EFFECT OF CONCENTRATION OF NEGATIVE ION AND TEMPERATURE OF BOTH IONS ON AMPLITUDE AND WIDTH FOR NON-THERMAL PLASMA

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ABSTRACT. In an unbounded, collisionless and unmagnetized plasma consisting of positive and negative ions together with non-thermal electrons, first and second order amplitudes and widths of the ion-acoustic solitary waves are discussed here properly along with the effect of the concentration of negative ion and temperature of both positive and negative ions.

1. INTRODUCTION

In presence of non-thermal electron together with positive and negative ions, ion-acoustic solitary waves along with their amplitudes and widths are the most important and excited phenomena at present. Many authors [1-4] have found various interesting results on non-thermal electrons. In a three component plasma consisting of positive ion, negative ion and non-thermal electron, compressive as well as rarefactive solitons [5-6] exist under certain condition. A recent observational data from Freja satellite [7] shows the existence of both compressive and rarefactive solitons in the auroral low frequency turbulence. Based on this, Cairns et al [8] have showed the coexistence of compressive and rarefactive solitons in such types of non-thermal plasma. Generally non-thermal electron distribution has been observed in laboratory and space plasmas. The large amplitude waves can also significantly modify the ion and electron distributions. In order to study the ion-acoustic solitary waves Mamun [9-10] considered the effect of non-thermal electrons. Again Bhattacharaya et al [11] studied theoretically the effects of non-thermal electrons with cold positive and negative ions on ion-acoustic solitary waves in a bounded plasma and obtained both compressive and rarefactive solitons without considering the ion temperature. Paul et al [12] studied the non-thermal plasma with warm positive ions in a new analytical way and obtained some necessary and sufficient conditions for the formation of ion-acoustic solitary waves with some special observations. In spite of this, they also found the sagdeev potential function $\Psi(\phi)$ for the same plasma and obtained the condition for the existence of a potential well with special attention of the critical values of the plasma parameters for solitary wave solutions. Chattopadhyay [13] considered the same non-thermal plasma with warm drifting positive and negative ions and found the critical negative ion concentration ($n_{je}$) and phase velocities under different physical conditions. The present author then analysed critically the first order K-dV soliton solution and also higher order M-KdV solitary wave solution with special discussion on the stability of the solitary wave solution in non-thermal plasma with warm positive and negative streaming ions. In a multicomponent plasma consisting of warm drifting positive and negative ions with non-thermal electrons, it is found that there exists a critical negative ion concentration ($n_{je}$) below which compressive soliton exists and above which rarefactive soliton exists [14]. With the discussion of the first order K-dV and higher order M-KdV solitary wave solution [13] the present author now analysed the solution with respect to the nature and behavior of their first and second order
amplitudes and widths under different physical situation and restriction of the concerned plasma parameter. It is mentioned distinctly in this context that the concentration of negative ion \((n_i)\), temperature of both positive and negative ions \((σ_i, σ_j)\), ratios of negative to positive ion masses \((Q)\) and non-thermal electron parameter \((β)\) play a great significant role on the amplitudes and widths of the non-thermal plasma compared with isothermal plasma[15] in some cases.

The plan of the present work is organized in the following way: In sec.2 we investigate mainly the first \((Φ_{01})\) and second \((Φ_{02})\) order amplitudes as well as the first \((δ_1)\) and second \((δ_2)\) order widths with sagdeev potential function \(ψ(Φ)\) and condition for the existence of a potential well. The entire problem is discussed critically in sec.3. Also in this section, the first \((Φ_{01})\) and second \((Φ_{02})\) order amplitudes as well as their first \((δ_1)\) and second \((δ_2)\) order widths with respect to non-thermal plasma are compared with isothermal case for three different mass ratios \((Q)\) shown by three tables 1–3. Concluding remarks are given in sec.4.

2. BASIC EQUATIONS WITH SAGDEEV POTENTIAL, AMPLITUDE AND WIDTH

We consider a collisionless unmagnetized plasma consisting of warm positive and negative ions with streaming motion in presence of non-thermal electrons. The normalized basic equations in unidirectional propagation for such types of unbounded plasmas [13] are

For positive ions

\[
\frac{∂n_i}{∂t} + \frac{∂}{∂x} \left( n_i u_i \right) = 0
\]

(1)

\[
\frac{∂u_i}{∂t} + u_i \frac{∂u_i}{∂x} + \frac{σ_i}{n_i} \frac{∂p_i}{∂x} = - \frac{∂ψ}{∂x}
\]

(2)

\[
\frac{∂p_i}{∂t} + u_i \frac{∂p_i}{∂x} + 3 p_i \frac{∂u_i}{∂x} = 0
\]

(3)

For negative ions

\[
\frac{∂n_j}{∂t} + \frac{∂}{∂x} \left( n_j u_j \right) = 0
\]

(4)

\[
\frac{∂u_j}{∂t} + u_j \frac{∂u_j}{∂x} + \frac{σ_j}{n_j} \frac{∂p_j}{∂x} = \frac{Z}{Q} \frac{∂ψ}{∂x}
\]

(5)

\[
\frac{∂p_j}{∂t} + u_j \frac{∂p_j}{∂x} + 3 p_j \frac{∂u_j}{∂x} = 0
\]

(6)

Poisson’s equation

\[
\frac{∂^2 ψ}{∂x^2} = n_e - n_i + Z n_j
\]

(7)

In this case the subscript i, j denote positive and negative ions. \(n_i, u_i, p_i, σ_i\) and \(n_j, u_j, p_j, σ_j\) are respectively the concentration, velocity, pressure and temperature of positive and negative ions. Again \(n_e\) is the concentration of non-thermal electrons connected with the parameter \(β\) which measures the deviation from thermalised state. Also \(α\) determines the presence of fast particles in the model, \(ψ\) denotes the electrostatic potential, \(Q (= m_j/m_i)\) is the ratio of the masses of negative to positive ions and \(Z\) is the charge.
The normalized electron density is

\[ n_e = (1 - \beta \Phi + \beta \Phi^2) e^\Phi \]  

(8)

Where \( \beta = \frac{4\alpha}{1+3\alpha} \)  

(9)

[with \( \alpha \geq 0 \) and \( 0 \leq \beta < \frac{4}{3} \)]

For solitary wave solution associated with the determination of first and second order amplitude and width, from equation (1) to (7) we use the Galelian transformation

\[ \eta = x - V t \]  

(10)

where \( V \) is the velocity of the solitary waves.

We also assume the boundary conditions

\[ n_i \to n_{i_0}, \ n_j \to n_{j_0}, u_i \to u_{i_0}, u_j \to u_{j_0}, p_i \to 1, p_j \to 1, \ \phi \to 0 \ \text{at} \ |x| \to \infty \]  

(11)

The charge neutrality condition of the plasma is \( n_{i_0} = 1 + Z n_{j_0} \)  

(12)

By using equation (10) and boundary conditions (11) we get finally from equation (7) as

\[ \frac{d^2 \Phi}{d\eta^2} = S_1 \Phi + S_2 \Phi^2 + S_3 \Phi^3 + S_4 \Phi^4 + S_5 \Phi^5 + \ldots \ldots = -\frac{\partial \Phi}{\partial \Phi} \]  

(13)

Where \( S_1 = (1 - \beta) - \frac{n_{i_0}}{(V - u_{i_0})^2 - \frac{\beta}{n_{i_0}} - \frac{2\beta}{q(V - u_{j_0})^2 - \frac{3\beta}{n_{j_0}}}} \)

\[ S_2 = -\frac{1}{2} \left[ 1 + \frac{n_{i_0}}{2\sqrt{3} \sigma_i} \left\{ (V - u_{i_0} + \frac{3\sigma_i}{n_{i_0}})^{-3} - (V - u_{i_0} - \frac{3\sigma_i}{n_{i_0}})^{-3} \right\} \right] \]

\[ S_3 = \frac{1}{6} \left[ \left( 1 + 3\beta \right) + \frac{3n_{i_0}^2}{2\sqrt{3} \sigma_i} \left\{ (V - u_{i_0} + \frac{3\sigma_i}{n_{i_0}})^{-5} - (V - u_{i_0} - \frac{3\sigma_i}{n_{i_0}})^{-5} \right\} \right] \]

\[ S_4 = \frac{1}{24} \left[ \left( 1 + 8\beta \right) + \frac{15n_{i_0}^2}{2\sqrt{3} \sigma_i} \left\{ (V - u_{i_0} - \frac{3\sigma_i}{n_{i_0}})^{-7} - (V - u_{i_0} + \frac{3\sigma_i}{n_{i_0}})^{-7} \right\} \right] \]

\[ S_5 = \frac{1}{120} \left[ \left( 1 + 15\beta \right) + \frac{105n_{i_0}^2}{2\sqrt{3} \sigma_i} \left\{ (V - u_{i_0} - \frac{3\sigma_i}{n_{i_0}})^{-9} - (V - u_{i_0} + \frac{3\sigma_i}{n_{i_0}})^{-9} \right\} \right] \]

\[ 
\begin{align*}
\frac{d^2 \Phi}{d\eta^2} &= S_1 \Phi + S_2 \Phi^2 + S_3 \Phi^3 + S_4 \Phi^4 + S_5 \Phi^5 + \ldots \ldots = -\frac{\partial \Phi}{\partial \Phi} \\
\end{align*} 

\]

The coefficients \( S_1, S_2, S_3 \) etc. have great significance on the formation of solitary wave solution for the non-thermal plasma in presence of warm ions.

For cold non-thermal plasma the above first three coefficients are reduced to the following forms:
These coefficients $S_1$, $S_2$ and $S_3$ in equation (14b) supports Ref. [16] for isothermal plasma (i.e. $\beta = 0$) with cold ions ($\sigma_i = 0 = \sigma_j$).

The function $\psi(\phi)$ is known as the sagdeev potential and can be written in the following form:

$$\psi(\phi) = (1 + 3\beta) - \left[ (1 + 3\beta) - 3\beta \phi + \beta \phi^2 \right] e^\phi - \frac{n_{io}^{3/2}}{6\sqrt{3} \sigma_l} \left[ \left( V - u_{io} + \frac{3\sigma_l}{\sqrt{n_{io}}} \right)^2 - 2\phi \right] - \left( V - u_{io} - \frac{3\sigma_l}{\sqrt{n_{io}}} \right)^3 + \left( V - u_{io} - \frac{3\sigma_l}{\sqrt{n_{io}}} \right)^3 - \left. \right| \left( V - u_{jo} + \frac{3\sigma_j}{\sqrt{Qn_{jo}}} \right)^2 - 2\phi \right) - \left( V - u_{jo} - \frac{3\sigma_j}{\sqrt{Qn_{jo}}} \right)^3 \right]$$

(15)

where $\phi$ satisfies the inequality

$$\frac{-Q}{\pi z} \left( V - u_{jo} - \frac{3\sigma_j}{\sqrt{Qn_{jo}}} \right)^2 < \phi < \frac{1}{2} \left( V - u_{io} - \frac{3\sigma_l}{\sqrt{n_{io}}} \right)^2$$

(16)

In order to find the amplitudes and the widths of the solitary waves, we have to discuss the condition for the existence of solitary wave solution. For solitary wave solution, the Sagdeev potential $\psi(\phi)$ must satisfy the following conditions:

i) $\psi(\phi) = 0 = \frac{\partial \psi}{\partial \phi}$ for all $V$ at $\phi = 0$.

ii) $\psi(\phi) = 0$ for some $\phi = \phi_m$, $\phi_m$ is some max. value of $\phi$.

iii) $\psi(\phi) < 0$ in $0 < |\phi| < |\phi_m|$

In addition to that for localized soliton solutions, the condition for the existence of a potential well is

$$\frac{\partial^2 \psi}{\partial \phi^2} < 0 \text{ at } \phi = 0$$

And this inequality may also be easily obtained from $S_1 > 0$ so that first ( $\delta_1$ ) and second order width ( $\delta_2$ ) are always real. For isothermal plasma (i.e. at $\beta = 0$) the function $\psi(\phi)$ in equation (15) reduces to Ref.[17] and for cold plasma it supports Ref.[16].
From equation (13) we can say that for small amplitude solitary wave solution, the first ($\Phi_{01}$) and second ($\Phi_{02}$) order amplitudes [16,17,18] of that solitary wave solution are respectively

$$\Phi_{01} = \frac{3S_1}{2S_2}$$

(17)

And

$$\Phi_{02} = \frac{6S_1}{2S_2 + 4S_2^2 - 16S_1S_2}$$

(18)

Moreover the first ($\delta_1$) and second ($\delta_2$) order width [16,17,18] of that solitary waves are respectively

$$\delta_1 = \frac{2}{\sqrt{S_2}}$$

(19)

And

$$\delta_2 = \delta_1 \cosh^{-1} \left\{ \left[ \frac{1.1981S_1}{4S_2^2 - 18S_1S_2} \right]^\frac{1}{3} + 1.6905 \right\}$$

(20)

3. DISCUSSION

On the basis of the formation of solitary waves by Sagdeev pseudopotential approach, we investigate first ($\Phi_{01}$) and second ($\Phi_{02}$) order amplitudes as well as first ($\delta_1$) and second ($\delta_2$) order widths of the solitary waves that are shown graphically in Figs. 1–6.

Fig.1 shows the first ($\Phi_{01}$) and second ($\Phi_{02}$) order amplitudes against negative ion concentration ($n_{jo}$) with the variation of the temperature of positive ($\sigma_i$) and negative ions ($\sigma_j$). The first order amplitude ($\Phi_{01}$) decreases when temperature of ions ($\sigma_i$, $\sigma_j$) increases [17,20] but the second order amplitude ($\Phi_{02}$) increases due to the increase of the temperature of ions ($\sigma_i$, $\sigma_j$) except for equal ion temperature (i.e. at $\sigma_i = \sigma_j = \frac{1}{30}$) in case of (He$^+$, O$^-$) plasma and that contradicts the result of Ref. [17] because of the presence of non-thermal electron. This shows the effect of non-thermal electron. It is seen from this figure that the first order amplitudes ($\Phi_{01}$) and second order amplitudes ($\Phi_{02}$) order amplitudes of the plasma (He$^+$, O$^-$) at Q = 4 for different ionic temperatures.

The first ($\Phi_{01}$) and second ($\Phi_{02}$) order amplitudes verses negative ion concentration ($n_{jo}$) with the variation of the non-thermal electron parameter (\(\beta\)) for (He$^+$, O$^-$) plasma are shown in Fig.2.

In this figure first order amplitude ($\Phi_{01}$) decreases for increasing value of the non-thermal electron parameter (\(\beta\)), but it is interesting to note that first ($\Phi_{01}$) order amplitude is greatest for lowest value of \(\beta\) (i.e. at $\beta = 0$ which is known as the isothermal case). It is also seen from this figure that the second ($\Phi_{02}$) order amplitude increases when thermal electron non parameter (\(\beta\)) increases from 0 to 0.2 except at $n_{jo} = 0.5$. Here ($a_2$, $a_2'$) represents the graph of first ($\Phi_{01}$) and second ($\Phi_{02}$) order amplitudes with the most interesting isothermal situation i.e. at $\beta = 0$ and ($b_2$, $b_2'$) at $\beta = 0.09$ and ($c_2$, $c_2'$) at $\beta = 0.2$ represent the graph of higher values of first ($\Phi_{01}$) and second ($\Phi_{02}$) order amplitudes.

Fig.3 shows the first ($\Phi_{01}$) and second ($\Phi_{02}$) order amplitudes against negative ion concentration ($n_{jo}$) with the variation of the ratios of negative to positive ion masses (Q) when the non-thermal electron parameter $\beta$ takes a definite value. As Q increases [i.e. from Q = 4, 8.875 and 16] the numerical value of first ($\Phi_{01}$) [shown in Fig.3(a)] and second ($\Phi_{02}$) [shown in Fig.3(b)] order amplitude increases. Again the numerical values of first ($\Phi_{01}$) [shown in Fig.3(a)] and second ($\Phi_{02}$) [shown in Fig.3(b)] order amplitudes decrease when the negative ion concentration ($n_{jo}$)
increases [19] for the mass ratios Q = 4 and 8.875 where \( \phi_{01} < \phi_{02} \), but for the mass ratio Q = 16 with the increasing values of the negative ion concentration \( n_{io} \), the numerical values of the first \( (\phi_{01}) \) order amplitude are increasing while that numerical values of second \( (\phi_{02}) \) order amplitude are decreasing here for higher concentration. These are the more general result than that of the isothermal plasma (i.e. at \( \beta = 0 \)) because we can easily deduce the results of isothermal plasma from non-thermal system. Actually in isothermal plasma (i.e. at \( \beta = 0 \)) at Q = 4 with increasing values of negative ion concentration \( n_{io} \), numerical values first \( (\phi_{01}) \) [shown in Fig.3(a)] order amplitude are increasing while that of second \( (\phi_{02}) \) order amplitude [shown in Fig.3(b)] are decreasing . This result supports Ref.[16] but for isothermal plasma the numerical values of first \( (\phi_{01}) \) order amplitude contradicts the result of first \( (\phi_{01}) \) order amplitude in non-thermal plasma for Q = 4.

Fig.4 shows the first \( (\phi_{01}) \) and second \( (\phi_{02}) \) order amplitudes against the non-thermal electron parameter(\( \beta \)) with the variation of negative ion concentration \( n_{io} \). As negative ion concentration \( n_{io} \) increases, the first \( (\phi_{01}) \) and second \( (\phi_{02}) \) order amplitudes decrease against the non-thermal electron parameter(\( \beta \)). Also the second \( (\phi_{02}) \) order amplitudes \( a'_{s} \) at \( n_{io} = 0.1 \), \( b'_{s} \) at \( n_{io} = 0.3 \) and \( c'_{s} \) at \( n_{io} = 0.5 \) are gradually decreasing whereas the first order amplitudes \( (\phi_{01}) a_{s} \), \( b_{s} \) and \( c_{s} \) decrease at the above mentioned negative ion concentration \( (n_{io}) \).

In Fig.5, the first \( (\delta_{1}) \)and second \( (\delta_{2}) \) order widths against negative ion concentration \( (n_{io}) \)for three different plasmas \((\text{He}^{+},\text{O}^{-})\), \((\text{He}^{+},\text{Cl}^{-})\)and \((\text{H}^{+},\text{O}^{-})\) are shown with the variation of the ratios of negative to positive ion masses \( Q \). The first \( (\delta_{1}) \) [shown in Fig.5(a)] and second \( (\delta_{2}) \) order width [shown in Fig.5(b)] varies inversely with the mass ratios \( Q \) for the above plasmas while the first \( (\delta_{1}) \) and second \( (\delta_{2}) \) order width increases with increasing values of the negative ion concentration \( (n_{io}) \) for \( Q = 8.875 \) and for the mass ratio \( Q = 16 \) with the increasing values of negative ion concentration \( (n_{io}) \), the numerical values of \( \delta_{1} \) and \( \delta_{2} \) are increasing where \( \delta_{1} < \delta_{2} \).

In Fig.6, the first \( (\delta_{1}) \) and second \( (\delta_{2}) \) order widths against the non-thermal electron parameter \( (\beta) \) for the plasma \((\text{He}^{+},\text{O}^{-})\) with the variation of negative ion concentration \( (n_{io}) \) are shown. As negative ion concentration \( (n_{io}) \) increases, the first \( (\delta_{1}) \) order widths \( a_{s} \), \( b_{s} \) and \( c_{s} \) and the second \( (\delta_{2}) \) order widths \( a'_{s} \), \( b'_{s} \) and \( c'_{s} \) are gradually increasing against the non-thermal electron parameter \( (\beta) \). It is also important to observe from this figure that the first order width \( (\delta_{1}) \) is always greater than the second \( (\delta_{2}) \) order width for some values of negative ion concentration \( (n_{io}) \).

### 4. COMPARISON OF FIRST AND SECOND ORDER AMPALITUDES \( (\phi_{01},\phi_{02}) \) AND WIDTHS \( (\delta_{1},\delta_{2}) \) BETWEEN ISOHERMAL \( (\beta=0) \) AND NON-TERMAL \( (\beta \neq 0) \) PLASMA WITH RESPECT TO DIFFERENT MASS RATIOS \( (Q) \).

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TABLE – 2 [(He$^+$,Cl$^-$)plasma with Q = 8.875]

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TABLE – 3 [(H$^+$,O$^-$)plasma with Q = 16]

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</table>

As Q increases from 4 to 16, the first order width($\delta_1$) decreases in isothermal plasma while for non-thermal plasma the first order width($\delta_1$) increases gradually for increasing values of negative ion concentration($n_{jo}$) under a particular value of negative to positive ion mass ratio(Q). It is also found from Tables 1, 2 and 3 that the first order width($\delta_1$) for non-thermal plasma decreases as Q takes values from 4 to higher values 16.

From Tables 1, 2 and 3, it is observed that for isothermal plasma, the numerical values of first order amplitudes($\phi_{01}$) are increasing for a particular Q with the variation of negative ion concentration($n_{jo}$). In case of non-thermal plasma, numerical values of first order amplitudes($\phi_{01}$) are decreasing for Q = 4 and 8.875 with increasing negative ion concentration($n_{jo}$) but for Q = 16, numerical values of first order amplitudes($\phi_{01}$) are increasing with increasing negative ion concentration($n_{jo}$).

For isothermal plasma, the second order amplitudes($\phi_{02}$) are increasing for Q = 4 to 16 but for non-thermal plasma the second order amplitudes($\phi_{02}$) are decreasing for increasing values of negative ion concentration($n_{jo}$). On the other hand, second order widths($\delta_2$) are increasing for increasing negative ion concentration($n_{jo}$) with three particular values of Q.
5. FIGURE CAPTIONS:

Fig.1 First ($\Phi_{01}$) and second ($\Phi_{02}$) order amplitudes verses negative ion concentration ($n_{jo}$) with the variation of the temperature of positive ($\sigma_i$) and negative ($\sigma_j$) ions for $V = 1.55$, $u_{io} = 0.02$, $u_{jo} = 0.40$, $Q = 4$, $n_{jo} = 0.1$ and $\beta = 0.2$.

Fig.2 First ($\Phi_{01}$) and second ($\Phi_{02}$) order amplitudes verses negative ion concentration ($n_{jo}$) with the variation of the non-thermal electron parameter ($\beta$) for $V = 1.55$, $u_{io} = 0.02$, $u_{jo} = 0.40$, $Q = 4$, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$.

Fig.3 First ($\Phi_{01}$) and second ($\Phi_{02}$) order amplitudes verses negative ion concentration ($n_{jo}$) with the variation of the ratios of negative to positive ion masses ($Q$) for $V = 1.55$, $u_{io} = 0.02$, $u_{jo} = 0.40$, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$ and $\beta = 0.2$.

Fig.4 First ($\Phi_{01}$) and second ($\Phi_{02}$) order amplitudes verses non-thermal electron parameter ($\beta$) with the variation of negative ion concentration ($n_{jo}$) for $V = 1.55$, $u_{io} = 0.02$, $u_{jo} = 0.40$, $Q = 4$, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$.

Fig.5 First ($\delta_{1}$) and Second ($\delta_{2}$) order widths verses negative ion concentration ($n_{jo}$) with the variation of the the ratios of negative to positive ion masses ($Q$) for $V = 1.55$, $u_{io} = 0.02$, $u_{jo} = 0.40$, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$ and $\beta = 0.2$.

Fig.6 Structures of first ($\delta_{1}$) and Second ($\delta_{2}$) order widths verses non-thermal electron parameter ($\beta$) with the variation of negative ion concentration ($n_{jo}$) for $V = 1.55$, $u_{io} = 0.02$, $u_{jo} = 0.40$, $Q = 4$, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{25}$.
Fig. 1

\[ \phi_{01} = 1^{st} \text{ order } = \text{ solid line} \]

\[ \phi_{02} = 2^{nd} \text{ order } = \text{ dotted line} \]

\[ (c_2, c'_2): \sigma_i = \sigma_j = \frac{1}{30} \]

\[ (b_2, b'_2): \sigma_i = \frac{1}{20}, \sigma_j = \frac{1}{10} \]

\[ (a_2, a'_2): \sigma_i = \frac{1}{30}, \sigma_j = \frac{1}{25} \]
Fig. 2

\[ \phi_{01} = 1^{st} \text{ order} = \text{solid line} \]
\[ \phi_{02} = 2^{nd} \text{ order} = \text{dotted line} \]

\[ [(a_3, a_3') : \beta = 0] \]
\[ [(b_3, b_3') : \beta = 0.09] \]
\[ [(c_3, c_3') : \beta = 0.2] \]
Solid line = 1st order amplitude = $\phi_{01}$

- $Q = 4$
- $Q = 8.875$
- $Q = 16$

Fig. 3(a)
Dotted lines = \ldots = 2^{nd} order amplitudes = \phi_{02}
Fig. 4
Solid line = 1st order width = $\delta_1$
Fig. 5(b) 

\[ Q = 4 \]
\[ Q = 8.875 \]
\[ Q = 16 \]

Dotted lines = 2\(^{\text{nd}}\) order width = \( \delta_2 \)
$\delta_1$ or $\delta_2$

$\delta_1 = 1^{st}$ order = solid line

$\delta_2 = 2^{nd}$ order = dotted line

$[(a_4, a_4') : n_{j_0} = 0.1]$

$[(b_4, b_4') : n_{j_0} = 0.3]$

$[(c_4, c_4') : n_{j_0} = 0.5]$

$\beta \rightarrow$
6. CONCLUDING REMARKS

In presence of non-thermal electrons we have theoretically investigated the ion-acoustic solitary waves with first ($\Phi_{01}$) and second order($\Phi_{02}$) amplitudes as well as first ($\delta_1$) and second order($\delta_2$) order widths in an unbounded plasma consisting of positive and negative ions. The effect of concentration of negative ion ($n_{jo}$) and temperature of both positive ($\sigma_i$) and negative ions ($\sigma_j$) on amplitude and width for non-thermal plasma is discussed here very carefully and represented graphically by the respective Figs.1 – 6.

The first ($\Phi_{01}$) and second order($\Phi_{02}$) amplitudes against negative ion concentration ($n_{jo}$) with the variation of temperature of positive ions ($\sigma_i$), negative ions ($\sigma_j$) and non-thermal electron parameters ($\beta$) are shown in Figs. 1 – 2.

From the Fig.1, it is seen that the numerical value of the first order amplitude($\Phi_{01}$) is less than the second order amplitude($\Phi_{02}$) for three different values of the temperature of positive ($\sigma_i$) and negative ions ($\sigma_j$). In case of cold non-thermal plasma, only first order amplitude($\Phi_{01}$) is obtained but no second order amplitude($\Phi_{02}$) is found. Also it is important to note from Fig.2 that the first order amplitudes($\Phi_{01}$) are negative or rarefactive in nature while second order amplitudes($\Phi_{02}$) are positive or compressive in nature against the negative ion concentration ($n_{jo}$) with the variation of non-thermal electron parameter ($\beta$). This type of nature is found also even if $\beta = 0$ i.e. for isothermal plasma.

In Fig.3, the first ($\Phi_{01}$) [shown in Fig.3(a)] and second ($\Phi_{02}$) [shown in Fig.3(b)] order amplitudes are decreasing when negative ion concentrations ($n_{jo}$) are increasing for the variation of the negative to positive ion mass ratios ($Q$) for a definite value of the non-thermal electron parameter($\beta$).

It is shown in Fig.4 that the first order amplitudes ($\Phi_{01}$) are increasing while second order amplitudes ($\Phi_{02}$) are decreasing for three different values of the negative ion concentration ($n_{jo}$) against the non-thermal electron parameter $\beta$. Also it is important to observe from this figure that first order amplitudes($\Phi_{01}$) represented by ($a_5, b_5$ and $c_5$) are smaller than that of second order($\Phi_{02}$) amplitudes ($a_5', b_5'$ and $c_5'$) which are shown by the graphs $a_5 < a_5'$ at $n_{jo} = 0.1$, $b_5 < b_5'$ at $n_{jo} = 0.3$ and $c_5 < c_5'$ at $n_{jo} = 0.5$. It is also found that the numerical values of first ($\Phi_{01}$) and second order($\Phi_{02}$) amplitudes are the highest for absence of negative ion i.e. at $n_{jo} = 0$.

Fig.5 shows the first ($\delta_1$) [shown in Fig.5(a)] and second($\delta_2$) [shown in Fig.5(b)] order widths verses negative ion concentration($n_{jo}$) with the variation of the ratios of negative to positive ion masses ($Q$). When $Q$ increases from 4 to 16 then it is found always from the graph that $\delta_1 < \delta_2$.

Again from Fig.6, it is seen that the first ($\delta_1$) and second($\delta_2$) order widths are increasing for increasing values of the non-thermal electron parameter ($\beta$) in three different values of the negative ion concentration($n_{jo}$). Also in this case first order widths ($\delta_1$) are smaller than the second order($\delta_2$) widths. It is evident from the figure that $a_4 < a_4'$ at $n_{jo} = 0.1$, $b_4 < b_4'$ at $n_{jo} = 0.3$ and $c_4 < c_4'$ at $n_{jo} = 0.5$. More over in absence of negative ion i.e. at $n_{jo} = 0$, the numerical values of first ($\delta_1$) and second order($\delta_2$) widths are less than that of the values of first ($\delta_1$) and second order widths($\delta_2$) at $n_{jo} = 0.1, 0.3$ and 0.5.

The principal aim or motivation of this problem is to discuss or compare the first and second order amplitudes together with their widths of the respective solitary waves and the determination of the generalized form of Sagdeev potential function for non-thermal plasma. From this amplitudes and widths of the waves, we can easily determine the nature of the solitary waves that are found in space plasma. Our future plan is to solve the same non-thermal plasma with relativistic form of positive and negative ion along with positron.
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