A New Approach to the Non Relativistic Schrödinger equation for an Energy-Depended Potential $V(r, E_{n,l}) = V_0(1 + \eta E_{n,l})r^2$ in Both Noncommutative three Dimensional spaces and phases

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ABSTRaCT: In present work we study the 3-dimensional non relativistic and noncommutative space-phase Schrödinger equation for modified potential $(V_0(1 + \eta E_{n,l})r^2 - V_0(1 + \eta E_{n,l})\vec{L}\vec{\Theta} + \frac{\vec{L}\vec{\Theta}}{2\mu})$ depends on energy and quadratic on the relative distance, we have obtained the exact modified bound-states solutions. It has been observed that, the energy spectra in ordinary quantum mechanics was changed, and replaced by degenerate new states, depending on new discrete quantum numbers: m, l, j and $s = \pm \frac{1}{2}$. We show the noncommutative new anisotropic Hamiltonian containing two new important terms, the first new term describe the spin-orbit interaction while the second describes the modified Zeeman effect.

1. INTRODUCTION

During the past several decades and few years, much effort has gone into studying the stationary non relativistic Schrödinger equation with central and non central potentials, in commutative and noncommutative two and three dimensional space and phase [1-32]. In recent years, the study of central and non central potentials were prolonged to the relativistic stationary Schrödinger equation and the relativistic Dirac equation, in particular, the spherical symmetry have received great attention because of their wide applications, a good example, we give the potential $V(r, E_{n,l}) = V_0(1 + \eta E_{n,l})r^2$ [7]. The purpose of this work is to present a new study of the previously potential in both noncommutative two and three dimensional spaces and phases, to discover the new spectrum. The notions of noncommutativity of space and phase based essentially on Seiberg-Witten map and Boopp's shift method and the star product, defined on the first order of two infinitesimal parameters antisymmetric $(\theta^{\mu\nu}, \bar{\theta}^{\mu\nu}) \equiv \frac{1}{2} \varepsilon^{ijk} (\theta_k, \bar{\theta}_k)$ as [23-43]:

$$f(x) * g(x) = f(x) * g(x) - \frac{i}{2} \theta^{\mu\nu} (\partial_\mu^x f(x)) (\partial_\nu^p g(x)) - \frac{i}{2} \bar{\theta}^{\mu\nu} (\partial_\mu^p f(x)) (\partial_\nu^x g(x)) \quad (1)$$

As a direct principal's result of the above equation, the two new non nulls commutators $[x_i, x_j]$ and $[\hat{p}_i, \hat{p}_j]_*$ in the notion of star product:

$$[x_i, x_j]_* = i\theta_{ij} \text{ and } [\hat{p}_i, \hat{p}_j]_* = i\bar{\theta}_{ij} \quad (2)$$

The Boopp's shift method instead of solving the (NC-3D) spaces and phases with star product, the Schrödinger equation will be treated by using directly star product procedure [31-43]:

$$[\hat{x}_i, \hat{x}_j] = i\theta_{ij} \text{ and } [\hat{p}_i, \hat{p}_j] = i\bar{\theta}_{ij} \quad (3)$$

The study of energy-depended potential have yielded acceptable results in the annals of particle and nuclear physics in addition to used in modification the scalar product to have a conserved norm

[7,8]. This study is based on our previous work [33-39]. The rest of present search is organized as follows: In next section, we briefly review the Schrödinger equation with an energy-depended potential in three dimensional spaces. In section 3, we review and applying Boopp's shift method to derive the deformed Hamiltonian for an energy-depended potential in noncommutative three dimensional space-phase ordinary three dimensional spaces. In the forth section we apply standard perturbation theory to find the exact quantum spectrum of the bound states in (NC-3D) space and phase for studied potential in first order of two infinitesimal parameters' Θ and $\bar{\theta}$. Finally, the important found results and the conclusions are discussed in the last section.

2. REVIEW OF ENERGY-DEPENDED POTENTIAL IN THREE DIMENSIONAL SPAPACES

In this section, we shall review the eigenvalues and eigenvalues for spherically symmetric quadratic depended on the special coordinates and linear one for the energy potential $V(r)$ [7]:

$$V(r, E_{n,l}) = V_0(1 + \eta E_{n,l})r^2 \quad (4)$$

$E_{n,l}$ Is the energy of a fermionic particle moving in this potential, the parameters V_0 and η are constants [7], the complex eigenfunctions $\Psi_{n,l,m}(r, \Omega)$ in 3-dimensional space for above potential satisfied the Schrödinger equation ($\hbar = c = 1$) is [7]:

$$\left(-\frac{\Delta}{2\mu} + V_0(1 + \eta E_{n,l})r^2 \right) \Psi_{n,l,m}(r, \Omega) = E_{n,l} \Psi_{n,l,m}(r, \Omega) \quad (5)$$

Where $\Psi_{n,l,m}(r, \Omega) = R_{n,l}(r)Y_l^m(\Omega)$, $R_{n,l}(r)$, $Y_l^m(\Omega)$ and l represent the hyperradial part, hyperspherical harmonics part and the orbital angular momentum, with: $-l \leq m \leq +l$. From reference [7], one can deduce immediately, that, the hyperradial part in 3-dimensions:

$$\frac{d^2 R_{n,l}(r)}{dr^2} + \frac{2}{r} \frac{dR_{n,l}(r)}{dr} - \frac{l(l+1)}{r^2} R_{n,l}(r) + 2\mu E_{n,l} R_{n,l}(r) - 2\mu V_0(1 + \eta E_{n,l})r^2 R_{n,l}(r) = 0 \quad (6)$$

The hyperradial part and the energy eigenvalues respectively in three dimensional spaces [7]:

$$R_{n,l}(r) = \exp(-\alpha_{n,l}r^2) r^{\frac{1}{2}+2\sqrt{\beta_l+\frac{1}{16}}} L_n^{2\sqrt{\beta_l+\frac{1}{16}}}(2\alpha_{n,l}r^2) \quad (7)$$

And

$$E_{n,l} = \frac{V_0\eta\{2(2n+1)+2l+1\}^2}{2\mu} \pm \sqrt{\frac{[V_0\eta\{2(2n+1)+2l+1\}^2]^2}{2\mu^2} + 2\frac{V_0\eta\{2(2n+1)+2l+1\}^2}{\mu}} \quad (8)$$

Where $L_n^{2\sqrt{\beta_l+\frac{1}{16}}}(2\alpha_{n,l}r^2)$ is the Laguerre polynomial, the factors $\alpha_{n,l}$ and β_l are determined by [7]:

$$\begin{aligned} \alpha_{n,l}^2 &= \frac{1}{2} V_0 \mu (1 + \eta E_{n,l}) \\ \beta_l &= \frac{l(l+1)}{4} \end{aligned} \quad (9)$$

3. NONCOMMUTATIVE PHASE-SPACE HAMILTONIAN FOR ENERGY –DEPENDENT POTENTIAL

3.1 FORMALISM OF BOOPP'S SHIFT METHOD

In this sub-section, we shall review some fundamental principles of the quantum noncommutative Schrödinger equation which resumed in the following steps for an energy-dependent potential [31-44]:

$$\begin{aligned} \text{Ordinary Hamiltonian : } \hat{H}(p_i, x_i) &\rightarrow \text{NC-Hamiltonian : } \hat{H}(\hat{p}_i, \hat{x}_i) \\ \text{Ordinary - complex wave function : } \Psi(\vec{r}) &\rightarrow \text{NC - complex wave function : } \hat{\Psi}(\vec{\hat{r}}) \\ \text{Ordinary - energy : } E &\rightarrow \text{NC - Energy : } E_{nc-edp} \end{aligned} \quad (10)$$

Ordinary - product $\rightarrow$ New star product - acting on phase and space : *

Which allow us to writing the three dimensional space-phase quantum noncommutative Schrödinger equations as follows:

$$\hat{H}(\hat{p}_i, \hat{x}_i) * \hat{\Psi}(\vec{\hat{r}}) = E_{nc-gds} \hat{\Psi}(\vec{\hat{r}}) \quad (11)$$

The Boopp's shift method permutes to reduce the above equation to the form:

$$H_{edp}(\hat{p}_i, \hat{x}_i) \psi(\vec{r}) = E_{nc-edp} \psi(\vec{r}) \quad (12)$$

Here the two $\hat{x}_i$ and $\hat{p}_i$ operators in (NC-3D) phase and space are given by [31-44]:

$$\hat{x}_i = x_i - \frac{\theta_{ij}}{2} p_j \quad \text{and} \quad \hat{p}_i = p_i - \frac{\bar{\theta}_{ij}}{2} x_j \quad (13)$$

based to our reference [39], we written the two operators $\hat{r}^2$ and $\hat{p}^2$ in (NC-3D) spaces and phases as follows:

$$\begin{aligned} \hat{r}^2 &= r^2 - \vec{L}\vec{\Theta} \\ \hat{p}^2 &= p^2 + \vec{L}\vec{\bar{\Theta}} \end{aligned} \quad (14)$$

Where $\vec{L}\vec{\Theta}$ and $\vec{L}\vec{\bar{\Theta}}$ denotes to $(L_x\Theta_{12} + L_y\Theta_{23} + L_z\Theta_{13})$ and $(L_x\bar{\Theta}_{12} + L_y\bar{\Theta}_{23} + L_z\bar{\Theta}_{13})$, with $\Theta \equiv \frac{\theta}{2}$.

Now, the global potential operators $V_{edp}(\hat{r})$ for an energy-dependent potential in both (NC-3D) phase and space will be written as:

$$V_{edp}(\hat{r}) = V_0(1 + \eta E_{n,l}) r^2 - V_0(1 + \eta E_{n,l}) \vec{L}\vec{\Theta} + \frac{\vec{L}\vec{\bar{\Theta}}}{2\mu} \quad (15)$$

It's clearly that, the first term in above equation are given the ordinary potential $V_{sd}(r)$ in 3D spaces, while the rest terms are proportional's with two infinitesimals parameters (Θ and $\bar{\Theta}$) and then gives the terms of perturbations $H_{\text{pert}}(r, \Theta, \bar{\Theta})$ for an energy-dependent potential in (NC-3D) real space and phase as:

$$H_{\text{pert}}(r, \Theta, \bar{\Theta}) = -V_0(1 + \eta E_{n,l}) \vec{L}\vec{\Theta} + \frac{\vec{L}\vec{\bar{\Theta}}}{2\mu} \quad (16)$$

3.2 THE SECOND PART OF NONCOMMUTATIVE PHASE-SPACE HAMILTONIAN FOR ENERGY –DEPENDENT POTENTIAL

Let us now to replace $\vec{L}\vec{\Theta}$ and $\vec{L}\vec{\bar{\Theta}}$ by $2\vec{\Theta}\vec{S}\vec{L}$ and $2\vec{\bar{\Theta}}\vec{S}\vec{L}$, the coupling between spin and orbital momentum respectively, to obtain the new forms of $H_{\text{pert}}(r, \Theta, \bar{\Theta})$ for an energy-dependent potential:

$$H_{\text{pert}}(r, \Theta, \bar{\Theta}) = 2 \left\{ -V_0(1 + \eta E_{n,l}) \vec{L}\vec{\Theta} + \frac{\vec{L}\vec{\bar{\Theta}}}{2\mu} \right\} \vec{L}\vec{S} \quad (17)$$

Here $\vec{S} = \frac{1}{2}$ denote the spin of electron, it's possible also to replace $(\vec{S}\vec{L})$ by $\frac{1}{2}(\vec{J}^2 - \vec{L}^2 - \vec{S}^2)$, which allow us to writing the perturbative terms for an energy-depended potential:

$$H_{\text{pert}}(r, \Theta, \bar{\theta}) = \left\{ -V_0 \Theta (1 + \eta E_{n,l}) + \frac{\bar{\theta}}{2\mu} \right\} G^2 \quad (18)$$

Where G^2 denote to the $\frac{1}{2}(\vec{J}^2 - \vec{L}^2 - \vec{S}^2)$. The $(\vec{J}^2, \vec{L}^2, \vec{S}^2$ and $s_z)$ formed complete basis on quantum mechanics for each potential, then the operator $(\vec{J}^2 - \vec{L}^2 - \vec{S}^2)$ will be gives 2-eigenvalues

$k_{\mp} \equiv \frac{1}{2} \left\{ \left(l \pm \frac{1}{2} \right) \left(l + \frac{1}{2} \pm 1 \right) + l(l+1) - \frac{3}{4} \right\}$, corresponding $j = l \pm \frac{1}{2}$ respectively. Then, one can form a diagonal matrix $H_{\text{edp-so}}(r, \vec{p}, \Theta, \bar{\theta})$ of order (3×3) , with non null elements $(H_{\text{edp-so}}(r, \vec{p}, \Theta, \bar{\theta}))_{11}$, $(H_{\text{edp-so}}(r, \vec{p}, \Theta, \bar{\theta}))_{22}$ and $(H_{\text{edp-so}}(r, \vec{p}, \Theta, \bar{\theta}))_{33}$ for an energy-depended potential in both (NC-3D) phase and space:

$$\begin{aligned} (H_{\text{edp-so}}(r, \vec{p}, \Theta, \bar{\theta}))_{11} &= k_+ \left\{ -V_0 \Theta (1 + \eta E_{n,l}) + \frac{\bar{\theta}}{2\mu} \right\} \text{ if } j = l + \frac{1}{2} \Rightarrow \text{spin up} \\ (H_{\text{edp-so}}(r, \vec{p}, \Theta, \bar{\theta}))_{22} &= k_- \left\{ -V_0 \Theta (1 + \eta E_{n,l}) + \frac{\bar{\theta}}{2\mu} \right\} \text{ if } j = l - \frac{1}{2} \Rightarrow \text{spin down} \\ (H_{\text{edp-so}}(r, \vec{p}, \Theta, \bar{\theta}))_{33} &= -\frac{\Delta}{2\mu} + V_0 (1 + \eta E_{n,l}) r^2 \rightarrow \text{Non - polarised - electron} \end{aligned} \quad (19)$$

3.3 THE THERID PART OF NONCOMMUTATIVE PHASE-SPACE HAMILTONIAN FOR ENERGY -DEPENDED POTENTIAL

On another hand, it's possible to consider the two at infinitesimals parameters $(\Theta$ and $\bar{\theta})$ are the sum of two infinitesimals parameters to each one as [33-38]:

$$\Theta = \Theta_1 + \Theta_2 \text{ and } \bar{\theta} = \bar{\theta}_1 + \bar{\theta}_2 \quad (20)$$

Furthermore, if we choose both Θ_2 and $\bar{\theta}_2$ are proportional's to an external magnetic field B as:

$$\Theta_2 = \alpha_2 B, \quad \bar{\theta}_2 = \bar{\varepsilon}_2 B \quad \text{and} \quad \vec{B} = B \vec{k} \quad (21)$$

Which allow us to obtain the following results:

$$-V_0 (1 + \eta E_{n,l}) \vec{L} \vec{\Theta}_2 + \frac{\vec{L} \vec{\bar{\theta}}_2}{2\mu} \rightarrow \left(-\alpha_2 V_0 (1 + \eta E_{n,l}) + \frac{\bar{\varepsilon}_2}{2\mu} \right) B L_z \quad (22)$$

Then, the second part of noncommutative magnetic Hamiltonian operator for an energy-depended potential $H_{\text{edp-mag}}(r, \vec{p}, \alpha_2, \bar{\varepsilon}_2)$ can be determined as follows:

$$H_{\text{edp-mag}}(r, \vec{p}, \alpha_2, \bar{\varepsilon}_2) = \left(-\alpha_2 V_0 (1 + \eta E_{n,l}) + \frac{\bar{\varepsilon}_2}{2\mu} \right) B L_z \quad (23)$$

3.4 THE GLOBAL NONCOMMUTATIVE PHASE-SPACE HAMILTONIAN FOR ENERGY -DEPENDED POTENTIAL

Regarding the previously obtained results, we can deduce the global diagonal noncommutative Hamiltonian matrix $H_{\text{nc-edp}}(r, \vec{p}, \Theta, \bar{\theta})$ of order (3×3) , with non null elements $(H_{\text{nc-edp}})_{11}$, $(H_{\text{nc-edp}})_{22}$ and $(H_{\text{nc-edp}})_{33}$ for an energy-depended potential in both (NC-3D) phase and space:

$$(H_{nc-edp})_{11} = -\frac{\Delta}{2\mu} + V_0(1 + \eta E_{n,l})r^2 + k_+ \left\{ -V_0\Theta(1 + \eta E_{n,l}) + \frac{\bar{\theta}}{2\mu} \right\} + \left(-\alpha_2 V_0(1 + \eta E_{n,l}) + \frac{\bar{\varepsilon}_2}{2\mu} \right) BL_z \text{ if } j = l + \frac{1}{2} \Rightarrow \text{spin up} \quad (24.1),$$

$$(H_{nc-edp})_{22} = -\frac{\Delta}{2\mu} + V_0(1 + \eta E_{n,l})r^2 + k_- \left\{ -V_0\Theta(1 + \eta E_{n,l}) + \frac{\bar{\theta}}{2\mu} \right\} + \left(-\alpha_2 V_0(1 + \eta E_{n,l}) + \frac{\bar{\varepsilon}_2}{2\mu} \right) BL_z \text{ if } j = l - \frac{1}{2} \Rightarrow \text{spin down} \quad (24.2),$$

And

$$(H_{nc-edp})_{33} = -\frac{\Delta}{2\mu} + V_0(1 + \eta E_{n,l})r^2 \rightarrow \text{Non-polarised-electron} \quad (24.3)$$

After profound straightforward calculation, one can show that, the radial function $R_{n,l}(r)$ satisfied the following differential equation, in (NC-3D: RSP) for an energy-depended potential:

$$\frac{d^2 R_{n,l}(r)}{dr^2} + \frac{2}{r} \frac{dR_{n,l}(r)}{dr} - \frac{l(l+1)}{r^2} R_{n,l}(r) + 2\mu E_{n,l} R_{n,l}(r) - 2\mu \left\{ V_0(1 + \eta E_{n,l})r^2 - \left\{ -V_0\Theta(1 + \eta E_{n,l}) + \frac{\bar{\theta}}{2\mu} \right\} G^2 - \left(-\alpha_2 V_0(1 + \eta E_{n,l}) + \frac{\bar{\varepsilon}_2}{2\mu} \right) BL_z \right\} R_{n,l}(r) = 0 \quad (25)$$

4. NONCOMMUTATIVE SPECTRUM FOR ENERGY DEPENDED -POTENTIAL

The purpose of this fundamental sub section is to draw the strategy of total exact energies: (E_{nc-up} , E_{nc-d} and E_{np}) for a particle fermionic with spin up, spin down and non polarized at first order of two infinitesimals parameters (Θ and $\bar{\theta}$) corresponding ($(H_{nc-edp})_{11}$, $(H_{ds-edp}(r, \vec{p}, \Theta, \bar{\theta}))_{22}$ and $(H_{nc-dedp})_{33}$) for an energy-depended potential in both (NC-3D) phase and space, respectively:

$$E_{nc-up} = \frac{V_0\eta\{2(2n+1)+2l+1\}^2}{2\mu} \pm \sqrt{\frac{[V_0\eta\{2(2n+1)+2l+1\}^2]^2}{2\mu^2} + 2\frac{V_0\eta\{2(2n+1)+2l+1\}^2}{\mu}} \quad (26.1),$$

$$+ E_{edp-up}(\Theta_1, \bar{\theta}_1) + E_{edp-mag}(\Theta_2, \bar{\theta}_2)$$

And

$$E_{nc-d} = \frac{V_0\eta\{2(2n+1)+2l+1\}^2}{2\mu} \pm \sqrt{\frac{[V_0\eta\{2(2n+1)+2l+1\}^2]^2}{2\mu^2} + 2\frac{V_0\eta\{2(2n+1)+2l+1\}^2}{\mu}} \quad (26.2),$$

$$+ E_{edp-d}(\Theta_1, \bar{\theta}_1) + E_{edp-mag}(\Theta_2, \bar{\theta}_2)$$

And

$$E_{np} = \frac{V_0\eta\{2(2n+1)+2l+1\}^2}{2\mu} \pm \sqrt{\frac{[V_0\eta\{2(2n+1)+2l+1\}^2]^2}{2\mu^2} + 2\frac{V_0\eta\{2(2n+1)+2l+1\}^2}{\mu}} \quad (26.3)$$

Where $E_{edp-up}(\Theta_1, \bar{\theta}_1)$, $E_{edp-d}(\Theta_1, \bar{\theta}_1)$ and $E_{edp-mag}(\Theta_2, \bar{\theta}_2)$ are the exact spin-orbital (up-down) and magnetic modifications for the studied potential.

4.1 NONCOMMUTATIVE SPIN-ORBITAL SPECTRUM FOR ENERGY DEPENDED -POTENTIAL

To obtain the exact noncommutative spin-orbital modifications of energy ($E_{edp-up}(\Theta_1, \bar{\theta}_1)$, $E_{edp-d}(\Theta_1, \bar{\theta}_1)$) for energy-depended potential, corresponding spin up and spin down, we apply the standard perturbations theory:

$$E_{edp-up}(\Theta_1, \bar{\theta}_1) = k_+ \left\{ -V_0\Theta(1 + \eta E_{n,l}) + \frac{\bar{\theta}}{2\mu} \right\} \int_0^{+\infty} \exp(-2\alpha_{n,l}r^2) r^{-\frac{1}{2}+2\sqrt{\beta_l+\frac{1}{16}}} \left[L_n^{2\sqrt{\beta_l+\frac{1}{16}}}(2\alpha_{n,l}r^2) \right]^2 r^2 dr \quad (27.1)$$

And

$$E_{edp-d}(\Theta_2, \bar{\Theta}_2) = k_- \left\{ -V_0 \Theta(1 + \eta E_{n,l}) + \frac{\bar{\Theta}}{2\mu} \right\} \int_0^{+\infty} \exp(-2\alpha_{n,l} r^2) r^{\frac{1}{2} + 2\sqrt{\beta_l + \frac{1}{16}}} \left[L_n^{2\sqrt{\beta_l + \frac{1}{16}}}(2\alpha_{n,l} r^2) \right]^2 r^2 dr \quad (27.2)$$

It's convenient to rewrite the above 2-eqs. (26.1) and (26.2) as follows:

$$E_{edp-up}(\Theta_1, \bar{\Theta}_1) = \frac{k_+}{2} \left\{ -V_0 \Theta(1 + \eta E_{n,l}) + \frac{\bar{\Theta}}{2\mu} \right\} \int_0^{+\infty} \exp(-\delta t) r^{\alpha-1} [L_n^\gamma(\delta t)]^2 dt \quad (27.3)$$

And

$$E_{edp-d}(\Theta_2, \bar{\Theta}_2) = \frac{k_-}{2} \left\{ -V_0 \Theta(1 + \eta E_{n,l}) + \frac{\bar{\Theta}}{2\mu} \right\} \int_0^{+\infty} \exp(-\delta t) r^{\alpha-1} \left[L_n^{2\sqrt{\beta_l + \frac{1}{16}}}(\delta t) \right]^2 dt \quad (27.4)$$

With $t = r^2$, $\alpha = \frac{5}{4} + \sqrt{\beta_l + \frac{1}{16}}$, $2\alpha_{n,l} = \delta$ and $\gamma = 2\sqrt{\beta_l + \frac{1}{16}}$. Applying the following special integration [45, 46]:

$$\int_0^{+\infty} \exp(-\delta t) r^{\alpha-1} L_m^\lambda(\delta t) L_n^\beta(\delta t) dt = \frac{\delta^{-\alpha} \Gamma(n - \alpha + \beta + 1) \Gamma(m + \lambda + 1)}{m! n! \Gamma(1 - \alpha + \beta) \Gamma(1 + \lambda)} {}_3F_2(-m, \alpha, \alpha - \beta; -n + \alpha - \beta, \lambda + 1; 1) \quad (28)$$

Where ${}_pF_q(\alpha_1, \dots, \alpha_p; \beta_1, \dots, \beta_q; z)$ (9.14- Index of Special Functions: xI) denote to the generalized hypergeometric series and Γ is Gamma function. After straightforward calculations, we can obtain the results:

$$\int_0^{+\infty} \exp(-\delta t) r^{\alpha-1} [L_n^\gamma(\delta t)]^2 dt = \frac{\delta^{-\alpha} \Gamma(n+1) \Gamma(n+\gamma+1)}{n!^2 \Gamma(1) \Gamma(1+\gamma)} {}_3F_2(-n, \alpha, \alpha - \gamma; -n + \alpha - \gamma, \gamma + 1; 1) \quad (29)$$

Substituting the Eq. (29) into the Eq. (26.1) and (26.2) to get spin-orbital modifications for an energy-depended potential

$$E_{edp-up}(\Theta_1, \bar{\Theta}_1) = \frac{k_+}{2} \left\{ -V_0 \Theta(1 + \eta E_{n,l}) + \frac{\bar{\Theta}}{2\mu} \right\} \frac{\delta^{-\alpha} \Gamma(n+1) \Gamma(n+\gamma+1)}{n!^2 \Gamma(1) \Gamma(1+\gamma)} {}_3F_2(-n, \alpha, \alpha - \gamma; -n + \alpha - \gamma, \gamma + 1; 1) \quad (30.1),$$

And

$$E_{edp-d}(\Theta_2, \bar{\Theta}_2) = \frac{k_-}{2} \left\{ -V_0 \Theta(1 + \eta E_{n,l}) + \frac{\bar{\Theta}}{2\mu} \right\} \frac{\delta^{-\alpha} \Gamma(n+1) \Gamma(n+\gamma+1)}{n!^2 \Gamma(1) \Gamma(1+\gamma)} {}_3F_2(-n, \alpha, \alpha - \gamma; -n + \alpha - \gamma, \gamma + 1; 1) \quad (30.2)$$

4.2 NONCOMMUTATIVE MAGNETIC SPECTRUM FOR ENERGY DEPENDENT - POTENTIAL

To obtain the exact noncommutative magnetic modifications of energy $E_{edp-mag}(\Theta_2, \bar{\Theta}_2)$ for $V(\hat{r})$ respectively, its sufficient to replace: k_+ , Θ_1 and $\bar{\Theta}_1$ in the Eqs.(30.1) and (30.2) by the following parameters: m , α_2 and $\bar{\varepsilon}_2$, respectively:

$$E_{edp-mag}(\Theta_1, \bar{\Theta}_1) = \frac{B}{2} m \left\{ -\alpha_2 V_0 (1 + \eta E_{n,l}) + \frac{\bar{\varepsilon}_2}{2\mu} \right\} \frac{\delta^{-\alpha} \Gamma(n+1) \Gamma(n+\gamma+1)}{n!^2 \Gamma(1) \Gamma(1+\gamma)} {}_3F_2(-n, \alpha, \alpha - \gamma; -n + \alpha - \gamma, \gamma + 1; 1) \quad (31)$$

Where m denote to eigenvalue of operator L_z and can be take $(2l+1)$ values. Let us resumed the energy-depended potential for a particle fermionic with spin up, spin down and non polarized at first order of two infinitesimals parameters (Θ and $\bar{\Theta}$) corresponding $(H_{nc-edp})_{11}$, $(H_{nc-edp})_{22}$ and $(H_{nc-edp})_{33}$ in both (NC-3D) phase and space:

$$E_{nc-up} = \frac{V_0 \eta \{2(2n+1) + 2l+1\}^2}{2\mu} \pm \sqrt{\frac{[V_0 \eta \{2(2n+1) + 2l+1\}^2]^2}{2\mu^2} + 2 \frac{V_0 \eta \{2(2n+1) + 2l+1\}^2}{\mu}} \quad (32.1),$$

$$+ M_{ab} \left\{ \frac{k_+}{2} \left\{ -V_0 \Theta(1 + \eta E_{n,l}) + \frac{\bar{\theta}}{2\mu} \right\} + \frac{B}{2} \left(-\alpha_2 V_0 (1 + \eta E_{n,l}) + \frac{\bar{\varepsilon}_2}{2\mu} \right) \right\}$$

And

$$E_{nc-d} = \frac{V_0 \eta \{2(2n+1) + 2l+1\}^2}{2\mu} \pm \sqrt{\frac{[V_0 \eta \{2(2n+1) + 2l+1\}^2]^2}{2\mu^2} + 2 \frac{V_0 \eta \{2(2n+1) + 2l+1\}^2}{\mu}} \quad (32.2),$$

$$+ M_{ab} \left\{ \frac{k_-}{2} \left\{ -V_0 \Theta(1 + \eta E_{n,l}) + \frac{\bar{\theta}}{2\mu} \right\} + \frac{B}{2} \left(-\alpha_2 V_0 (1 + \eta E_{n,l}) + \frac{\bar{\varepsilon}_2}{2\mu} \right) \right\}$$

And

$$E_{np} = \frac{V_0 \eta \{2(2n+1) + 2l+1\}^2}{2\mu} \pm \sqrt{\frac{[V_0 \eta \{2(2n+1) + 2l+1\}^2]^2}{2\mu^2} + 2 \frac{V_0 \eta \{2(2n+1) + 2l+1\}^2}{\mu}} \quad (32.3)$$

Where the factor M_{ab} is given by

$$M_{ab} = \frac{\delta^{-\alpha} \Gamma(n+1) \Gamma(n+\gamma+1)}{n!^2 \Gamma(1) \Gamma(1+\gamma)} {}_3F_2(-n, \alpha, \alpha-\gamma; -n+\alpha-\gamma, \gamma+1; 1) (\Theta_2, \bar{\theta}_2) \quad (33)$$

On another hand for the effect spin-orbital interactions we have two possible values for each value of total momentums $j = l \pm \frac{1}{2}$ (spin up-spin down), thus every state in usually 3D of energy-depended potential in NC (3D-SP) phase and space will be: $2(2l+1)$ sub-states. Let us applied our obtained results to the particularly case $\eta = 0$ and $V_0 = \frac{1}{2} \mu \omega^2$, we find:

$$E_{nc-up} = \omega \left\{ 2n + l + \frac{3}{2} \right\} + \frac{1}{2} M_{ab} \left\{ -\frac{1}{2} \mu \omega^2 (k_+ \Theta - B \alpha_2) + \frac{1}{2\mu} (\bar{\theta} + \bar{\varepsilon}_2 B) \right\} \quad (34.1),$$

And

$$E_{nc-d} = \omega \left\{ 2n + l + \frac{3}{2} \right\} + \frac{1}{2} M_{ab} \left\{ -\frac{1}{2} \mu \omega^2 (k_- \Theta - B \alpha_2) + \frac{1}{2\mu} (\bar{\theta} + \bar{\varepsilon}_2 B) \right\} \quad (34.2),$$

And

$$E_{np} = \omega \left\{ 2n + l + \frac{3}{2} \right\} \quad (34.3)$$

The first part is the famous energy eigenvalues relation for the harmonic oscillator in 3-dimensions [7], while the rest of terms are represents the contributions of noncommutativity of space and phase.

5. CONCLUSION

We have obtained three dimensions noncommutative phase-space Hamiltonian of energy-depended potential and we have also solving Schrödinger equations using both Boopp's shift method and perturbation theory. We find the eigenvalues of modified potential. Our spectrum results corresponding three modes (spin: up and spin down) depended with discrete quantum numbers m , l and l_B , in addition to four infinitesimal parameters α_2 , $\bar{\varepsilon}_2$, Θ_1 and $\bar{\theta}_1$, while the third mode correspond the usual spectrum in commutative space.

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