

New exact solution of the bound states for the potential family

$V(r) = \frac{A}{r^2} - \frac{B}{r} + Cr^k$ ($k = 0, -1, -2$) in both noncommutative three dimensional spaces

and phases: Non relativistic quantum mechanics

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ABSTRACT. In present work we obtain the modified bound-states solutions for central family $V(r) = \frac{A}{r^2} - \frac{B}{r} + Cr^k$ ($k = 0, -1, -2$) in both noncommutative three dimensional spaces and phases. It has been observed that the energy spectra in ordinary quantum mechanics was changed, and replaced degenerate new states, depending on two infinitesimals parameters Θ and $\bar{\theta}$ corresponding the noncommutativity of space and phase, in addition to the discrete atomic quantum numbers: j , l , $s_z = \pm 1/2$ and m corresponding to the two spins states of electron by (up and down) and non polarized electron. The deformed anisotropic Hamiltonian formed by three operators: the first describes usual the usual family potential, the second describe spin-orbit interaction while the last one describes the modified Zeeman effect (containing ordinary Zeeman effect).

1. INTRODUCTION:

Although, the two fundamentals equations of Klein-Gordon and Dirac are satisfied very large susifall at high energy for describing scalar particle with spin zero and fermionic particle with spin $1/2$ respectively, the Schrödinger equation rest as a big revolution like general relativity and special relativity for describing physics phenomena at microscopic (Planks scales) and macroscopic scales (the planets movements). In last few years among different forms of physical central and non central potentials which appear in the operator of Hamiltonian, those received great attention the recent years in commutative and noncommutative spaces-phases at two and three dimensional spaces and phases [1-43]. In 1947 Mr.: H. Snyder introduces a new physical notion in quantum mechanics know by the noncommutative geometry at small length scales to obtains an profound interpretations to physics and chemical and another field [24]. The notions of noncommutativity of space and phase based essentially on Seiberg-Witten map and Boopp's shift method and the star product [23-43]:

$$\delta(f(x)*g(x)) = -\frac{i}{2}\theta^{\mu\nu}(\partial_\mu^x f(x))(\partial_\nu^p g(x)) - \frac{i}{2}\bar{\theta}^{\mu\nu}(\partial_\mu^p f(x))(\partial_\nu^p g(x)) \quad (1)$$

The antisymmetric parameters ($\theta^{\mu\nu}$ and $\bar{\theta}^{\mu\nu}$) are elements of matrixes (order $(N \times N)$), θ_{ij} and $\bar{\theta}_{ij}$ are equals $\frac{1}{2}\varepsilon^{ijk}\theta_k$ and $\frac{1}{2}\varepsilon^{ijk}\bar{\theta}_k$, respectively and both (μ and ν) are variants from one to dimensions of the space N . As a direct principal's result of the above equation:

$$\left[x_i, x_j \right]_* = i\theta_{ij} \text{ and } \left[\hat{p}_i, \hat{p}_j \right]_* = i\bar{\theta}_{ij} \quad (2)$$

In this present work, we want to study the family potential ($V(r) = \frac{A}{r^2} - \frac{B}{r} + Cr^k$ ($k = 0, -1, -2$)) in noncommutative 3D space and phase to discover the new symmetries which satisfied by Boopp's shift method instead of solving the (NC-3D) spaces and phases with star product, the Schrödinger equation will be treated by using directly star product procedure:

$$[\hat{x}_i, \hat{x}_j] = i\theta_{ij} \text{ and } [\hat{p}_i, \hat{p}_j] = i\bar{\theta}_{ij} \quad (3)$$

The star product replaced by usual product together with a Boopp's shift [31-43]:

$$\begin{aligned} \hat{x}_i &= x_i - \frac{\theta_{ij}}{2} p_j, \hat{p}_i = p_i - \frac{\bar{\theta}_{ij}}{2} x_j \\ [x_i, x_j] &= 0, [p_i, p_j] = 0 \text{ and } [x_i, p_j] = i\delta_{ij} \end{aligned}$$

The rest of present search is organized as follows: In next section, we briefly review the Schrödinger equation with family potential $V(r) = \frac{A}{r^2} - \frac{B}{r} + Cr^k$ ($k = 0, -1, -2$) in ordinary three dimensional spaces. The Section 3, by applying both Boopp's shift method we derive the deformed Hamiltonians of the Schrödinger equation with family potential $V(r)$ and we find the exact quantum spectrum of the bound states in (NC-3D) space and phase for studied potential. Finally, the important found results and the conclusions are discussed in the four and last section.

2. THE ORDINARY SOLUTIONS FOR THE POTENTIAL $V(r) = \frac{A}{r^2} - \frac{B}{r} + Cr^k$ ($k = 0, -1, -2$) IN THREE DIMENSIONAL SPACES

We starting this section by a brief review of time independent Schrödinger equation for an electron of rest mass m_0 and it's energy E_{nl} moving in the family potential ($V(r) = \frac{A}{r^2} - \frac{B}{r} + Cr^k$ ($k = 0, -1, -2$)), this potential for $C=0$ has been used to describe molecular structure and interactions [7-11], and also this potential has raised great interest in atomic and molecular physics for $A = 0$ and $k = 1$ [7,11], in another hand it is applied to examine the Zeeman quadratic effect and the magnetic field effect in the hydrogen atom [7,12]. On based to the principal reference [7], we can write the following differential equation for the radial function $R_{nl}(r)$ in ordinary 3D space:

$$\frac{d^2 R_{nl}(r)}{dr^2} + 2m_0 \left[E - \frac{A}{r^2} + \frac{B}{r} - Cr^k - \frac{l(l+1)}{2mr^2} \right] R_{nl}(r) = 0 \quad (5)$$

Where n and l are radial and orbital angular momentum quantum numbers. A , B and C are strictly positive constants [7]. By using the following ansatz:

$$-\varepsilon^2 = 2m_0 E, -\zeta_{nl}^2 = \varepsilon_{nl}^2 - \tilde{C}, \tilde{A} = 2m_0 A, \tilde{B} = 2m_0 B, \tilde{C} = 2m_0 C \quad (6)$$

Equation (5) becomes [7]:

$$\frac{d^2 R_{nl}(r)}{dr^2} + \left[-\varepsilon^2 - \frac{\tilde{A}}{r^2} + \frac{\tilde{B}}{r} - \tilde{C}r^k - \frac{l(l+1)}{r^2} \right] R_{nl}(r) = 0 \quad (7)$$

Now, we present the analytical solutions for the potential

$$V(r) = \frac{A}{r^2} - \frac{B}{r} + Cr^k \quad (k = 0, -1, -2) \quad [7-10, 13]$$

k	Eigenvalues	Eigenfunctions
0	$E_{nl} = C - \frac{m_0 B^2}{2} \left(n + \frac{1}{2} + \sqrt{\left(l + \frac{1}{2} \right)^2 + 2m_0 A} \right)^{-2}$	$R_{nl}(r) = N \exp(-\varepsilon_{nl}^{k=0} r) {}_1F_1(-n, 2\Lambda^{k=0} + 2; 2\varepsilon_{nl}^{k=0} r)$ (8.1)
-1	$E_{nl} = -\frac{m_0(B-C)^2}{2} \left(n + \frac{1}{2} + \sqrt{\left(l + \frac{1}{2} \right)^2 + 2m_0 A} \right)^{-2}$	$R_{nl}(r) = N \exp(-\varepsilon_{nl}^{k=-1} r) {}_1F_1(-n, 2\Lambda^{k=-1} + 2; 2\varepsilon_{nl}^{k=-1} r)$ (8.2)
-2	$E_{nl} = -\frac{m_0 B^2}{2} \left(n + \frac{1}{2} + \sqrt{\left(l + \frac{1}{2} \right)^2 + 2m_0(A+C)} \right)^{-2}$	$R_{nl}(r) = N \exp(-\varepsilon_{nl}^{k=-2} r) {}_1F_1(-n, 2\Lambda^{k=-2} + 2; 2\varepsilon_{nl}^{k=-2} r)$ (8.3)

With [7-10, 13]:

$$\begin{aligned}
 \varepsilon_{nl}^{k=0} &= \frac{\tilde{B}}{2(n+\Lambda+1)}, & \Lambda^{k=0} &= -\frac{1}{2} + \sqrt{\left(l + \frac{1}{2} \right)^2 + \tilde{A}} \\
 \varepsilon_{nl}^{k=-1} &= \frac{\tilde{B}-\tilde{C}}{2(n+\Lambda+1)}, & \Lambda^{k=-1} &= -\frac{1}{2} + \sqrt{\left(l + \frac{1}{2} \right)^2 + \tilde{A}} \\
 \varepsilon_{nl}^{k=-2} &= \frac{\tilde{B}}{2(n+\Lambda+1)}, & \Lambda^{k=-2} &= -\frac{1}{2} + \sqrt{\left(l + \frac{1}{2} \right)^2 + \tilde{A} + \tilde{C}}
 \end{aligned}
 \tag{9}$$

3. NONCOMMUTATIVE PHASE-SPACE HAMILTONIAN FOR FAMILY POTENTIAL

$$V(r) = \frac{A}{r^2} - \frac{B}{r} + Cr^k \quad (k = 0, -1, -2) :$$

Know, we shall present the fundamental principles of the quantum noncommutative Schrödinger equation which resumed in the following steps [31-43]:

- Ordinary Hamiltonian : $\hat{H}(p_i, x_i) \rightarrow$ NC - Hamiltonian : $\hat{H}(\hat{p}_i, \hat{x}_i)$
- Ordinary - complex wave function : $\Psi(\vec{r}) \rightarrow$ NC - complex wave function : $\tilde{\Psi}(\vec{r})$
- Ordinary - energy : $E \rightarrow$ NC - Energy : E_{nc-f}
- Ordinary - product \rightarrow New star product - acting on phase and space : *

Which allow us to writing the three dimensional space-phase quantum noncommutative Schrödinger equations as follows:

$$\hat{H}(\hat{p}_i, \hat{x}_i) * \tilde{\Psi}(\vec{r}) = E_{nc-ih} \tilde{\Psi}(\vec{r})
 \tag{11}$$

The Boopp's shift method permutes to reduce the above equation to the form:

$$H(\hat{p}_i, \hat{x}_i) \psi(\vec{r}) = E_{nc-f} \psi(\vec{r}) \Rightarrow H(\hat{p}_i, \hat{x}_i) R_{nl}(r) = E_{nc-f} R_{nl}(r)
 \tag{12}$$

Here the two \hat{x}_i and \hat{p}_i operators in (NC-3D) phase and space are given by [31-43]:

$$\hat{x}_i = x_i - \frac{\theta_{ij}}{2} p_j \quad \text{and} \quad \hat{p}_i = p_i - \frac{\bar{\theta}_{ij}}{2} x_j
 \tag{13}$$

It's convenient to introduce to notations: $\hat{x} \equiv \hat{x}_1, \hat{y} \equiv \hat{x}_2, \hat{z} \equiv x_2, \hat{p}_x \equiv p_1, \hat{p}_y \equiv p_2$ and $\hat{p}_z \equiv p_3$, apply equation (13) to obtains g, in (NC-3D) space and phase, the important 6-operators :

$$\begin{aligned}
 \hat{x} &= x - \frac{\theta_{12}}{2} p_y - \frac{\theta_{13}}{2} p_z, & \hat{y} &= y + \frac{\theta_{12}}{2} p_x + \frac{\theta_{13}}{2} p_z \\
 \hat{z} &= z + \frac{\theta_{13}}{2} p_x + \frac{\theta_{23}}{2} p_y, & \hat{p}_x &= p_x + \frac{\bar{\theta}_{21}}{2} y + \frac{\bar{\theta}_{31}}{2} z \\
 \hat{p}_y &= p_y + \frac{\bar{\theta}_{12}}{2} y + \frac{\bar{\theta}_{23}}{2} z \quad \text{and} \quad \hat{p}_z = p_z + \frac{\bar{\theta}_{13}}{2} x + \frac{\bar{\theta}_{23}}{2} y
 \end{aligned}
 \tag{14}$$

After straightforward calculations, one can derive, the two operators $\frac{1}{\hat{r}}$ and \hat{p}^2 in (NC-3D) spaces and phases as follows:

$$\frac{1}{\hat{r}} = \frac{1}{r} + \frac{\vec{\mathbf{L}}\vec{\Theta}}{4r^3}$$

$$\hat{p}^2 = p^2 + \vec{\mathbf{L}}\vec{\Theta}$$
(15)

Where $\mathbf{L}\Theta$ and $\vec{\mathbf{L}}\vec{\Theta}$ are given by, respectively:

$$\mathbf{L}\Theta \equiv L_x\Theta_{12} + L_y\Theta_{23} + L_z\Theta_{13}$$

$$\vec{\mathbf{L}}\vec{\Theta} \equiv L_x\bar{\theta}_{12} + L_y\bar{\theta}_{23} + L_z\bar{\theta}_{13}$$
(16)

Where $\Theta \equiv \frac{\Theta}{2}$, based, on the eq. (15), in the first order of two infinitesimal parameters Θ and $\bar{\theta}$, the four important terms which will be used to determine the noncommutative new family potential can be written explicitly, in (NC-3D) spaces and phases as:

$$\frac{A}{\hat{r}^2} = \frac{A}{r^2} + \frac{A}{r^4}\mathbf{L}\Theta$$

$$-\frac{B}{r'} = -\frac{B}{r} - \frac{\mathbf{B}\vec{\mathbf{L}}\vec{\Theta}}{4r^3}$$

$$C\hat{r}^k = Cr^k - k\frac{\mathbf{C}\vec{\mathbf{L}}\vec{\Theta}}{4r^{2-k}}$$

$$\frac{\hat{p}^2}{2m} = -\frac{\Delta}{2m} + \frac{\vec{\mathbf{L}}\vec{\Theta}}{2m}$$
(17)

Now, the global potential operator $H_{\text{pert}}(\hat{r})$ for the new family $V(\hat{r}) = \frac{A}{\hat{r}^2} - \frac{B}{\hat{r}} + C\hat{r}^k$ in both (NC-3D) phase and space will be written as:

$$V(\hat{r}) = \frac{A}{r^2} - \frac{B}{r} + Cr^k + \left(\frac{A}{r^4} - \frac{\mathbf{B}}{4r^3} - \frac{k\mathbf{C}}{4r^{2-k}} \right) \vec{\mathbf{L}}\vec{\Theta} + \frac{\vec{\mathbf{L}}\vec{\Theta}}{2m}$$
(18)

It's clearly, the three first terms are given the ordinary $V(r) = \frac{A}{r^2} - \frac{B}{r} + Cr^k$ in 3D spaces, while the rest terms are proportional's with two infinitesimals parameters (Θ and $\bar{\theta}$) and then gives the terms of perturbations $H_{\text{pert}}(r, \Theta, \bar{\theta})$ in (NC-3D) real space and phase as:

$$H_{\text{pert}}(r, \Theta, \bar{\theta}) = \left(\frac{A}{r^4} - \frac{\mathbf{B}}{4r^3} - \frac{k\mathbf{C}}{4r^{2-k}} \right) \vec{\mathbf{L}}\vec{\Theta} + \frac{\vec{\mathbf{L}}\vec{\Theta}}{2m}$$
(19)

Now we replace $\vec{\mathbf{L}}\vec{\Theta}$ and $\vec{\mathbf{L}}\vec{\Theta}$ by $2\Theta\vec{\mathbf{S}}\vec{\mathbf{L}}$ and $2\bar{\theta}\vec{\mathbf{S}}\vec{\mathbf{L}}$, respectively, to obtain the new forms of $H_{\text{pert}}(r, \Theta, \bar{\theta})$:

$$H_{\text{pert}}(r, \Theta, \bar{\theta}) = 2\left(\Theta \left(\frac{A}{2r^4} - \frac{\mathbf{B}}{8r^3} - \frac{k\mathbf{C}}{8r^{2-k}} \right) + \frac{\bar{\theta}}{4m} \right) \vec{\mathbf{L}}\vec{\mathbf{S}}$$
(20)

With $\vec{\mathbf{S}} = \frac{\vec{\mathbf{1}}}{2}$, it's possible also to replace $(\vec{\mathbf{S}}\vec{\mathbf{L}})$ by $\frac{1}{2}(\vec{\mathbf{J}}^2 - \vec{\mathbf{L}}^2 - \vec{\mathbf{S}}^2)$, which allow us to writing the perturbative terms $H_{\text{pert}}(r, \Theta, \bar{\theta})$ as follows:

$$H_{\text{pert}}(r, \Theta, \bar{\theta}) = \left(\Theta \left(\frac{A}{2r^4} - \frac{\mathbf{B}}{8r^3} - \frac{k\mathbf{C}}{8r^{2-k}} \right) + \frac{\bar{\theta}}{4m} \right) (\vec{\mathbf{J}}^2 - \vec{\mathbf{L}}^2 - \vec{\mathbf{S}}^2)$$
(21)

As it's known, this operator traduces the coupling between spin and orbital momentum. The $(\vec{\mathbf{J}}^2, \vec{\mathbf{L}}^2, \vec{\mathbf{S}}^2$ and $s_z)$ formed complete basis on quantum mechanics, then the operator $(\vec{\mathbf{J}}^2 - \vec{\mathbf{L}}^2 - \vec{\mathbf{S}}^2)$ will be gives two eigenvalues $L(j, l, s) = \frac{1}{2}$ and $L'(j, l, s) = -\frac{l+1}{2}$, corresponding $j = l + \frac{1}{2}$ (spin up) and $j = l - \frac{1}{2}$ (spin down), respectively. Then, one can form a diagonal matrix of order (3×3) , with

non null elements $(H(r, \vec{p}, \Theta, \bar{\theta}))_{11}$, $(H(r, \vec{p}, \Theta, \bar{\theta}))_{22}$ and $(H(r, \vec{p}, \Theta, \bar{\theta}))_{33}$ for new family

$V(\hat{r}) = \frac{A}{\hat{r}^2} - \frac{B}{\hat{r}} + C\hat{r}^k$ in both (NC-3D) phase and space:

$$\begin{aligned}
 (H(r, \vec{p}, \Theta, \bar{\theta}))_{11} &= -\frac{\Lambda}{2m_0} + \frac{A}{r^2} - \frac{B}{r} + Cr^k + \frac{l}{2} \left(\Theta \left(\frac{A}{2r^4} - \frac{\mathbf{B}}{8r^3} - \frac{k\mathbf{C}}{8r^{2-k}} \right) + \frac{\bar{\theta}}{4m_0} \right) \\
 \text{if } j &= l + \frac{1}{2} \Rightarrow \text{spin up} \\
 (H(r, \vec{p}, \Theta, \bar{\theta}))_{22} &= -\frac{\Lambda}{2m_0} + \frac{A}{r^2} - \frac{B}{r} + Cr^k - \frac{l+1}{2} \left(\Theta \left(\frac{A}{2r^4} - \frac{\mathbf{B}}{8r^3} - \frac{k\mathbf{C}}{8r^{2-k}} \right) + \frac{\bar{\theta}}{4m_0} \right) \\
 \text{if } j &= l - \frac{1}{2} \Rightarrow \text{spin down} \\
 (H(r, \vec{p}, \Theta, \bar{\theta}))_{33} &= -\frac{\Lambda}{2m_0} + \frac{A}{r^2} - \frac{B}{r} + Cr^k \rightarrow \text{Non-polarised-electron}
 \end{aligned}
 \tag{22}$$

After profound straightforward calculation, one can show that, the radial function $R_{nl}(r)$ satisfied the following the equation, in (NC-3D: RSP):

$$\frac{d^2 R_{nl}(r)}{dr^2} + 2m \left[E_{nc} - \frac{A}{r^2} + \frac{B}{r} - Cr^k - \left(\Theta \left(\frac{A}{2r^4} - \frac{\mathbf{B}}{8r^3} - \frac{k\mathbf{C}}{8r^{2-k}} \right) + \frac{\bar{\theta}}{4m} \right) \left(\vec{J}^2 - \vec{L}^2 - \vec{S}^2 \right) - \frac{l(l+1)}{2m_0 r^2} \right] R_{nl}(r) = 0. \tag{23}$$

4. NONCOMMUTATIVE SPECTRUM FOR NEW FAMILY $V(\hat{r}) = \frac{A}{\hat{r}^2} - \frac{B}{\hat{r}} + C\hat{r}^k$ IN (3D-NC : SP) :

4.1 NONCOMMUTATIVE SPIN-ORBITALSPECTRUM FOR NEW FAMILY

$V(\hat{r}) = \frac{A}{\hat{r}^2} - \frac{B}{\hat{r}} + C\hat{r}^k$ IN (3D-NC : SP) :

Know, we want to obtain the energies: E_{ncup} , E_{ncd} and E_{np} for a particle fermionic with spin up, spin down and non polarized at first order of two infinitesimals parameters (Θ and $\bar{\theta}$) corresponding $(H(r, \vec{p}, \Theta, \bar{\theta}))_{11}$, $(H(r, \vec{p}, \Theta, \bar{\theta}))_{22}$ and $(H(r, \vec{p}, \Theta, \bar{\theta}))_{33}$, respectively, by applying standard perturbation theory for new family potential $V(\hat{r}) = \frac{A}{\hat{r}^2} - \frac{B}{\hat{r}} + C\hat{r}^k$ in (NC-3D) phase and space, one can obtain the following noncommutative energy $(E_{ncup}(n, l, k = 0), E_{ncud}(n, l, k = 0), E_{np}(n, l, k = 0))$, $E_{ncup}(n, l, k = -1)$, $E_{ncud}(n, l, k = -1)$, $E_{np}(n, l, k = -1)$ and $(E_{ncup}(n, l, k = -2), E_{ncud}(n, l, k = -2), E_{np}(n, l, k = -2))$ for three studied case $k = 0$, $k = -1$ and $k = -2$, respectively:

k	Eigenvalues
0	$ \begin{aligned} E_{ncup}(n, l, k = 0) &= C - \frac{mB^2}{2} \left(n + \frac{1}{2} + \sqrt{\left(l + \frac{1}{2} \right)^2 + 2mA} \right)^{-2} + E_{pu}(k = 0) \\ E_{ncud}(n, l, k = 0) &= C - \frac{mB^2}{2} \left(n + \frac{1}{2} + \sqrt{\left(l + \frac{1}{2} \right)^2 + 2mA} \right)^{-2} + E_{pd}(k = 0) \\ E_{np}(n, l, k = 0) &= C - \frac{mB^2}{2} \left(n + \frac{1}{2} + \sqrt{\left(l + \frac{1}{2} \right)^2 + 2mA} \right)^{-2} \end{aligned} \tag{24} $

-1	$E_{ncup}(n, l, k = -1) = -\frac{m(B-C)^2}{2} \left(n + \frac{1}{2} + \sqrt{\left(l + \frac{1}{2} \right)^2 + 2mA} \right)^{-2} + E_{pu}(k = -1)$ $E_{ncud}(n, l, k = -1) = -\frac{m(B-C)^2}{2} \left(n + \frac{1}{2} + \sqrt{\left(l + \frac{1}{2} \right)^2 + 2mA} \right)^{-2} + E_{pd}(k = -1) \quad (25)$ $E_{np}(n, l, k = -1) = -\frac{m(B-C)^2}{2} \left(n + \frac{1}{2} + \sqrt{\left(l + \frac{1}{2} \right)^2 + 2mA} \right)^{-2}$
-2	$E_{ncup}(n, l, k = -2) = -\frac{mB^2}{2} \left(n + \frac{1}{2} + \sqrt{\left(l + \frac{1}{2} \right)^2 + 2m(A+C)} \right) + E_{pu}(k = -2)$ $E_{ncud}(n, l, k = -2) = -\frac{mB^2}{2} \left(n + \frac{1}{2} + \sqrt{\left(l + \frac{1}{2} \right)^2 + 2m(A+C)} \right) + E_{pd}(k = -2) \quad (26)$ $E_{np}(n, l, k = -2) = -\frac{mB^2}{2} \left(n + \frac{1}{2} + \sqrt{\left(l + \frac{1}{2} \right)^2 + 2m(A+C)} \right)$

Where $E_{pu}(k)$ And $E_{pd}(k)$ are the modifications to the energy levels, associate with spin up and spin down, respectively, at first order of $(\Theta$ and $\bar{\theta})$ obtained by applying the perturbation theory, as follows:

$$E_{pu}(k) = \frac{l}{2} \left(\Theta T_s(k) + \frac{\bar{\theta}}{2m} T_p(k) \right) \quad (27)$$

$$E_{pd}(k) = -\frac{l+1}{2} \left(\Theta T_s(k) + \frac{\bar{\theta}}{2m} T_p(k) \right)$$

Where $T_s(k)$ and $T_p(k)$ are given by:

k	$T_s(k)$ and $T_p(k)$
0	$T_s(k = 0) = N ^2 \int_0^{+\infty} \exp(-2\varepsilon_{nl}^{k=0} r) [F_1(-n, 2\Lambda^{k=0} + 2; 2\varepsilon_{nl}^{k=0} r)]^2 \left(\frac{A}{2r^4} - \frac{B}{8r^3} \right) r^2 dr$ $T_p(k = 0) = N ^2 \int_0^{+\infty} \exp(-2\varepsilon_{nl}^{k=0} r) [F_1(-n, 2\Lambda^{k=0} + 2; 2\varepsilon_{nl}^{k=0} r)]^2 r^2 dr \quad (28)$
-1	$T_s(k = -1) = N ^2 \int_0^{+\infty} \exp(-2\varepsilon_{nl}^{k=-1} r) [F_1(-n, 2\Lambda^{k=-1} + 2; 2\varepsilon_{nl}^{k=-1} r)]^2 \left(\frac{A}{2r^4} - \frac{B}{8r^3} + \frac{C}{8r^3} \right) r^2 dr$ $T_p(k = -1) = N ^2 \int_0^{+\infty} \exp(-2\varepsilon_{nl}^{k=-1} r) [F_1(-n, 2\Lambda^{k=-1} + 2; 2\varepsilon_{nl}^{k=-1} r)]^2 r^2 dr \quad (29)$
-2	$T_s(k = -2) = N ^2 \int_0^{+\infty} \exp(-2\varepsilon_{nl}^{k=-2} r) [F_1(-n, 2\Lambda^{k=-2} + 2; 2\varepsilon_{nl}^{k=-2} r)]^2 \left(\frac{A}{2r^4} - \frac{B}{8r^3} + \frac{C}{4r^4} \right) r^2 dr$ $T_p(k = -2) = N ^2 \int_0^{+\infty} \exp(-2\varepsilon_{nl}^{k=-2} r) [F_1(-n, 2\Lambda^{k=-2} + 2; 2\varepsilon_{nl}^{k=-2} r)]^2 r^2 dr \quad (30)$

A direct simplification gives:

k	$T_s(k)$ and $T_p(k)$
0	$T_s(k=0) = N ^2 \int_0^{+\infty} \exp(-2\varepsilon_{nl}^{k=0} r) [F_1(-n, 2\Lambda^{k=0} + 2; 2\varepsilon_{nl}^{k=0} r)]^2 \left(\frac{A}{2} r^{-1-1} - \frac{\mathbf{B}}{8} r^{0-1} \right) dr$ $T_p(k=0) = N ^2 \int_0^{+\infty} \exp(-2\varepsilon_{nl}^{k=0} r) [F_1(-n, 2\Lambda^{k=0} + 2; 2\varepsilon_{nl}^{k=0} r)]^2 r^{3-1} dr$ (31)
-1	$T_s(k=-1) = N ^2 \int_0^{+\infty} \exp(-2\varepsilon_{nl}^{k=-1} r) [F_1(-n, 2\Lambda^{k=-1} + 2; 2\varepsilon_{nl}^{k=-1} r)]^2 \left(\frac{A}{2} r^{-1-1} - \frac{\mathbf{B} + \mathbf{C}}{8} r^{0-1} \right) dr$ $T_p(k=-1) = N ^2 \int_0^{+\infty} \exp(-2\varepsilon_{nl}^{k=-1} r) [F_1(-n, 2\Lambda^{k=-1} + 2; 2\varepsilon_{nl}^{k=-1} r)]^2 r^{3-1} dr$ (32)
-2	$T_s(k=-2) = N ^2 \int_0^{+\infty} \exp(-2\varepsilon_{nl}^{k=-2} r) [F_1(-n, 2\Lambda^{k=-2} + 2; 2\varepsilon_{nl}^{k=-2} r)]^2 \left(\frac{2A + C}{4} r^{-1-1} - \frac{\mathbf{B}}{8} r^{0-1} \right) dr$ $T_p(k=-2) = N ^2 \int_0^{+\infty} \exp(-2\varepsilon_{nl}^{k=-2} r) [F_1(-n, 2\Lambda^{k=-2} + 2; 2\varepsilon_{nl}^{k=-2} r)]^2 r^{3-1} dr$ (33)

It's convenient to writing the above table to the following form:

k	$T_s(k)$ and $T_p(k)$
0	$T_s(k=0) = \frac{ N ^2}{2\varepsilon_{nl}^{k=0}} \int_0^{+\infty} \exp(-v_1) [F_1(-n, \gamma_1; v_1)]^2 \left(\frac{A(2\varepsilon_{nl}^{k=0})^2}{2} v_1^{-1-1} - \frac{\mathbf{B}\varepsilon_{nl}^{k=0}}{4} v_1^{0-1} \right) dv_1$ $T_p(k=0) = \frac{ N ^2}{(2\varepsilon_{nl}^{k=0})^3} \int_0^{+\infty} \exp(-v_1) [F_1(-n, \gamma_1; v_1)]^2 v_1^{3-1} dv_1$ (34)
-1	$T_s(k=-1) = \frac{ N ^2}{2\varepsilon_{nl}^{k=-1}} \int_0^{+\infty} \exp(-v_2) [F_1(-n, \gamma_2; v_2)]^2 \left(\frac{A(2\varepsilon_{nl}^{k=-1})^2}{2} v_2^{-1-1} - \frac{(\mathbf{B} + \mathbf{C})\varepsilon_{nl}^{k=-1}}{4} v_2^{0-1} \right) dv_2$ $T_p(k=-1) = \frac{ N ^2}{(2\varepsilon_{nl}^{k=-1})^3} \int_0^{+\infty} \exp(-v_2) [F_1(-n, \gamma_2; v_2)]^2 v_2^{3-1} dv_2$ (35)
-2	$T_s(k=-2) = \frac{ N ^2}{2\varepsilon_{nl}^{k=-2}} \int_0^{+\infty} \exp(-v_3) [F_1(-n, \gamma_3; v_3)]^2 \left(\frac{(2A + C)\varepsilon_{nl}^{k=-2}}{2} r^{-1-1} - \frac{\mathbf{B}\varepsilon_{nl}^{k=-2}}{4} r^{0-1} \right) dv_3$ $T_p(k=-2) = \frac{ N ^2}{(2\varepsilon_{nl}^{k=-2})^3} \int_0^{+\infty} \exp(-v_3) [F_1(-n, \gamma_3; v_3)]^2 v_3^{3-1} dv_3$ (36)

Where $(v_1 = 2\varepsilon_{nl}^{k=0} r, \gamma_1 = 2\Lambda^{k=0} + 2), (v_2 = 2\varepsilon_{nl}^{k=-1} r, \gamma_2 = 2\Lambda^{k=-1} + 2)$ and $(v_3 = 2\varepsilon_{nl}^{k=-2} r, \gamma_2 = 2\Lambda^{k=-2} + 2)$

Applying the following special integration [44]:

$$\int_0^{+\infty} v^{\alpha-1} \exp(-v) [F_1(-n, \gamma, v)]^2 dv = \frac{n! \Gamma(\alpha)}{\gamma(\gamma+1) \dots (\gamma+n-1)} + 1 + \frac{n(\gamma-\alpha-1)(\gamma-\alpha)}{1^2 \gamma} + \frac{n(n-1)(\gamma-\alpha-2)(\gamma-\alpha-1)(\gamma-\alpha)(\gamma-\alpha+1)}{1^2 2^2 \gamma(\gamma+1)}$$
(37)

After straightforward calculations, we can obtain the results:

k	$T_s(k)$ and $T_p(k)$
0	$T_s(k=0) = \frac{ N ^2 \Theta A (2\varepsilon_{nl}^{k=0})^2}{4\varepsilon_{nl}^{k=0}} T_1 + \frac{ N ^2 \Theta B \varepsilon_{nl}^{k=0}}{8\varepsilon_{nl}^{k=0}} T_2$ $T_p(k=0) = \frac{ N ^2 \bar{\theta}}{(2\varepsilon_{nl}^{k=0})^3 m} T_3$ (38)
-1	$T_s(k=-1) = \frac{ N ^2}{2\varepsilon_{nl}^{k=-1}} \left(\frac{A(2\varepsilon_{nl}^{k=-1})^2}{2} T_4 - \frac{(B+C)\varepsilon_{nl}^{k=-1}}{4} T_5 \right)$ $T_p(k=-1) = \frac{ N ^2}{(2\varepsilon_{nl}^{k=-1})^3} T_6$ (39)
-2	$T_s(k=-2) = \frac{ N ^2}{2\varepsilon_{nl}^{k=-2}} T_7 \frac{(2A+C)\varepsilon_{nl}^{k=-2}}{2} - \frac{ N ^2}{2\varepsilon_{nl}^{k=-2}} \frac{B\varepsilon_{nl}^{k=-2}}{4} T_8$ $T_p(k=-2) = \frac{ N ^2}{(2\varepsilon_{nl}^{k=-2})^3} T_9$ (40)

With:

$$T_1 = \frac{n! \Gamma(-1)}{\gamma_1(\gamma_1+1) \dots (\gamma_1+n-1)} + 1 + \frac{n(\gamma_1-2)(\gamma_1-1)}{\gamma_1} + \frac{n(n-1)(\gamma_1-3)(\gamma_1-2)(\gamma_1-1)(\gamma_1)}{1^2 2^2 \gamma(\gamma+1)}$$

$$T_2 = \frac{n! \Gamma(0)}{\gamma_1(\gamma_1+1) \dots (\gamma_1+n-1)} + 1 + 1 + \frac{n(\gamma_1-2)(\gamma_1-1)}{\gamma_1} + \frac{n(n-1)(\gamma_1-3)(\gamma_1-2)(\gamma_1-1)(\gamma_1)}{1^2 2^2 \gamma(\gamma+1)}$$

$$T_3 = \frac{n! \Gamma(-1)}{\gamma(\gamma_1+1) \dots (\gamma_1+n-1)} + 1 + 1 + \frac{n(\gamma_1-2)(\gamma_1-1)}{\gamma_1} + \frac{n(n-1)(\gamma_1-3)(\gamma_1-2)(\gamma_1-1)(\gamma_1)}{1^2 2^2 \gamma(\gamma+1)}$$
(41.1)

And

$$T_4 = \frac{n! \Gamma(-1)}{\gamma_2(\gamma_2+1) \dots (\gamma_2+n-1)} + 1 + \frac{n(\gamma_2-2)(\gamma_2-1)}{\gamma_2} + \frac{n\gamma_2^2(n-1)(\gamma_2-3)(\gamma_2-1)}{4\gamma_2(\gamma_2+1)}$$

$$T_5 = \frac{n! \Gamma(0)}{\gamma_2(\gamma_2+1) \dots (\gamma_2+n-1)} + 1 + \frac{n(\gamma_2-2)(\gamma_2-1)}{\gamma_2} + \frac{n\gamma_2^2(n-1)(\gamma_2-3)(\gamma_2-1)}{4\gamma_2(\gamma_2+1)}$$

$$T_6 = \frac{n! \Gamma(3)}{\gamma_2(\gamma_2+1) \dots (\gamma_2+n-1)} + 1 + \frac{n(\gamma_2-2)(\gamma_2-1)}{\gamma_2} + \frac{n\gamma_2^2(n-1)(\gamma_2-3)(\gamma_2-1)}{4\gamma_2(\gamma_2+1)}$$
(41.2)

And

$$T_7 = \frac{n! \Gamma(1)}{\gamma_3(\gamma_3+1) \dots (\gamma_3+n-1)} + 1 + \frac{n(\gamma_3-2)(\gamma_3-1)}{\gamma_3} + \frac{n\gamma_3^2(n-1)(\gamma_3-3)(\gamma_3-1)}{4\gamma_3(\gamma_3+1)}$$

$$T_8 = \frac{n! \Gamma(0)}{\gamma_3(\gamma_3+1) \dots (\gamma_3+n-1)} + 1 + \frac{n(\gamma_3-2)(\gamma_3-1)}{\gamma_3} + \frac{n\gamma_3^2(n-1)(\gamma_3-3)(\gamma_3-1)}{4\gamma_3(\gamma_3+1)}$$

$$T_9 = \frac{n! \Gamma(3)}{\gamma_3(\gamma_3+1) \dots (\gamma_3+n-1)} + 1 + \frac{n(\gamma_3-2)(\gamma_3-1)}{\gamma_3} + \frac{n\gamma_3^2(n-1)(\gamma_3-3)(\gamma_3-1)}{4\gamma_3(\gamma_3+1)}$$
(41.3)

The modification of the energy levels, associate with spin up , spin down and non polarized electron, at first order of Θ and $\bar{\theta}$ for new family potential $V(\hat{r}) = \frac{A}{\hat{r}^2} - \frac{B}{\hat{r}} + C\hat{r}^k$ in (NC-3D) phase and space are obtained from Eqs.(38), (39) and (40) as follows:

k	Modification of Eigenvalues
0	$E_{pu}(k=0) = \frac{l N ^2}{2} \left(\Theta \frac{A(2\varepsilon_{nl}^{k=0})^2}{4\varepsilon_{nl}^{k=0}} T_1 + \Theta \frac{\mathbf{B}\varepsilon_{nl}^{k=0}}{8\varepsilon_{nl}^{k=0}} T_2 + \frac{\bar{\theta}}{2(2\varepsilon_{nl}^{k=0})^3 m_0} T_3 \right) \quad (42)$ $E_{pd}(k=0) = -\frac{(l+1) N ^2}{2} \left(\Theta \frac{A(2\varepsilon_{nl}^{k=0})^2}{4\varepsilon_{nl}^{k=0}} T_1 + \Theta \frac{\mathbf{B}\varepsilon_{nl}^{k=0}}{8\varepsilon_{nl}^{k=0}} T_2 + \frac{\bar{\theta}}{2(2\varepsilon_{nl}^{k=0})^3 m_0} T_3 \right)$
-1	$E_{pu}(k=-1) = \frac{l N ^2}{2} \left(\Theta \frac{A(2\varepsilon_{nl}^{k=-1})^2}{4\varepsilon_{nl}^{k=-1}} T_4 - \Theta \frac{(\mathbf{B}+\mathbf{C})\varepsilon_{nl}^{k=-1}}{8\varepsilon_{nl}^{k=-1}} T_5 + \frac{\bar{\theta}}{(2\varepsilon_{nl}^{k=-1})^3 m_0} T_6 \right) \quad (43)$ $E_{pd}(k=-1) = -\frac{(l+1) N ^2}{2} \left(\Theta \frac{A(2\varepsilon_{nl}^{k=-1})^2}{4\varepsilon_{nl}^{k=-1}} T_4 - \Theta \frac{(\mathbf{B}+\mathbf{C})\varepsilon_{nl}^{k=-1}}{8\varepsilon_{nl}^{k=-1}} T_5 + \frac{\bar{\theta}}{(2\varepsilon_{nl}^{k=-1})^3 m_0} T_6 \right)$
-2	$E_{pu}(k=-2) = \frac{l N ^2}{2} \left(\frac{\Theta(2A+C)}{4} T_7 - \frac{\Theta}{8} T_8 + \frac{\bar{\theta}}{(2\varepsilon_{nl}^{k=-2})^3 m_0} T_9 \right) \quad (44)$ $E_{pd}(k=-2) = -\frac{(l+1) N ^2}{2} \left(\frac{\Theta(2A+C)}{4} T_7 - \frac{\Theta}{8} T_8 + \frac{\bar{\theta}}{(2\varepsilon_{nl}^{k=-2})^3 m_0} T_9 \right)$

The first two terms in above six equations are proportional's with infinitesimal parameter Θ , which represent the modifications of Spectrum for noncommutative space while the rest parts are proportional's to second infinitesimal parameter $\bar{\theta}$ which represent the modifications of Spectrum for noncommutative phase. We conclude, from Eqs. (24), (25), (26), (42), (43) and (44) that, the total energy of electron with spin up and down ($E_{\text{ncup}}(n, l, k=0)$, $E_{\text{ncud}}(n, l, k=0)$, $E_{\text{np}}(n, l, k=0)$), $E_{\text{ncup}}(n, l, k=-1)$, $E_{\text{ncud}}(n, l, k=-1)$, $E_{\text{np}}(n, l, k=-1)$) and ($E_{\text{ncup}}(n, l, k=-2)$, $E_{\text{ncud}}(n, l, k=-2)$, $E_{\text{np}}(n, l, k=-2)$) corresponding $(H(r, \vec{p}, \Theta, \bar{\theta}))_{11}$, $(H(r, \vec{p}, \Theta, \bar{\theta}))_{22}$ and $(H(r, \vec{p}, \Theta, \bar{\theta}))_{33}$ respectively for new family potential $V(\hat{r}) = \frac{A}{\hat{r}^2} - \frac{B}{\hat{r}} + C\hat{r}^k$ in (NC-3D) phase and space:

k	Non Commutative Spin-Orbital Eigenvalues
0	$E_{\text{ncup}}(n, l, k=0) = C - \frac{m_0 B^2}{2} \left(n + \frac{1}{2} + \sqrt{\left(l + \frac{1}{2} \right)^2 + 2m_0 A} \right)^{-2} + \frac{l N ^2}{2} \left(\Theta \frac{A(2\varepsilon_{nl}^{k=0})^2}{4\varepsilon_{nl}^{k=0}} T_1 + \Theta \frac{\mathbf{B}\varepsilon_{nl}^{k=0}}{8\varepsilon_{nl}^{k=0}} T_2 + \frac{\bar{\theta}}{2(2\varepsilon_{nl}^{k=0})^3 m_0} T_3 \right) \quad (45)$ $E_{\text{ncud}}(n, l, k=0) = C - \frac{m_0 B^2}{2} \left(n + \frac{1}{2} + \sqrt{\left(l + \frac{1}{2} \right)^2 + 2m_0 A} \right)^{-2} - \frac{(l+1) N ^2}{2} \left(\Theta \frac{A(2\varepsilon_{nl}^{k=0})^2}{4\varepsilon_{nl}^{k=0}} T_1 + \Theta \frac{\mathbf{B}\varepsilon_{nl}^{k=0}}{8\varepsilon_{nl}^{k=0}} T_2 + \frac{\bar{\theta}}{2(2\varepsilon_{nl}^{k=0})^3 m_0} T_3 \right)$ $E_{\text{np}}(n, l, k=0) = C - \frac{m_0 B^2}{2} \left(n + \frac{1}{2} + \sqrt{\left(l + \frac{1}{2} \right)^2 + 2m_0 A} \right)^{-2}$
-1	$E_{\text{ncup}}(n, l, k=1) = -\frac{m_0(B-C)^2}{2} \left(n + \frac{1}{2} + \sqrt{\left(l + \frac{1}{2} \right)^2 + 2m_0 A} \right)^{-2} + \frac{l N ^2}{2} \left(\Theta \frac{A(2\varepsilon_{nl}^{k=-1})^2}{4\varepsilon_{nl}^{k=-1}} T_4 - \Theta \frac{(\mathbf{B}+\mathbf{C})\varepsilon_{nl}^{k=-1}}{8\varepsilon_{nl}^{k=-1}} T_5 + \frac{\bar{\theta}}{(2\varepsilon_{nl}^{k=-1})^3 m_0} T_6 \right) \quad (46)$ $E_{\text{ncud}}(n, l, k=1) = -\frac{m_0(B-C)^2}{2} \left(n + \frac{1}{2} + \sqrt{\left(l + \frac{1}{2} \right)^2 + 2m_0 A} \right)^{-2} - \frac{(l+1) N ^2}{2} \left(\Theta \frac{A(2\varepsilon_{nl}^{k=-1})^2}{4\varepsilon_{nl}^{k=-1}} T_4 - \Theta \frac{(\mathbf{B}+\mathbf{C})\varepsilon_{nl}^{k=-1}}{8\varepsilon_{nl}^{k=-1}} T_5 + \frac{\bar{\theta}}{(2\varepsilon_{nl}^{k=-1})^3 m_0} T_6 \right)$ $E_{\text{np}}(n, l, k=1) = -\frac{m_0(B-C)^2}{2} \left(n + \frac{1}{2} + \sqrt{\left(l + \frac{1}{2} \right)^2 + 2m_0 A} \right)^{-2}$

-2	$E_{\text{ncup}}(n, l, k = -2) = -\frac{m_0 B^2}{2} \left(n + \frac{1}{2} + \sqrt{\left(l + \frac{1}{2} \right)^2 + 2m_0(A+C)} \right) + \frac{ N ^2}{2} \left(\Theta \frac{(2A+C)}{4} T_7 - \frac{\Theta}{8} T_8 + \frac{\bar{\theta}}{(2\varepsilon_{nl}^{k=-2})^3 m_0} T_9 \right)$ $E_{\text{ncud}}(n, l, k = -2) = -\frac{m_0 B^2}{2} \left(n + \frac{1}{2} + \sqrt{\left(l + \frac{1}{2} \right)^2 + 2m_0(A+C)} \right) - \frac{(l+1) N ^2}{2} \left(\Theta \frac{(2A+C)}{4} T_7 - \frac{\Theta}{8} T_8 + \frac{\bar{\theta}}{(2\varepsilon_{nl}^{k=-2})^3 m_0} T_9 \right) \quad (47)$ $E_{\text{np}}(n, l, k = -2) = -\frac{m_0 B^2}{2} \left(n + \frac{1}{2} + \sqrt{\left(l + \frac{1}{2} \right)^2 + 2m_0(A+C)} \right)$
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4.2 NONCOMMUTATIVE MAGNETICSPECTRUM FOR NEW FAMILY

$V(\hat{r}) = \frac{A}{\hat{r}^2} - \frac{B}{\hat{r}} + C\hat{r}^k$ IN (3D-NC : SP) :

In another hand, it's possible to consider the two at infinitesimals parameters (Θ and $\bar{\theta}$) are the sum of two infinitesimals parameters to each one as [33-37]:

$$\Theta = \Theta_1 + \Theta_2 \text{ and } \bar{\theta} = \bar{\theta}_1 + \bar{\theta}_2 \quad (48)$$

Furthermore, if we choose both Θ_2 and $\bar{\theta}_2$ are proportional's to an external magnetic field as [33-37]:

$$\Theta_2 = \alpha_2 B \text{ , } \bar{\theta}_2 = \bar{\varepsilon}_2 B \text{ and } \vec{B} = B\vec{k} \quad (49)$$

Which allow us to obtain the following results:

$$\left(\frac{A}{r^4} - \frac{\mathbf{B}}{4r^3} - \frac{k\mathbf{C}}{4r^{2-k}} \right) \vec{L}\vec{\Theta}_2 + \frac{\vec{L}\vec{\bar{\theta}}_2}{2m} \rightarrow \left(\alpha_2 \left(\frac{A}{r^4} - \frac{\mathbf{B}}{4r^3} - \frac{k\mathbf{C}}{4r^{2-k}} \right) + \frac{\bar{\varepsilon}_2}{2m_0} \right) BL_z \quad (50)$$

Here α_2 and $\bar{\varepsilon}_2$ are just infinitesimal real proportional constants, the magnetic moment $\vec{\mu} \equiv \frac{1}{2}$ and $(-\vec{s}\vec{B})$ denote to the ordinary Hamiltonian of Zeeman Effect. A similarly calculations give the spectrum produced by the effect of the external magnetic field B as:

k	Non commutative Magnetic Eigenvalues
0	$E_{\text{pmag}}(k = 0) = N ^2 m \left(\Theta_2 \frac{A(2\varepsilon_{nl}^{k=0})^2}{4\varepsilon_{nl}^{k=0}} T_1 + \Theta_2 \frac{\mathbf{B}\varepsilon_{nl}^{k=0}}{8\varepsilon_{nl}^{k=0}} T_2 + \frac{\bar{\theta}_2}{2(2\varepsilon_{nl}^{k=0})^3 m_0} T_3 \right) \quad (51)$
-1	$E_{\text{pmag}}(k = -1) = N ^2 m \left(\Theta_2 \frac{A(2\varepsilon_{nl}^{k=-1})^2}{4\varepsilon_{nl}^{k=-1}} T_4 - \Theta_2 \frac{(\mathbf{B} + \mathbf{C})\varepsilon_{nl}^{k=-1}}{8\varepsilon_{nl}^{k=-1}} T_5 + \frac{\bar{\theta}_2}{(2\varepsilon_{nl}^{k=-1})^3 m_0} T_6 \right) \quad (52)$
-2	$E_{\text{pmag}}(k = -2) = N ^2 m \left(\Theta_2 \frac{(2A+C)}{4} T_7 - \frac{\Theta_2}{8} T_8 + \frac{\bar{\theta}_2}{(2\varepsilon_{nl}^{k=-2})^3 m_0} T_9 \right) \quad (53)$

Where $(-l \leq m \leq +l)$ denote to the quantum number of the operator L_z . Now, it is possible to resume the global noncommutative spectrum $(E_{\text{Tncup}}(n, l, k = 0), E_{\text{Tncud}}(n, l, k = 0), E_{\text{np}}(n, l, k = 0))$, $(E_{\text{Tncup}}(n, l, k = -1), E_{\text{Tncud}}(n, l, k = -1))$ and $(E_{\text{Tncup}}(n, l, k = -2), E_{\text{Tncud}}(n, l, k = -2), E_{\text{np}}(n, l, k = -2))$ for new family potential $V(\hat{r}) = \frac{A}{\hat{r}^2} - \frac{B}{\hat{r}} + C\hat{r}^k$ in (NC-3D) phase and space:

k	Global Non commutative Spectrum Eigenvalues
0	$E_{\text{Tncup}}(n, l, k = 0) = C - \frac{mB^2}{2} \left(n + \frac{1}{2} + \sqrt{\left(l + \frac{1}{2} \right)^2 + 2mA} \right)^{-2} + E_{pu}(k = 0) + E_{pmag}(k = 0)$ $E_{\text{Tncud}}(n, l, k = 0) = C - \frac{mB^2}{2} \left(n + \frac{1}{2} + \sqrt{\left(l + \frac{1}{2} \right)^2 + 2mA} \right)^{-2} + E_{pd}(k = 0) + E_{pmag}(k = 0)$ $E_{np}(n, l, k = 0) = C - \frac{mB^2}{2} \left(n + \frac{1}{2} + \sqrt{\left(l + \frac{1}{2} \right)^2 + 2mA} \right)^{-2}$ <div style="text-align: right;">(54)</div>
-1	$E_{\text{Tncup}}(n, l, k = -1) = -\frac{m(B-C)^2}{2} \left(n + \frac{1}{2} + \sqrt{\left(l + \frac{1}{2} \right)^2 + 2mA} \right)^{-2} + E_{pu}(k = -1) + E_{pmag}(n, l, k = -1)$ $E_{\text{Tncud}}(n, l, k = -1) = -\frac{m(B-C)^2}{2} \left(n + \frac{1}{2} + \sqrt{\left(l + \frac{1}{2} \right)^2 + 2mA} \right)^{-2} + E_{pd}(k = -1) + E_{pmag}(n, l, k = -1)$ $E_{np}(n, l, k = -1) = -\frac{m(B-C)^2}{2} \left(n + \frac{1}{2} + \sqrt{\left(l + \frac{1}{2} \right)^2 + 2mA} \right)^{-2}$ <div style="text-align: right;">(55)</div>
-2	$E_{\text{Tncup}}(n, l, k = -2) = -\frac{mB^2}{2} \left(n + \frac{1}{2} + \sqrt{\left(l + \frac{1}{2} \right)^2 + 2m(A+C)} \right)^{-2} + E_{pu}(k = -2) + E_{pmag}(n, l, k = -2)$ $E_{\text{Tncud}}(n, l, k = -2) = -\frac{mB^2}{2} \left(n + \frac{1}{2} + \sqrt{\left(l + \frac{1}{2} \right)^2 + 2m(A+C)} \right)^{-2} + E_{pd}(k = -2) + E_{pmag}(n, l, k = -2)$ $E_{np}(n, l, k = -2) = -\frac{mB^2}{2} \left(n + \frac{1}{2} + \sqrt{\left(l + \frac{1}{2} \right)^2 + 2m(A+C)} \right)^{-2}$ <div style="text-align: right;">(56)</div>

4.3 GLOBAL QUNTUM NONCOMMUTATUVE HAMILTONIAN OPERATOR FOR NEW

FAMILY $V(\hat{r}) = \frac{A}{\hat{r}^2} - \frac{B}{\hat{r}} + C\hat{r}^k$ IN (3D-NC : SP) :

Regarding the usual spectrum and the obtained two parts of new spectrum corresponding spin-orbital interaction and new magnetic field, one can form the new noncommutative quantum Hamiltonian H_{NC-nf} for new family potential $V(\hat{r}) = \frac{A}{\hat{r}^2} - \frac{B}{\hat{r}} + C\hat{r}^k$ in NC (3D-SP) as:

$$H_{NC-nf} = H_{c-f} + H_{so-nf} + H_{nfm} \tag{57}$$

Where H_{c-f} , H_{so-nf} and H_{nfm} are determined from, the following relation, respectively:

$$H_{(h.p.i)} = \left(-\frac{A}{2m_0} + \frac{A}{r^2} - \frac{B}{r} + Cr^k \right) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$H_{so-(h.p.i)} = \left(\Theta \left(\frac{A}{2r^4} - \frac{B}{8r^3} - \frac{kC}{8r^{2-k}} \right) + \frac{\bar{\theta}}{4m} \right) \begin{pmatrix} \frac{l}{2} & 0 & 0 \\ 0 & -\frac{l+1}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$H_{nfm} = \left(\alpha_2 \left(\frac{A}{r^4} - \frac{B}{4r^3} - \frac{kC}{4r^{2-k}} \right) + \frac{\bar{\epsilon}_2}{2m_0} \right) BL_z$$

(58)

Finally, the complete noncommutative quantum Hamiltonian will be anisotropic diagonal matrix of order (3×3) , with non null elements $(H_{ncf}(r, \vec{p}, \Theta, \bar{\theta}))_{11}$, $(H_{ncf}(r, \vec{p}, \Theta, \bar{\theta}))_{22}$ and $(H_{ncf}(r, \vec{p}, \Theta, \bar{\theta}))_{33}$

for new family $V(\hat{r}) = \frac{A}{\hat{r}^2} - \frac{B}{\hat{r}} + C\hat{r}^k$ in both (NC-3D) phase and space:

$$\begin{aligned} (H_{ncf}(r, \vec{p}, \Theta, \bar{\theta}))_{11} &= -\frac{\Delta}{2m_0} + \frac{A}{r^2} - \frac{B}{r} + Cr^k + \frac{l}{2} \left(\Theta \left(\frac{A}{2r^4} - \frac{B}{8r^3} - \frac{kC}{8r^{2-k}} \right) + \frac{\bar{\theta}}{4m_0} \right) + \left(\alpha_2 \left(\frac{A}{r^4} - \frac{B}{4r^3} - \frac{kC}{4r^{2-k}} \right) + \frac{\bar{\varepsilon}_2}{2m_0} \right) BL_z \\ \text{if } j = l + \frac{1}{2} &\Rightarrow \text{spin up} \\ (H_{ncf}(r, \vec{p}, \Theta, \bar{\theta}))_{22} &= -\frac{\Delta}{2m_0} + \frac{A}{r^2} - \frac{B}{r} + Cr^k - \frac{l+1}{2} \left(\Theta \left(\frac{A}{2r^4} - \frac{B}{8r^3} - \frac{kC}{8r^{2-k}} \right) + \frac{\bar{\theta}}{4m_0} \right) + \left(\alpha_2 \left(\frac{A}{r^4} - \frac{B}{4r^3} - \frac{kC}{4r^{2-k}} \right) + \frac{\bar{\varepsilon}_2}{2m_0} \right) BL_z \\ \text{if } j = l - \frac{1}{2} &\Rightarrow \text{spin down} \\ (H_{ncf}(r, \vec{p}, \Theta, \bar{\theta}))_{33} &= -\frac{\Delta}{2m_0} + \frac{A}{r^2} - \frac{B}{r} + Cr^k \rightarrow \text{Non-polarised-electron} \end{aligned} \quad (59)$$

We have seen that, $-l \leq m \leq +l$, then can be take $(2l+1)$ values correspond the magnetic effect and for the effect spin-orbital interaction and we have two possible values $j = l \pm \frac{1}{2}$, thus every state in usually 3D of energy for new family potential $V(\hat{r}) = \frac{A}{\hat{r}^2} - \frac{B}{\hat{r}} + C\hat{r}^k$ in NC (3D-SP) will be in (NC-3D) phase and space: $2(2l+1)$ sub-states, this is similarly to my work [34].

5. CONCLUSION:

In this work, the effect of the non commutativity was studied for the new family potential $V(\hat{r}) = \frac{A}{\hat{r}^2} - \frac{B}{\hat{r}} + C\hat{r}^k$ in NC (3D-SP) in (NC-3D) real space and phase, we shown that the NC global quantum Hamiltonian was anisotropic diagonal matrix of order (3×3) and the corresponding quantum atomic spectrum changed totally, every state in usually 3D was replaced by $2(2l+1)$ sub-states, describing three modes of electron: electron with spin up, electron with spin down and non-polarized electron,. Thus, the applications of new family potential $V(\hat{r}) = \frac{A}{\hat{r}^2} - \frac{B}{\hat{r}} + C\hat{r}^k$ in (NC-3D) phase and space are prolonged to be very large at height energy and also for low energy.

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