are represented by red colour and the lowest values by blue. Since E_z is analogous to the magnitude of the pressure these figures describe the behavior of an electric field distribution in z-direction in a magnetic metamaterial or equivalently the pressure distribution of an acoustic wave inside an acoustic metamaterial for the refractive index $n \approx -1$. Figures show that the planar metamaterial medium turns the diverging wave vectors towards the beam axis and, hence, acts as a lens to focus the beam. With the source distance and the slab depth equal, the foci of the beam in the metamaterial slab and

in the background medium beyond the metamaterial slab coincide for $n \approx -1$ at a distance equal to the depth of the slab. That is, the focal plane of the beam occurs at the rear face of the slab. This means that the metamaterial slab should ideally reproduce the beam at its rear face as it exists from the metamaterial. Therefore the metamaterial slab in this case acts as an acoustic lens.

For the refractive index $n \approx -6$, the simulations for z-component of electric field E_z or equivalently the pressure P of the acoustic wave at time steps 1000, 3000 and 5000 are presented in figures 6, 7 and 8 respectively. The highest values are represented by red colour and the lowest values by blue. When the beam interacts with the matched metamaterial slab with $n \approx -6$, little focusing is observed. The negative angles of refraction dictated by Snell's law are shallower for this higher magnitude of the refractive index. Rather than a strong focusing, the medium

Figure 3: Electric field intensity distribution in z direction for electromagnetic slab or magnitude of pressure distribution for acoustic slab at 500 time step. DNG slab is shown by the black rectangle.

Figure 4: Electric field intensity distribution in z direction for electromagnetic slab or magnitude of pressure distribution for acoustic slab at 1500 time step. DNG slab is shown by the black rectangle.

Figure 5: Electric field intensity distribution in z direction for electromagnetic slab or magnitude of pressure for acoustic slab at 5000 time step. DNG slab is shown by the black rectangle

channels power from the wings of the beam towards its axis, hence, maintaining its amplitude as it propagates into the metamaterial medium. The fact that the wings of the beam feed its center portion can be perceived by the converging wave fronts shown in Figure 6 at the edges of the beam in the metamaterial slab.

5. CONCLUSIONS

In order to illustrate the characteristics of an acoustic metamaterial network, the finite difference time dependent (FDTD) method and Drude model in transverse magnetic mode was adopted and the results were simulated. These models incorporate terms such as, induced electric and magnetic currents, magnetic and electric plasma frequencies. For acoustic metamaterial parameters, such terms are not useable. To avoid confusions, the derivations and simulations were carried on for the electromagnetic metamaterials, to show the final output and then to compare the results for the acoustic metamaterials as the results are completely identical. Namely under the variable exchange the behaviour of magnitude of z -component of electric field E_z is analogous to the behaviour of magnitude of pressure P in an acoustic metamaterial slab. Also the boundary condition for electromagnetic metamaterial slab is that the normal component of E_z is continuous and in acoustic metamaterials, the normal component of pressure is continuous.

Figure 6: Electric field intensity distribution in z direction for electromagnetic slab or magnitude of pressure for acoustic slab at 1000 time step. DNG slab is shown by the black rectangle

Figure 7: Electric field intensity distribution in z direction for electromagnetic slab or magnitude of pressure for acoustic slab at 3000 time step. DNG slab is shown by the black rectangle

Figure 8: Electric field intensity distribution in z direction for electromagnetic slab or magnitude of pressure for acoustic slab at 5000 time step. DNG slab is shown by the black rectangle

The phenomenon known as negative refraction is visible in these simulations. The propagation of sound is reverse to the incident waves within the metamaterial. For the negative refractive index of one, the sub-wavelength imaging is apparent, as the sound waves from source diverges into two sources, one inside and the other outside the slab. In the case of negative refractive index of six the reverse propagation is much significant than for the refractive index of minus one and shows that

the group velocity is much larger inside the metamaterial by the closely packed wave front and the wave pattern continuation is more significant though the metamaterial in to the normal space. In all the metamaterial cases the beam appears to diverge significantly once it leaves the metamaterial slab. The properties of the metamaterial medium hold the beam together as it propagates through the slab. If the metamaterial slab focuses the beam as it enters, the same physics will cause the beam to diverge as it exits. The rate of divergence of the exiting beam will be determined by its original value and the properties and size of the metamaterial medium. The beam focusing into a metamaterial slab generating a diverging beam within the slab and a converging beam as it exits from the slab is confirmed with the FDTD simulator.

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