International Letters of Chemistry, Physics and Astronomy
ISSN: 2299-3843, Vol. 61, pp 38-48
doi:10.18052/www.scipress.com/ILCPA.61.38
2015 SciPress Ltd, Switzerland Online: 2015-11-03

A New study to the Schrödinger equation for Modified Potential $V(r) = ar^2 + br^{-4} + cr^{-6}$ in Nonrelativistic three dimensional real spaces and phases

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Keywords: Schrödinger equation, Star product, Boopp's shift method, harmonic and anharmonic potential, noncommutative space and noncommutative phase.

ABSTRCT: In present work, by applying Boopp's shift method and standard perturbation theory we have generated exact nonrelativistic bound states solution for a modified potential $V_{is}(\hat{r}) = ar^2 + br^{-4} + cr^{-6} + V_{pert-is}(r,\Theta,\bar{\theta})$ in both three dimensional noncommutative space and phase (NC: 3D-RSP) at first order of two two infinitesimal parameters antisymmetric $(\theta^{\mu\nu}, \bar{\theta}^{\mu\nu})$, we have also derived the corresponding noncommutative Hamiltonian.

1. INTRODUCTION:

It is well-known that the exact solutions the non relativitivstic Schrödinger equation and relativitivstic two equations Klein-Gordon and Dirac with central potentials play principal and important roles in different fields of sciences like atomic, nuclear, molecular, an harmonic and harmonic spectroscopy [1-23]. In the last few years, the provisos study were extended to the noncommutative space-phase at two, three and N generalized dimensions [24-45]. The notions of noncommutativity of space and phase based essentially on Seiberg-Witten map and Boopp's shift method and the star product, defined on the first order of two infinitesimal parameters antisymmetric $(\theta^{\mu\nu}, \bar{\theta}^{\mu\nu}) \equiv \frac{1}{2}\varepsilon^{ijk}(\theta_k, \bar{\theta}_k)$ as [29-41]:

$$f(x) * g(x) = f(x) * g(x) - \frac{i}{2}\theta^{\mu\nu}\left(\partial_\mu^x f(x)\right)\left(\partial_\nu^p g(x)\right) - \frac{i}{2}\bar{\theta}^{\mu\nu}\left(\partial_\mu^p f(x)\right)\left(\partial_\nu^p g(x)\right) \tag{1}$$

Which allow us to obtaining the two new non nulls commutators $\left[x_i, x_j\right]$ and $\left[\hat{p}_i, \hat{p}_j\right]_*$:

$$\left[x_i, x_j\right]_* = i\theta_{ij} \text{ and } \left[\hat{p}_i, \hat{p}_j\right]_* = i\bar{\theta}_{ij} \tag{2}$$

The Boopp's shift method will be apply in this paper instead of solving the (NC-3D) spaces and phases with star product, the Schrödinger equation will be treated by using directly the two commutators, in addition to usual commutator on quantum mechanics [29-41]:

$$\left[\hat{x}_i, \hat{x}_j\right] = i\theta_{ij} \text{ and } \left[\hat{p}_i, \hat{p}_j\right] = i\bar{\theta}_{ij} \tag{3}$$

The main goal of this work is to extend our study in reference [34] for the potential $V(r) = ar^2 + br^{-4} + cr^{-6}$ into noncommutative three dimensional spaces and phases on based to the principal reference [23] to discover the new spectrum and a possibility of obtain new applications to the modified potential $V_{is}(\hat{r}) = ar^2 + br^{-4} + cr^{-6} + V_{pert-is}(r,\Theta,\bar{\theta})$ in different fields.

The rest of present search is organized as follows: In next section, we briefly review the Schrödinger equation with $V(r) = ar^2 + br^{-4} + cr^{-6}$ in three dimensional spaces. In section 3, we review and applying Boopp's shift method to derive: the deformed potential and noncommutative spin-orbital Hamiltonian. In section 4, we apply perturbation theory to find the spectrum for ground stat and first excited states and then we deduce the spectrum produced automatically by the external magnetic field. In section 5, we resume the global spectrum for modified potential $V_{nc-is}(r)$ and we conclue the corresponding global noncommutative Hamiltonian in (NC-3D) space and phase in first order of two infinitesimal parameters' Θ and $\bar{\theta}$. Finally, the important found results and the conclusions are discussed in the last section.

2. REVIEW THE $V(r) = ar^2 + br^{-4} + cr^{-6}$ POTENTIAL IN THREE DIMENSIONAL SPAPACES

In this section, we shall review the eigenvalues and eigenvalues for spherically symmetric for the potential $V(r)$[23]:

$$V(r) = ar^2 + br^{-4} + cr^{-6} \tag{4}$$

The three parameters: $a\rangle$, $b\rangle$ and c are constants, the complex eignenfunctions $\Psi(r,\theta,\varphi)$ in 3-dimensional space for above potential satisfied the Schrödinger equation in polar coordinates ($\hbar = 2m = c = 1$) is [23]:

$$\left(-\Delta + ar^2 + br^{-4} + cr^{-6}\right)\Psi(r,\theta,\varphi) = E\Psi(r,\theta,\varphi) \tag{5}$$

Where $\Psi(r,\theta,\varphi) = r^{-1}R_l(r)Y_l^m(\theta,\varphi)$, l and E represent angular momentum and the energy while $-l \le m \le +l$. The radial part $R_l(r)$ satisfis in 3-dimensions space [23]:

$$\frac{d^2R_l(r)}{dr^2} + \left[E - ar^2 + br^{-4} + cr^{-6} - \frac{l(l+1)}{r^2}\right]R_l(r) = 0 \tag{6}$$

The radial part $R_l(r)$ in terms of a Laurent-series [23]:

$$R_l(r) = Nr^k \exp\left(-\tfrac{1}{2}\sqrt{a}r^2 - \tfrac{1}{2}\sqrt{c}r^{-2}\right)\sum_{m=-M}^{N} h_m r^{2m} \tag{7}$$

For the ground state and the first existed states, the corresponding radial function $R_0(r)$ and $R_1(r)$, respectively, are given by [23]:

$$\begin{aligned} R_0(r) &= N_0 r^{k_0} \exp\left(-\tfrac{1}{2}\sqrt{a}r^2 - \tfrac{1}{2}\sqrt{c}r^{-2}\right) \\ R_1(r) &= N_1\left(1 + \beta r^2 + \gamma r^{-2}\right)r^{k_1} \exp\left(-\tfrac{1}{2}\sqrt{a}r^2 - \tfrac{1}{2}\sqrt{c}r^{-2}\right) \end{aligned} \tag{8}$$

Where $\beta \neq 0$ and $\gamma \neq 0$, N_0 and N_1 are the normalizations constants, the two parameters k_0 and k_1 are given by:

$$k_0 = \frac{b + 3\sqrt{c}}{2\sqrt{c}}, \quad k_0 = \frac{b + 7\sqrt{c}}{2\sqrt{c}} \tag{9}$$

Then, the complete normalized wave functions and corresponding energies for the ground state and the first existed states, respectively [23]:

$$\Psi^{(0)}(\vec{r}) = N_0 r^{k_0 - 1} \exp\left(-\tfrac{1}{2}\sqrt{a}r^2 - \tfrac{1}{2}\sqrt{c}r^{-2}\right)Y_l^0(\theta,\varphi) \quad \text{and} \quad E_0 = \sqrt{\frac{a}{c}}\left(b + 4\sqrt{c}\right) \tag{10}$$

$$\Psi^{(1)}(\vec{r}) = N_1\left(1 + \beta r^2 + \gamma r^{-2}\right)r^{k_1 - 1} \exp\left(-\tfrac{1}{2}\sqrt{a}r^2 - \tfrac{1}{2}\sqrt{c}r^{-2}\right)Y_l^m(\theta,\varphi) \quad \text{and} \quad E_1 = \sqrt{\frac{a}{c}}\left(b + 12\sqrt{c}\right) \tag{11}$$

3. NONCOMMUTATIVE PHASE-SPACE HAMILTONIAN FOR MODIFIED POTENTIAL $V_{is}(\hat{r}) = ar^2 + br^{-4} + cr^{-6} + V_{pert-is}(r,\Theta,\bar{\theta})$:

3.1 FORMALISM OF BOOPP'S SHIFT METOD

Know, we shall review some fundamental principles of the quantum noncommutative Schrödinger equation which resumed in the following steps for modified potential $V_{is}(\hat{r})$ [25-42]:

$$\begin{aligned} &\text{Ordinary Hamiltonian}: \hat{H}(p_i, x_i) \to \text{NC - Hamiltonian}: \hat{H}(\hat{p}_i, \hat{x}_i) \\ &\text{Ordinary - complex wave function}: \Psi(\vec{r}) \to \text{NC - complex wave function}: \hat{\Psi}(\vec{r}) \\ &\text{Ordinary - energy}: \mathrm{E} \to NC - Energy: E_{nc-is} \\ &\text{Ordinary - product} \to \text{New star producr - acting on phace and space}: * \end{aligned} \tag{12}$$

Which allow us to writing the three dimensional space-phase quantum noncommutative Schrödinger equations as follows:

$$\hat{H}(\hat{p}_i,\hat{x}_i)*\hat{\Psi}(\vec{\hat{r}})=E_{nc-is}\hat{\Psi}(\vec{\hat{r}}) \tag{13}$$

The Boopp's shift method permutes to reduce the above equation to simplest the form:

$$H_{nc-3is}(\hat{p}_i,\hat{x}_i)\psi(\vec{r})=E_{nc-is}\psi(\vec{r}) \tag{14}$$

The new modified Hamiltonian $H_{nc-3is}(\hat{p}_i,\hat{x}_i)$ defied as a function of $\hat{x}_i$ and $\hat{p}_i$:

$$H_{nc-3is}(\hat{p}_i,\hat{x}_i)=\hat{p}_i^{\,2}+V_{is}(\hat{r}) \tag{15}$$

And the modified three dimensional potential $V_{is}(\hat{r})$:

$$V_{is}(\hat{r})=a\hat{r}^2+b\hat{r}^{-4}+\hat{r}^{-6} \tag{16}$$

The two $\hat{x}_i$ and $\hat{p}_i$ operators in (NC-3D) phase and space are given by [25-42]:

$$\hat{x}_i=x_i-\frac{\theta_{ij}}{2}p_j \quad \text{and} \quad \hat{p}_i=p_i-\frac{\overline{\theta}_{ij}}{2}x_j \tag{17}$$

On based to our references [37-40], we can write the two operators $\hat{r}^2$ and $\hat{p}^2$ in noncommutative three dimensional spaces and phases as follows:

$$\hat{r}^2=r^2-\vec{\mathbf{L}}\vec{\Theta} \quad \text{and} \quad \hat{p}^2=p^2+\vec{\mathbf{L}}\vec{\overline{\boldsymbol{\theta}}} \tag{18}$$

Where $\mathbf{L}\Theta$ and $\vec{\mathbf{L}}\vec{\overline{\boldsymbol{\theta}}}$ denotes to $(L_x\Theta_{12}+L_y\Theta_{23}+L_z\Theta_{13})$ and $(L_x\overline{\theta}_{12}+L_y\overline{\theta}_{23}+L_z\overline{\theta}_{13})$, with $\Theta\equiv\frac{\boldsymbol{\theta}}{2}$, respectively. After straightforward calculations one can obtains the different terms in noncommutative three dimensional spaces and phases as follows:

$$\begin{cases} a\hat{r}^2=ar^2-a\vec{\mathbf{L}}\vec{\Theta} \\ \dfrac{b}{\hat{r}^4}=\dfrac{b}{r^4}+\dfrac{2b}{r^6}\vec{\mathbf{L}}\vec{\Theta} \\ \dfrac{c}{\hat{r}^6}=\dfrac{c}{r^6}+\dfrac{3c}{r^8}\vec{\mathbf{L}}\vec{\Theta} \end{cases} \tag{19}$$

Which allow us to writing the modified three dimensional studied potential $V_{is}(\hat{r})$in noncommutative three dimensional spaces and phases as follows:

$$V_{is}(\hat{r})=ar^2+br^{-4}+cr^{-6}+V_{pert-is}(r,\Theta,\overline{\theta}) \tag{20}$$

It's clearly that, the first 3-terms in eq. (20) represent the ordinary potential while the rest term is produced by the deformation of space and phase. The global perturbative potential operators $V_{pert-is}(r,\Theta,\overline{\theta})$ for studied potential $V_{nc-is}(r)$ in both noncommutative three dimensional spaces and phases will be written as:

$$V_{pert-is}(r,\Theta,\overline{\theta})=\left(\frac{3c}{r^8}+\frac{2b}{r^6}-a\right)\vec{\mathbf{L}}\vec{\Theta}+\vec{\mathbf{L}}\vec{\overline{\boldsymbol{\theta}}} \tag{21}$$

4. THE NONCOMMUTATIVE HAMILTONIAN FOR MODIFIED POTENTIAL $V_{is}(\hat{r})$:

4.1 THE NONCOMMUTATIVE SPIN-ORBITAL HAMILTONIAN FOR MODIFIED POTENTIAL $V_{is}(\hat{r})$ IN (NC: 3D- RSP):

We replace $\vec{\mathbf{L}}\vec{\Theta}$ and $\vec{\mathbf{L}}\vec{\overline{\boldsymbol{\theta}}}$ by $2\Theta\vec{S}\vec{L}$ and $2\overline{\theta}\vec{S}\vec{L}$, to obtain the new forms of $H_{\text{pert}}(r,\Theta,\overline{\theta})$ for modified potential $V_{is}(\hat{r})$:

$$H_{\text{pert}}\left(r,\Theta,\bar{\theta}\right)=2\left\{\Theta\left(\frac{3c}{r^8}+\frac{2b}{r^6}-a\right)+\bar{\theta}\right\}\vec{\mathbf{L}}\vec{S} \tag{22}$$

Here $\vec{S}=\frac{\vec{1}}{2}$ denote the spin of electron; it's possible also to rewriting eq. (22) to equivalent physical form:

$$H_{\text{pert}}\left(r,\Theta,\bar{\theta}\right)=\left\{\Theta\left(\frac{3c}{r^8}+\frac{2b}{r^6}-a\right)+\bar{\theta}\right\}G^2 \tag{23}$$

Where G^2 denote to the $\frac{1}{2}\left(\overleftrightarrow{J}^2-\overleftrightarrow{L}^2-\overleftrightarrow{S}^2\right)$. As it well known, ($\overleftrightarrow{J}^2$, $\overleftrightarrow{L}^2$, $\overleftrightarrow{S}^2$ and s_z) formed complete basis on quantum mechanics, then the operator $\left(\overleftrightarrow{J}^2-\overleftrightarrow{L}^2-\overleftrightarrow{S}^2\right)$ will be gives 2-eigenvalues $k_{\mp}\equiv\frac{1}{2}\left\{\left(l\pm\frac{1}{2}\right)(l+\frac{1}{2}\pm1)+l(l+1)-\frac{3}{4}\right\}$, corresponding $j=l\pm\frac{1}{2}$ respectively. Then, one can form a diagonal matrix $H_{so-is}\left(r,\vec{p},\Theta,\bar{\theta}\right)$ of order (3×3), with non null elements $\left(H_{so-is}\right)_{11}$, $\left(H_{so-is}\right)_{22}$ and $\left(H_{so-is}\right)_{33}$ for in both (NC-3D) space and phase:

$$\begin{aligned}\left(H_{so-is}\right)_{11}&=k_{+}\left\{\Theta\left(\frac{3c}{r^8}+\frac{2b}{r^6}-a\right)+\bar{\theta}\right\}\text{if } j=l+\tfrac{1}{2}\Rightarrow\text{spin up}\\ \left(H_{so-is}\right)_{22}&=k_{-}\left\{\Theta\left(\frac{3c}{r^8}+\frac{2b}{r^6}-a\right)+\bar{\theta}\right\}\text{if } j=l-\tfrac{1}{2}\Rightarrow\text{spin down}\\ \left(H_{so-is}\right)_{33}&=0\end{aligned} \tag{24}$$

After profound straightforward calculation, one can show that, the radial function $R_l(r)$ satisfied the following differential equation, in (NC-3D: RSP) for studied potential $V_{is}(\hat{r})$:

$$\frac{d^2R_l(r)}{dr^2}+\left[\begin{array}{l}E_{nc-is}-ar^2-br^{-4}-cr^{-6}-\dfrac{l(l+1)}{r^2}-\left\{\Theta\left(\dfrac{3c}{r^8}+\dfrac{2b}{r^6}-a\right)+\bar{\theta}\right\}G^2\\ -\left(\alpha\left(\dfrac{3c}{r^8}+\dfrac{2b}{r^6}-a\right)+\bar{\varepsilon}\right)BL_z\end{array}\right]R_l(r)=0 \tag{25}$$

4-2 THE EXACT SPIN-ORBITAL SPECTRUM MODIFICATIONS FOR MODIFIED POTENTIAL $V_{nc-is}(r)$ FOR FUNDAMENTAL STAES IN (NC: 3D- RSP):

The modifications to the energy levels for fundamental states $E_{\text{u-is}}$ and $E_{\text{d-is}}$ for spin up and spin down, respectively, at first order of two parameters θ and $\bar{\theta}$ obtained by applying the standard perturbation theory:

$$E_{\text{u-is}}=\left|N_0\right|^2k_{+}\int_0^{+\infty}r^{2k_0}\exp\left(-\sqrt{a}r^2-\sqrt{c}r^{-2}\right)\left(\Theta\left(\frac{3c}{r^8}+\frac{2b}{r^6}-a\right)+\bar{\theta}\right)dr\,. \tag{26.a}$$

$$E_{\text{d-is}}=\left|N_0\right|^2k_{-}\int_0^{+\infty}r^{2k_0}\exp\left(-\sqrt{a}r^2-\sqrt{c}r^{-2}\right)\left(\Theta\left(\frac{3c}{r^8}+\frac{2b}{r^6}-a\right)+\bar{\theta}\right)dr \tag{26.b}$$

A direct simplification gives:

$$E_{\text{u-is}}=\left|N_0\right|^2k_{+}\left\{\theta\sum_{i=1}^{3}T_i+\bar{\theta}T_4\right\}\,. \tag{27.a}$$

$$E_{\text{d-is}}=\left|N_0\right|^2k_{-}\left\{\theta\sum_{i=1}^{4}T_i+\frac{\bar{\theta}}{2m_0}T_5\right\} \tag{27.b}$$

Where, the five terms $T_i\left(i=\overline{1,5}\right)$ are given by:

$$T_1=3c\int_0^{+\infty} r^{(2k-3)-1}\exp\left(-\sqrt{a}r^2-\sqrt{c}r^{-2}\right)dr, T_2=2b\int_0^{+\infty} r^{(2k-1)-1}\exp\left(-\sqrt{a}r^2-\sqrt{c}r^{-2}\right)dr$$
$$T_3=-a\int_0^{+\infty} r^{(2k+1)-1}\exp\left(-\tfrac{1}{}\sqrt{a}r^2-\sqrt{c}r^{-2}\right)dr, T_4=\int_0^{+\infty} r^{(2k+1)-1}\exp\left(-\sqrt{a}r^2-\sqrt{c}r^{-2}\right)dr \tag{28}$$

Applying the following special integration [46]:

$$\int_0^{+\infty} x^{\nu-1}\exp\left(-\beta x^p-\gamma x^{-p}\right)dx=\frac{2}{p}\left(\frac{\gamma}{\beta}\right)^{\frac{\nu}{2p}}K_{\nu/p}\left(2\sqrt{\beta\gamma}\right) \tag{29}$$

Where $K_{\nu/p}$ denote to the Bateman's function, $\operatorname{Re}l(\beta)\rangle 0$ and $\operatorname{Re}l(\gamma\rangle 0)$. After straightforward calculations, we can obtain the explicitly results:

$$T_1=3c\left(\frac{c}{a}\right)^{\frac{2k-5}{8}}K_{(2k-5)/2}\left(2(ac)^{1/4}\right), T_2=2b\left(\frac{c}{a}\right)^{\frac{2k-3}{8}}K_{(2k-3)/2}\left(2(ac)^{1/4}\right)$$
$$T_3=-a\left(\frac{c}{a}\right)^{\frac{2k+3}{8}}K_{(2k+3)/2}\left(2(ac)^{1/4}\right) \text{ and } T_4=\left(\frac{c}{a}\right)^{\frac{2k+3}{8}}K_{(2k+3)/2}\left(2(ac)^{1/4}\right) \tag{30}$$

Which allow us to obtaining the exact modifications of ground states $E_{\text{u-is}}$ and $E_{\text{d-is}}$ produced by spin-orbital effect:

$$E_{\text{u-is}}=\left|N_0\right|^2 k_+\left\{\theta T_{nc-0sis}+\bar{\theta}T_{nc-0pis}\right\} \tag{31}$$

$$E_{\text{d-is}}=\left|N_0\right|^2 k_-\left\{\theta T_{nc-0sis}+\bar{\theta}T_{nc-0pis}\right\} \tag{32}$$

Where $T_{nc-0sis}$ and $T_{nc-0pis}$ are given by:

$$T_{nc-0sis}\equiv\left(3c\left(\frac{c}{a}\right)^{\frac{2k-1}{8}}K_{(2k-3)/2}\left(2(ac)^{1/4}\right)+2b\left(\frac{c}{a}\right)^{\frac{2k-3}{8}}K_{(2k-1)/2}\left(2(ac)^{1/4}\right)-a\left(\frac{c}{a}\right)^{\frac{2k+3}{8}}K_{(2k+1)/2}\left(2(ac)^{1/4}\right)\right) \tag{33.a}$$

$$T_{nc-0pis}\equiv\left(\frac{c}{a}\right)^{\frac{2k+3}{8}}K_{(2k+1)/2}\left(2(ac)^{1/4}\right) \tag{33.b}$$

The two terms $T_{nc-0sis}$ and $T_{nc-0pis}$ are repesent the noncommutative geometry of space and phase, respectively.

4-3 THE EXACT SPIN-ORBITAL MODIFICATIOBS FOR MODIFIED POTENTIAL $V_{nc-is}(r)$ IN (NC: 3D- RSP):

Now, we turn to the modifications to the energy levels for first excited states $E_{\text{u1-ip}}$ and $E_{\text{d1-ip}}$ corresponding spin up and spin down, respectively, at first order of two parameters θ and $\bar{\theta}$, which obtained by applying the standard perturbation theory:

$$E_{\text{u-1ip}}=\left|N_1\right|^2 k_+\int_0^{+\infty}\left((1+2\gamma\beta)+2\beta r^2+\beta^2r^4+2\gamma r^{-2}+\gamma^2r^{-4}\right)r^{2k_1}\exp\left(-\sqrt{a}r^2-\sqrt{c}r^{-2}\right)\left(\Theta\left(\frac{3c}{r^8}+\frac{2b}{r^6}-a\right)+\bar{\theta}\right)dr \tag{34.a}$$

$$E_{\text{d-1ip}}=\left|N_1\right|^2 k_-\int_0^{+\infty}\left((1+2\gamma\beta)+2\beta r^2+\beta^2r^4+2\gamma r^{-2}+\gamma^2r^{-4}\right)r^{2k_1}\exp\left(-\sqrt{a}r^2-\sqrt{c}r^{-2}\right)\left(\Theta\left(\frac{3c}{r^8}+\frac{2b}{r^6}-a\right)+\bar{\theta}\right)dr \tag{34.b}$$

A direct simplification gives:

$$E_{\text{u-1ip}}=\left|N_1\right|^2 k_+\left\{\theta\sum_{i=1}^{15}L_i+\bar{\theta}\sum_{i=16}^{20}T_i\right\} \tag{35.a}$$

$$E_{\text{d-1ip}} = |N_1|^2 k_- \left\{ \theta \sum_{i=1}^{15} L_i + \bar{\theta} \sum_{i=16}^{20} T_i \right\} \tag{35.b}$$

Where, the 20- terms $L_i \left(i = \overline{1,20} \right)$ are given by:

$$L_1 = 3(1+2\gamma\beta)c \int_0^{+\infty} r^{(2k_1-7)-1} \exp\left(-\sqrt{a}r^2 - \sqrt{c}r^{-2}\right)dr, L_2 = 2(1+2\gamma\beta)b \int_0^{+\infty} r^{(2k_1-5)-1} \exp\left(-\sqrt{a}r^2 - \sqrt{c}r^{-2}\right)dr$$

$$L_3 = -a \int_0^{+\infty} r^{(2k_1)-1} \exp\left(-\sqrt{a}r^2 - \sqrt{c}r^{-2}\right)dr, L_4 = 6\beta c \int_0^{+\infty} r^{(2k_1-5)-1} \exp\left(-\sqrt{a}r^2 - \sqrt{c}r^{-2}\right)dr \tag{36.a}$$

$$L_5 = 4\beta b \int_0^{+\infty} r^{(2k_1-3)-1} \exp\left(-\sqrt{a}r^2 - \sqrt{c}r^{-2}\right)dr, L_6 = -2\beta a \int_0^{+\infty} r^{(2k_1+3)-1} \exp\left(-\sqrt{a}r^2 - \sqrt{c}r^{-2}\right)dr$$

$$L_7 = 3\beta^2 c \int_0^{+\infty} r^{(2k_1-3)-1} \exp\left(-\sqrt{a}r^2 - \sqrt{c}r^{-2}\right)dr, L_8 = 2\beta^2 b \int_0^{+\infty} r^{(2k_1-1)-1} \exp\left(-\sqrt{a}r^2 - \sqrt{c}r^{-2}\right)dr$$

$$L_9 = -\beta^2 a \int_0^{+\infty} r^{(2k_1+5)-1} \exp\left(-\sqrt{a}r^2 - \sqrt{c}r^{-2}\right)dr, L_{10} = 6\gamma \int_0^{+\infty} r^{(2k_1-9)-1} \exp\left(-\sqrt{a}r^2 - \sqrt{c}r^{-2}\right)dr \tag{36.b}$$

$$L_{11} = 4\gamma b \int_0^{+\infty} r^{(2k_1-7)-1} \exp\left(-\sqrt{a}r^2 - \sqrt{c}r^{-2}\right)dr, L_{12} = -2a\gamma \int_0^{+\infty} r^{(2k_1-1)-1} \exp\left(-\sqrt{a}r^2 - \sqrt{c}r^{-2}\right)dr$$

$$L_{13} = 3\gamma^2 c \int_0^{+\infty} r^{(2k_1-11)-1} \exp\left(-\sqrt{a}r^2 - \sqrt{c}r^{-2}\right)dr, L_{14} = \mathbf{2}b\gamma^2 \int_0^{+\infty} r^{(2k_1-9)-1} \exp\left(-\sqrt{a}r^2 - \sqrt{c}r^{-2}\right)dr$$

$$L_{15} = -a\gamma^2 \int_0^{+\infty} r^{(2k_1-3)-1} \exp\left(-\sqrt{a}r^2 - \sqrt{c}r^{-2}\right)dr, L_{16} = (1+2\gamma\beta) \int_0^{+\infty} r^{(2k_1+1)-1} \exp\left(-\sqrt{a}r^2 - \sqrt{c}r^{-2}\right)dr \tag{36.c}$$

$$L_{17} = 2\beta \int_0^{+\infty} r^{(2k_1+3)-1} \exp\left(-\sqrt{a}r^2 - \sqrt{c}r^{-2}\right)dr, L_{18} = \beta^2 \int_0^{+\infty} r^{(2k_1+5)-1} \exp\left(-\sqrt{a}r^2 - \sqrt{c}r^{-2}\right)dr$$

$$L_{19} = 2\gamma \int_0^{+\infty} r^{(2k_1-1)-1} \exp\left(-\sqrt{a}r^2 - \sqrt{c}r^{-2}\right)dr, L_{20} = \gamma^2 \int_0^{+\infty} r^{(2k_1-3)-1} \exp\left(-\sqrt{a}r^2 - \sqrt{c}r^{-2}\right)dr \tag{36.d}$$

Now we apply the special integral which represents by eq. (29) to obtain:

$$L_1 = 3(1+2\gamma\beta)c\left(\frac{c}{a}\right)^{\frac{2k_{-1}-7}{4}} K_{(2k_1-7)/2}\left(2(ac)^{1/4}\right), L_2 = 2(1+2\gamma\beta)b\left(\frac{c}{a}\right)^{\frac{2k_{11}-5}{4}} K_{(2k_1-5)/2}\left(2(ac)^{1/4}\right)$$

$$L_3 = -a\left(\frac{c}{a}\right)^{\frac{k_1}{2}} K_{k_1}\left(2(ac)^{1/4}\right), L_4 = 6\beta c\left(\frac{c}{a}\right)^{\frac{2k_1-5}{4}} K_{(2k_1-5)/2}\left(2(ac)^{1/4}\right) \tag{37.a}$$

$$L_5 = 4\beta b\left(\frac{c}{a}\right)^{\frac{2k_1+3}{24}} K_{(2k_1+3)/2}\left(2(ac)^{1/4}\right), L_6 = -2\beta a\left(\frac{c}{a}\right)^{\frac{2k_1-3}{4}} K_{(2k_1-3)/2}\left(2(ac)^{1/4}\right)$$

$$L_7 = 3\beta^2 c\left(\frac{c}{a}\right)^{\frac{2k_1-3}{4}} K_{(2k_1-3)/2}\left(2(ac)^{1/4}\right), L_8 = 2\beta^2 b\left(\frac{c}{a}\right)^{\frac{2k_1-1}{4}} K_{(2k_1-1)/2}\left(2(ac)^{1/4}\right)$$

$$L_9 = -\beta^2 a\left(\frac{c}{a}\right)^{\frac{2k_1+5}{4}} K_{(2k_1+5)/2}\left(2(ac)^{1/4}\right), L_{10} = 6\gamma\left(\frac{c}{a}\right)^{\frac{2k_1-9}{4}} K_{(2k_1-9)/2}\left(2(ac)^{1/4}\right) \tag{37.b}$$

$$L_{11} = 4\gamma b\left(\frac{c}{a}\right)^{\frac{2k_1-7}{4}} K_{(2k_1-7)/2}\left(2(ac)^{1/4}\right), L_{12} = -2a\gamma\left(\frac{c}{a}\right)^{\frac{2k_1-1}{4}} K_{(2k_1-1)/2}\left(2(ac)^{1/4}\right)$$

$$L_{13} = 3\gamma^2 c\left(\frac{c}{a}\right)^{\frac{2k_1-11}{4}} K_{(2k_1-11)/2}\, 2\left(2(ac)^{1/4}\right), L_{14} = \mathbf{2}b\gamma^2\left(\frac{c}{a}\right)^{\frac{2k_1-9}{4}} K_{(2k_1-9)/2}\left(2(ac)^{1/4}\right)$$

$$L_{15} = -a\gamma^2\left(\frac{c}{a}\right)^{\frac{2k_1-3}{4}} K_{(2k_1-3)/2}\left(2(ac)^{1/4}\right), L_{16} = (1+2\gamma\beta)\left(\frac{c}{a}\right)^{\frac{2k_1+1}{4}} K_{2k_1+1/2}\left(2(ac)^{1/4}\right) \tag{37.c}$$

$$L_{17} = 2\beta\left(\frac{c}{a}\right)^{\frac{2k_1+3}{4}} K_{(2k_1+3)/2}\left(2(ac)^{1/4}\right), L_{18} = \beta^2\left(\frac{c}{a}\right)^{\frac{2k_1+5}{4}} K_{(2k_1+5)/2}\left(2(ac)^{1/4}\right)$$

$$L_{19} = 2\gamma\left(\frac{c}{a}\right)^{\frac{2k_1-1}{4}} K_{(2k_1-1)/2}\left(2(ac)^{1/4}\right) \quad \text{and} \quad L_{20} = \gamma^2\left(\frac{c}{a}\right)^{\frac{2k_1-3}{4}} K_{(2k_1-3)/2}\left(2(ac)^{1/4}\right) \tag{37.d}$$

Which allow us to obtaining the exact modifications $E_{\text{u1-is}}$ and $E_{\text{d1-is}}$ of degenerated first excited states corresponding two polarized states produced for spin-orbital effect:

$$E_{\text{u-1is}} = |N_1|^2 k_+ \left\{\theta L_{nc-1sis} + \frac{\bar{\theta}}{2m_0} L_{nc-1pis}\right\} \tag{38.a}$$

$$E_{\text{d-1is}} = |N_1|^2 k_- \left\{\theta L_{nc-1sis} + \frac{\bar{\theta}}{2m_0} L_{nc-1pis}\right\} \tag{38.b}$$

Where $L_{nc-1sis}$ and $L_{nc-1pis}$ are given by:

$$L_{nc-1sip} \equiv \sum_{i=1}^{15} L_i \quad \text{and} \quad L_{nc-1pis} \equiv \sum_{i=16}^{20} L_i \tag{39}$$

4.4 THE EXACT MAGNETIC SPECTRUM MODIFICATIOBS FOR MODIFIED POTENTIAL $V_{nc-is}(r)$ IN (NC: 3D- RSP):

On another hand, it's possible to found another automatically symmetry for modified potential $V_{is}(\hat{r})$ related to the influence of an external uniform magnetic field, it's deduced by the following replacements:

$$\Theta \to \chi B \quad \text{and} \quad \bar{\theta} \to \chi B \tag{40}$$

Here χ and $\bar{\sigma}$ are infinitesimal real proportional's constants and we choose the magnetic field $\overleftrightarrow{B} = B\overleftrightarrow{k}$, then we can make the following translation:

$$\left\{\Theta\left(\frac{3c}{r^8} + \frac{2b}{r^6} - a\right) + \bar{\theta}\right\}\vec{B}\vec{L} \to \left(\chi\left(\frac{3c}{r^8} + \frac{2b}{r^6} - a\right) + \bar{\sigma}\right)BL_z \tag{41}$$

Which allow us to introduce the modified new magnetic Hamiltonian H_{m-is} in noncommutative three dimensional spaces and phases as:

$$H_{m-is} = \left(\chi\left(\frac{3c}{r^8} + \frac{2b}{r^6} - a\right) + \bar{\sigma}\right)\left(\vec{B}\vec{J} - \overleftrightarrow{S}\overleftrightarrow{B}\right) \tag{42}$$

Here $\left(-\overleftrightarrow{S}\overleftrightarrow{B}\right)$ denote to the ordinary Hamiltonian of Zeeman Effect. To obtain the exact noncommutative magnetic modifications of energy ($E_{\text{mag-0ip}}$, $E_{\text{mag-1ip}}$) for modified potential $V_{is}(\hat{r})$, we replace: k_+, Θ_1 and $\bar{\theta}_1$ in the Eqs.(32.a) and (35.a) by the following parameters: m, χ and $\bar{\sigma}$, respectively:

$$E_{\text{mag-0is}} = |N_0|^2 Bm\left\{\chi T_{nc-0sis} + \bar{\sigma} T_{nc-0pis}\right\} \tag{43.a}$$

$$E_{\text{mag-1is}} = |N_1|^2 Bm\left\{\chi T_{nc-0sis} + \bar{\sigma} T_{nc-0pis}\right\} \tag{43.b}$$

Where $E_{\text{mag-0is}}$ and $E_{\text{mag-1is}}$ are the exact magnetic modifications of spectrum corresponding the fundamental states and first excited states, respectively and $-l \le m \le +l$.

5. THE EXACT MODIFIED ENERGY OF THE LOWEST EXCITATIONS SPECTRUM FOR MODIFIED POTENTIAL $V_{nc-is}(r)$ IN (NC: 3D- RSP):

The total modified energies ($E_{\text{nc u0-is}}$ - $E_{\text{nc d0-is}}$) and ($E_{\text{nc u1-is}}$ - $E_{\text{nc d1-is}}$) of a particle fermionic with spin up and spin down corresponding fundamental states and first excited states, respectively, for modified potential $V_{is}(\hat{r})$ in (NC: 3D-RSP) are dtermined on based to the Eqs. (10), (11), (31), (32), (38.a), (38.b), (43.a) and (43.b):

$$E_{\text{nc u0-is}} = -\sqrt{\frac{a}{c}}\left(b+4\sqrt{c}\right)+\left|N_1\right|^2 k_+\left\{\theta T_{nc-sis}+\bar{\theta}T_{nc-pis}\right\}+\left|N_0\right|^2 Bm\left\{\chi T_{nc-0sis}+\bar{\sigma}T_{nc-0pis}\right\} \quad (44.a)$$

$$E_{\text{nc d0-is}} = \sqrt{\frac{a}{c}}\left(b+4\sqrt{c}\right)+\left|N_1\right|^2 k_-\left\{\theta T_{nc-sis}+\bar{\theta}T_{nc-pis}\right\}+\left|N_0\right|^2 Bm\left\{\chi T_{nc-0sis}+\bar{\sigma}T_{nc-0pis}\right\} \quad (44.b)$$

And

$$E_{\text{nc u1-is}} = \sqrt{\frac{a}{c}}\left(b+12\sqrt{c}\right)+\left|N_0\right|^2 k_+\left\{\theta L_{nc-sis}+\bar{\theta}L_{nc-pis}\right\}+\left|N_1\right|^2 Bm\left\{\chi L_{nc-1sis}+\bar{\sigma}L_{nc-1pis}\right\} \quad (44.c)$$

$$E_{\text{nc d1-is}} = \sqrt{\frac{a}{c}}\left(b+12\sqrt{c}\right)+\left|N_0\right|^2 k_-\left\{\theta L_{nc-sis}+\bar{\theta}L_{nc-pis}\right\}+\left|N_1\right|^2 Bm\left\{\chi L_{nc-1sis}+\bar{\sigma}L_{nc-1pis}\right\} \quad (44.d)$$

The quantum number m can be takes ($2l+1$) values and we have also two values for $j=l\pm\frac{1}{2}$, thus every state in usually three dimensional space of modified potential $V_{nc-is}(r)$ will be in (NC: 3D-RSP): $2(2l+1)$sub-states. It's clearly, that the obtained eigenvalues of energies are reals and then the noncommutative diagonal Hamiltonian H_{nc-3is} is Hermitian. Regarding the previous obtained results, we can deduce the global diagonal noncommutative Hamiltonian matrix H_{nc-3is} of order (3×3), with elements $(H_{nc-3is})_{11}$, $(H_{nc-3is})_{22}$ and $(H_{com-i3s})_{33}$ in both (NC-3D) phase and space:

$$(H_{nc-3is})_{11} = -\Delta + ar^2 + br^{-4} + cr^{-6} + k_+\left\{\Theta\left(\frac{3c}{r^8}+\frac{2b}{r^6}-a\right)+\bar{\theta}\right\}+\left(\chi\left(\frac{3c}{r^8}+\frac{2b}{r^6}-a\right)+\bar{\sigma}\right)BL_z \quad \text{if } j=l+\tfrac{1}{2}\Rightarrow \text{spin up} \quad (45.a),$$

$$(H_{nc-3is})_{22} = -\Delta + ar^2 + br^{-4} + cr^{-6} + k_-\left\{\Theta\left(\frac{3c}{r^8}+\frac{2b}{r^6}-a\right)+\bar{\theta}\right\}+\left(\chi\left(\frac{3c}{r^8}+\frac{2b}{r^6}-a\right)+\bar{\sigma}\right)BL_z \quad \text{if } j=l-\tfrac{1}{2}\Rightarrow \text{spin - down} \quad (45.b),$$

And

$$(H_{com-3is})_{33} = -\Delta + ar^2 + br^{-4} + cr^{-6} \rightarrow \text{Non}-\text{polarised}-\text{electron} \quad (45.c)$$

6. CONCLUSIONS

We have obtained exact degenerated bound state solution in both three dimensional space and phase by applying perturbation theory and Boopp's Shift method instead to solving Schrödinger equation directly with star product for modified potential $V_{is}(\hat{r}) = ar^2 + br^{-4} + cr^{-6} + V_{pert-is}\left(r,\Theta,\bar{\theta}\right)$. We showed the obtained degenerated spectrum for the modified studied potential corresponding three modes of fermionic particles, the first one with spin up and the second with spin down while the last is non polarized particle.

ACKNOWLEDGMENTS

This work was supported with search laboratory of: Physique et Chimie des matériaux, in university of M'sila, Algeria.

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