Mixed Convection of Heat Transfer in a Square Lid-Driven Cavity

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ABSTRACT. Three dimensional steady state mixed convection in a lid driven cubical cavity heating from below has been investigated numerically. Two sided walls are maintained at a constant ambient temperature $T_{\text{top}} > T_{\text{bottom}}$, while the vertical walls are thermally insulated. Governing equations expressing in a dimensionless form are solved by using finite element method. The Reynolds number is fixed at $Re=100$, while the Richardson number is varied from 0.001 to 10. Parametric studies focusing on the effect of the Richardson number on the fluid flow and heat transfer have been performed. The flow and heat transfer characteristics, expressed in terms of streamlines, isotherms and average wall Nusselt number are presented for the entire range of Richardson number considered. Multiple correlations in terms of the heat transfer rate and Richardson number has been established.

1. INTRODUCTION

In recent years the mixed convection in rectangular or square cavities has been investigated by many researchers. This attempt is due to the fact that heat transfer in a cavity can be found in many industrial and engineering applications such as electronic component cooling, food drying process, nuclear reactors etc... Flow and heat transfer phenomena caused by buoyancy and shear forces in enclosures have been studied extensively in the literature. For example, Iwatsu and Hyun [1] studied numerically three dimensional flows in cubical containers. The top moving wall is maintained at a higher temperature than the bottom wall. Numerical solutions are obtained over a wide range of physical parameters, $102 \leq Re \leq 2 \times 10^3$,

$0 \leq Ri \leq 10$ and $Pr = 0.71$. Numerical flow visualizations demonstrate the explicit effects of Ri as well as Re . Mohamed and Viskanta [2] investigated the effects of a sliding lid on the fluid flow and thermal structures in a lid-driven cavity. Moallemi and Jang [3] studied numerically mixed convective flows in a bottom heated square lid-driven enclosure. They investigated the effect of Prandtl number on the flow and heat transfer process. They found that the effects of buoyancy are more pronounced for higher values of Prandtl numbers, and they also derived a correlation for the average Nusselt number in terms of the Prandtl number, Reynolds number and Richardson number. Prasad and Koseff [4] performed an experimental investigation of mixed convection flow in a lid-driven cavity for different Richardson numbers, ranging from 0.1 to 1000. Their results indicate that the overall heat transfer rate is a very weak function of the Grashof number for the examined range of Reynolds numbers. They have also analyzed the mean heat flux values over the entire boundary to produce Nusselt number and Stanton number correlations which are very useful for design applications. Sharif [5] performed a numerical investigation with supplementary flow visualization of laminar mixed convective heat transfer in two-dimensional shallow rectangular driven cavities of aspect ratio 10. The top moving plate of the cavity is set at a higher temperature than the bottom stationary plate. Computations are reported for Rayleigh numbers ranging from 105 to 107 while keeping the Reynolds number fixed at 408.21, thus encompassing the wide spectrum of dominating forced convection, mixed convection, and dominating natural convection flow

regimes. The fluid Prandtl number is taken as 6, representative of water. The effects of inclination of such a cavity on the flow and thermal fields are also investigated for inclination angles ranging from 0° to 30° . The author observed that the local Nusselt number at the heated moving plate starts with a high value and decreases rapidly to a small value towards the right side. The local Nusselt number at the cold plate reveals an oscillatory behavior near the right side due to the presence of a separation bubble at the cold surface in that location.

In the present work, the effect of temperature gradient orientation on the fluid flow and heat transfer in a lid-driven square cavity is investigated numerically.

2. MATHEMATICAL FORMULATION

2.1 Governing equations

For laminar, incompressible and three-dimensional mixed convection, after invoking the Boussinesq approximation and neglecting the viscous dissipation, can be expressed in the dimensionless form as:

Continuity equation:

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (1)$$

Three momentum equations:

$$\frac{\partial u_i}{\partial t} + \frac{\partial (u_i u_i)}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \frac{1}{\text{Re}} \left(\frac{\partial^2 u_i u_i}{\partial x_i \partial x_i} \right) + \text{Ri} \theta (\delta_{i2} \sin \gamma + \delta_{i3} \cos \gamma) \quad (2)$$

Energy equation:

$$\frac{\partial \theta}{\partial t} + \frac{\partial (u_i \theta)}{\partial x_i} = \frac{1}{\text{Re Pr}} \left(\frac{\partial^2 \theta}{\partial x_i \partial x_i} \right) \quad (3)$$

Where, u , v and w are the velocity components in the x , y and z directions, respectively, θ is the temperature and P is the pressure. ρ is the mass density and g is the gravitational acceleration. In Eq. (2), the symbol δ stands for the Kronecker delta. The chosen scales in Eqs. (1)–(3) are the length H , the reference velocity $u_0 = \sqrt{g\beta H \Delta T}$, the time $t_0 = \frac{H}{u_0}$ and

the pressure $P_0 = \rho u_0^2$. Further, the dimensionless temperature is defined by

$\theta = (T - T_r)(T_h - T_c)$, where the reference temperature is $T_r = \frac{(T_h + T_c)}{2}$. Correspondingly, the

dimensionless numbers that emerge are the Grashof number, Gr , Reynolds number, Re , Prandtl number, Pr , and and Richardson number Ri , which are defined as:

$$Gr = \frac{g\beta \Delta T L^3}{\nu^2}, Re = \frac{u_0 L}{\nu}, Pr = \frac{\nu}{\alpha} \text{ and } Ri = \frac{Gr}{Re^2}, \text{ respectively.}$$

2.2 Initial and Boundary conditions

No slip condition at and side walls. The upper lid has a constant velocity, u_0 . The upper lid wall has an isothermal condition with temperature, $T = T_c$. The bottom wall is at rest and isotherm, i.e., $T = T_h$ ($T_c < T_h$). Finally, the remaining walls are adiabatic (see Fig.1).

2.3 Numerical procedure

Numerical computations were carried out with an in-house computer code written in the FORTRAN programming language.

In the FORTRAN code, the unsteady Navier–Stokes and energy equations are discretized by a second-order time stepping finite difference procedure. The procedure adopted here deserves a detailed explanation. First, the non-linear terms in Eqs. (2) are treated explicitly with a second-order Adams–Bashforth scheme. Second, the convective terms in Eq. (3) are treated semi implicitly. Third, the diffusion terms in Eqs. (2) and (3) are treated implicitly. In order to avoid the difficulty that the strong velocity-pressure coupling brings forward, we selected a projection method described in the work of Peyret and Taylor [6], Achdou and Guermond [7].

A finite-volume method is implemented to discretize the Navier–Stokes and energy equations (Patankar [8], Moukhalled and Darwish [9], Kobayachi and Pereira [10]). In this method, the solution domain is subdivided into small finite control volumes (CV). The grid used is more refined than that in areas in need and bigger in other zones.

In fact, it is necessary to use a fine mesh at the top and bottom wall, capable of accurately modeling the heat transfer and flow. With this approach, we can reduce the number of grid points without losing accuracy of calculation.

The advective terms in Eqs. (2) are discretized using a QUICK third-order scheme whereas a second-order central differencing (Hayase, Humphrey and Greif [11]) is applied in Eq. (3). The discretized momentum and energy equations are solved employing the red and black successive over relaxation method (RBSOR) [12], while the Poisson pressure correction equation is solved utilizing a full multi-grid method (Hortmann, Peric and Scheuerer [13], M.S. Mesquita and M.J.S. de Lemos [14], E. Nobile [15]). If specific details about the computational methodology are needed, the reader is directed to Ben-Cheikh et al. [16].

Here, ϕ represents a dependent variable u , v , w , or θ , the indexes i , j , k indicate a grid point, and the index m is the current iteration at the grid level. The convergence criterion was set to 10^{-6} .

Fig1: Physical model and boundary conditions

3. CODE VALIDATION

The present numerical code is validated against a documented numerical study. Namely, the numerical solution reported by Iwatsu and Hyun [1]. The findings of the comparisons are documented in Table 2 for the average Nusselt number. The comparisons illustrate close proximity in the predictions made between the various solutions. These validation cases boost up the confidence in the numerical outcome of the present work.

Table 1: Grid independency results for $Re = 100$, $\gamma = 0^\circ$

	Ri=0.001		Ri=1		Ri=10	
Grid	48^3	64^3	48^3	64^3	48^3	64^3
Nusselt number	1.8371	1.8367	1.3487	1.3487	1.0927	1.0928

Table 2: Comparison of the computed average Nusselt number at the top wall.

Re	Ri=0.001			Ri=1			Ri=10		
	Ref[1]	Ref[17]	Pres.work	Ref[1]	Ref[17]	Pres.work	Ref[1]	Ref[17]	Pres.work
100	1.82	1.836	1.836	1.33	1.348	1.348	1.08	1.092	1.092
400	3.99	3.964	3.963	1.50	1.528	1.539	1.17	1.130	1.152
1000	7.03	7.284	7.295	1.80	1.856	1.863	1.37	1.143	1.143

The effect of grid resolution was also examined in order to select the appropriate grid density. Table 1 presents the results of a grid independency study showing the effects of number of grid points on Nusselt. A 48^3 non uniform grid is found to meet the requirements of both the grid independency study and the computational time costs.

4. RESULTS AND DISCUSSION

The effect of the downward temperature gradient on the streamline and isotherm patterns for $Ri = 0.001$, 1, and 10 is shown in Fig. 2. When $Ri = 10$, a single central primary vortex is observed covering the cavity domain. The vortex is driven by the moving lid. From subfigure 2c. the isotherms are symmetrical and clamped in the lower part of the plane (YOZ) and also we observed the appearance of the plume is believed to be the reason for the more enhancements in heat transfer.

Fig.2. Computed isotherms (a,b and c) and streamlines (d, e and f) contours for $Ri=0.001$, 1 and 10.

The temperature and flow fields in the cavity for $Ri = 1$ are presented in subfigures 3b and 3e. It can be seen that the distribution of streamlines for $Ri = 1$ is similar to that of $Ri = 10$. When Ri is further decreased to 0.001 (subfigures 3a and 3d), the effect of the mechanically driven top lid, with $Ri=0.001$, dominates the entire cavity and generates a primary recirculating vortex which was observed for the other Richardson number.

The average Nusselt number for different values of Ri as shown in Figs.3. The average heat transfer rate increase with the Richardson number. This is due to the increased buoyancy effects in the lower portions of the cavity. Referring to the literature [18], multiple correlations in terms of the heat transfer rate and Richardson number has been established.

The average Nusselt number along the heated wall is correlated in term of Richardson number ($0.001 \leq Ri \leq 10$). Using the numerical results, the correlation can be expressed as: $Nu_{avg} = 2.243 \times \log(2.204 + Ri^{0.64})$

Fig. 3: Average Nusselt number along the hot surface for $Re=100$

Comparisons of the average Nusselt number between the numerical results and those obtained by the correlation are reported in Figs.3. As a result, the average Nusselt number computed from the above equations together with the numerical results agree well.

5. CONCLUSION

Numerical simulations of mixed convection in a lid-driven square cavity are made to investigate the effects of temperature gradient on the flow field and heat transfer characteristics for five different values of the Richardson number ($Ri = 0.001 \leq Ri \leq 10$). From the numerical results, the following conclusions may be drawn:

When large Ri is united with low Re , a single central primary vortex is observed covering the cavity domain. This vortex decreases slightly when Ri decreased. The rate of heat transfer increases with the Richardson number. A correlation in terms of the heat transfer rate and Richardson number has been established also in this case.

Numerical results demonstrate that the heat transfer rate increase due to the increased buoyancy effects in the lower portion of the cavity.

References

- [1] R. Iwatsu and J.M. Hyun, Three-dimensional driven-cavity flows with a vertical temperature gradient.
- [2] A.A.Mohamed, R.Viskanta, Flow and heat transfer in a lid-driven cavity filled with a stably stratified fluid, *Appl.Math. Modelling* 19 (1995) 465-472.
- [3] M. K. Moallemi and K. S. Jang, Prandtl number effects on laminar mixed convection heat transfer in a lid-driven cavity, *hf. J. Heat Mass Transfer*. Vol. 35, No. 8, pp. 1881-1892, 1992.
- [4] A. K. Prasad and J.R. Koseff, Combined forced and natural convection heat transfer in a deep lid-driven cavity flow, *Int.J..Heat Fluid Flow*, Vol.17, pp. 460-467, 1996;
- [5] M.A.R.Sharif, Laminar mixed convection in shallow inclined driven cavities with hot moving lid on top and cooled from bottom, *Applied Thermal Engineering* 27 (2007) 1036-1042.
- [6] R. Peyret, T.D. Taylor, *Computational Methods for Fluid Flow*, Springer-Verlag, Berlin, Germany, 1983.
- [7] Y. Achdou, J.L. Guermond, Convergence analysis of a finite element projection/Lagrange Galerkin method for the incompressible Navier–Stokes equations, *SIAM J. Numer. Anal.* 37 (2000) 799–826.
- [8] S.V.Patankar, A calculation procedure for two-dimensional elliptic situations, *Numer. Heat Transfer* 34 (1981) 409–425.
- [9] F. Moukhalled, M. Darwish, A unified formulation of the segregated class of algorithm for fluid flow at all speeds, *Numer. Heat Transfer, Part B: Fundamentals* 37 (2000) 103– 139.
- [10] M.H. Kobayachi, J.M.C. Pereira, J.C.F. Pereira, A conservative finite-volume second-order-accurate projection method on hybrid unstructured grids, *J. Comput. Phys.* 150 (1999) 40–75.
- [11] T. Hayase, J.A.C. Humphrey, R. Greif, A consistently formulated QUICK scheme for fast and stable convergence using finite-volume iterative calculation procedures, *J. Comput. Phys.* 98 (1992) 108 -118.
- [12] W.H. Press, et al., second edition, *Numerical Recipes in Fortran 77: The Art of Scientific Computing*, vol. 1, Cambridge Press, London, UK, 1997.
- [13] M. Hortmann, M. Peric, G. Scheuerer, Finite volume multigrid prediction of laminar natural convection: benchmark solutions, *Int. J. Numer. Meth. Fluids* 11 (1990) 189– 207.
- [14] M.S. Mesquita, M.J.S. de Lemos, Optimal multigrid solutions of dimensional convection–conduction problems, *Appl. Math. Comput.* 152 (2004) 725–742.
- [15] E. Nobile, Simulation of time-dependent flow in cavities with the correction multigrid method, Part I: Mathematical formulation, *Numer. Heat Transfer, Part B: Fundamentals* 30 (1996) 341–350.

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- [16]] N. Ben-Cheikh, B. Ben-Beya, T. Lili, Benchmark solution for time-dependent natural convection flows with an accelerated full-multigrid method, *Num. Heat Trans. (B)*, 52 (2007) 131-151.
- [17] Nasreddine Ouertatani, Nader Ben Cheikh, Brahim BenBeya, Taieb Lili, Antonio Campo Mixed convection in a double lid-driven cubic cavity, *Int. J. Therm. Sciences*. Vol. 48, pp. 1265–1272, 2009.
- [18] Numerical Study of Mixed Convection Heat Transfer and Fluid Flow in Cubical Lid-Driven Cavity, *Eur. J. Sci. Research*, Vol. 72 No. 3 (2012), pp. 460-473.