

Estimation of Verdet constant for KCl, KBr, MgCl₂ using Faraday rotator

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ABSTRACT

Nowadays, the rapid development of optics technology, especially for laser technology and scientific fields is important. One of this optics theory is Faraday rotation. Way of light propagation in a material medium can be influenced by an external field. The applied magnetic field causes rotation of the plane of polarized light. This effect is due to the optical activity, a property of the material that causes the rotation of the electric field of an incident linear wave. In this paper, we present an experiment to measure the Verdet constant through Faraday Effect using the Faraday rotator. Verdet constant is a factor for relation of magnetic field and the length of the path. When this effect is used in spectrographic studies, we can find information on the properties of higher energy states. Used materials for this experiment are KCl, KBr, and MgCl₂. We used three lasers with different wavelengths 405nm, 450nm, and 515nm. We shows that the linear polarized light rotates after passing the applied magnetic field, when it's parallel with the magnetic field vector. Verdet constant are calculated about 382, 429.125, and 311.625 in 450nm for KCl, KBr, MgCl₂ solutions

Keywords: Faraday Effect; Optics; Polarized light; Verdet constant

1. INTRODUCTION

In recent years, optics technology has found its way to the forefront of scientific thinking. Considerable work has been done in the area and a series of reasons are glaring to hope for wonderful achievements in the future horizons. This science owing to validity, history and interpretation built on the foundation of electromagnetic theory structure, has never lost its authority. Optics concepts have been developed, throughout the history of physics. Thus, optical devices are now increasingly used in the fields of medicine and health, industry, agriculture and many others. Electric power industry is among the industries in which optical devices have been widely used in recent years. In this industry it is common for data transmission to use optical fiber and the use of optical elements for measuring current and voltage is also expanding.

2. THEORY

In 1845, Michael Faraday discovered that when a piece of glass is placed in a magnetic field, it is optically activated. To observe the Faraday Effect, light should be illuminated in

parallel with the direction of the field. $K \parallel H$. This case is called Faraday geometry. When a polarization plane goes along the applied magnetic field the polarization plane begins to rotate. Angle of rotation of the plane is directly related to the applied field and the length of the material in which the light has been transmitted [1, 2].

According to Faraday's law, we have [1, 2]:

$$\theta_f = V \int H \cdot dl \quad (1)$$

Because in most cases, physical arrangement of Faraday environment is as $H \cdot dl = H \times L$, so Eq. 1 can be written as [1, 2]:

$$\theta_f = VHL \quad (2)$$

Where, L is the length of the Faraday environment.

V is Verdet Constant that is the amount of rotation per field unit and per length unit. Verdet Constant depends on material, frequency and temperature.

2.1. Light

Light can be explained by electric field and magnetic field fluctuations perpendicular to the wave propagation direction and also to each other. According to the definition of polarization, spatial and temporal characteristics of the electric vector of the light wave, determine the light polarization type. If the electric vector of the light is always in one plane, it is called linear polarized light. In this case, the electric vector of light moves on a fixed line and its magnitude and its sign change. The plane, in which the electric vector is fluctuating, is called vibration plane. This plane includes propagation vector K as well as the electric vector. The page is also called the polarization plane. It is worth noting that light polarization is expressed in electric field E direction, since the intensity of field E is larger than that of field B in the light wave [1].

Now suppose that we have two linearly polarized light waves with the same frequency and they are moving in the same direction. If the electric vectors of the two waves are in one direction, the combination of the two waves is a wave with linear polarization. If the electric fields of the waves are perpendicular, the polarization of the resultant wave depends on the relative phase difference between the waves and their amplitude. To understand this, suppose that the waves can be written as follows [2]:

$$E_x(z, t) = E_{0x} \cos(\omega t - kz) \quad (3)$$

$$E_y(z, t) = E_{0y} \cos(\omega t - kz - \varphi) \quad (4)$$

Here E_{0x} and E_{0y} represent the amplitude of the waves and k is the propagation constant that is equal to $2\pi / \lambda$, and λ is the wavelength of light. φ is the relative phase difference between the waves and $\omega = 2\pi f$, where f is the frequency of light. The equation indicates waves that are moving in the direction of Z axis. In this case, the resultant wave is equal to:

$$\mathbf{E}(z, t) = i\mathbf{E}_x(z, t) + j\mathbf{E}_y(z, t) \quad (5)$$

Where, i and j are the unit vectors on the X and Y axis. The resultant wave according to Eq. 5 can have linear or circular or elliptical polarization. These cases will be examined next.

2.1.1. Linear polarized light

If in the Eq. 2-3, ϕ is zero or an integer multiple of 2π , two waves are in phase. Thus, the resultant wave is a wave with linear polarization, which is equal to [2, 3, 4]:

$$\mathbf{E}(z, t) = (i\mathbf{E}_x(z, t) + j\mathbf{E}_y(z, t)) \cos(\omega t - kz) \quad (6)$$

In this case, the polarization vector makes an angle to the x-axis as:

$$\theta = \arctan\left(\frac{E_{0x}}{E_{0y}}\right) \quad (7)$$

The magnitude of the resultant wave is as follows:

$$E = \sqrt{E_{0x}^2 + E_{0y}^2} \quad (8)$$

2.1.2. Linear polarized light

If Eq. 3 and Eq. 4 has the following:

$$\phi = -\frac{\pi}{2} + 2m\pi \quad (9)$$

$$E_{0x} = E_{0y} = E_0 \quad (10)$$

Then the resultant wave is equal to [3, 4]:

$$\mathbf{E}(z, t) = E_0(i\cos(\omega t - kz) + j\sin(\omega t - kz)) \quad (11)$$

Here the amplitude of E is constant but its direction changes with time and movement is not limited to one plane. Such a wave is called clockwise circular polarized wave. In this case the electric vector end moves on a circle.

3. METHODS AND MATERIALS

A linear polarized wave is the combination of two right and left circular polarized waves. By applying a magnetic field, linear polarization rotation is proportional to the refractive index n difference between clockwise and counterclockwise light [5].

The rotation angle β , by the pathway length and the refractive index difference between the two beams caused by the magnetic field, is obtained from Eq. 12 [5, 6]:

$$\beta = \frac{\pi L}{\lambda} (n_l - n_r) \quad (12)$$

Where, λ is light wavelength, L is the length of the interaction between light and magnetic field, n_l is the refractive index for counterclockwise circular wave and n_r is the refractive index for clockwise circular wave.

Some complex combinations can naturally rotate the polarization plane by birefringence. But the birefringence effect can also be made by the electric field or pressure or magnetism in other materials. Faraday rotation is the magnetic state of birefringence effect.

Monochromatic linearly polarized light passing through an optical component rotates to an angle of θ in the magnetic field shown in the following example:

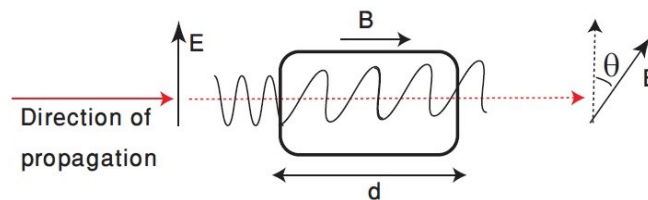


Figure 1. Change of polarization angle as light passes through an optical component.

If the optical component is integrated, Eq. 13 is used [1-4]:

$$\beta = VBd \quad (13)$$

For non-integrated components we use Eq. 14:

$$\beta = V \int_0^d B(z) dz \quad (14)$$

V ratio is related to the material structure and is called Verdet Constant that is a function of light wavelength, temperature and refractive index of the material. Faraday rotation in per-unit includes field per-unit equations. In other words, it is forced to birefringence phenomenon [5].

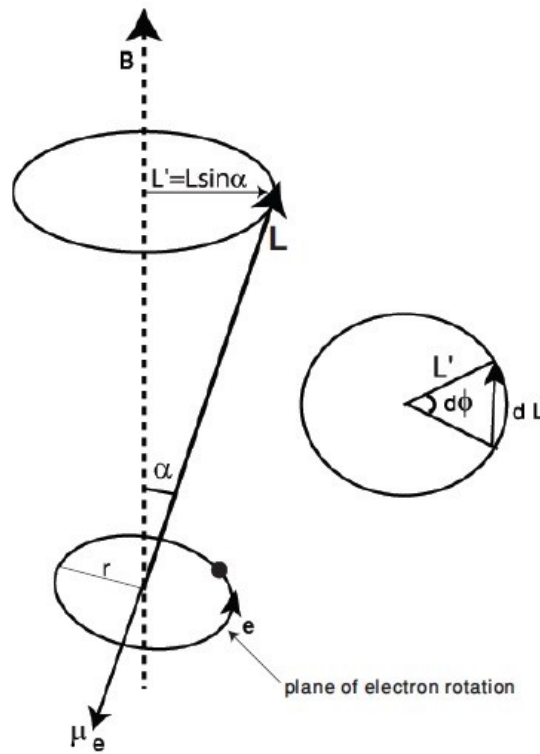


Figure 2. Change axis of angular momentum vector about the direction of magnetic field.

In describing Faraday rotation method, first a brief and basic explanation is presented. Suppose an electron rotating in a plane in a circular path of radius R and makes angle α to magnetic field B . Negatively charged electron consists of angular momentum L and magnetic moment μ in the opposite directions. In this case, the magnetic field applies moment τ to the dipole field [7].

$$\tau = \mu \times B = \mu B \sin \alpha \quad (15)$$

According to Newton's second law, angular momentum causes the change in angular moment:

$$\tau dt = dL \quad (16)$$

Vector L rotates counterclockwise. The expected procession by the tip of vector L is shown in Figure 2. Rotation angle of angular momentum image on the axis of the field is called L and the movement with time is called dt [7]:

$$d\phi = \frac{dL}{L'} = \tau dt / L \sin \alpha \quad (17)$$

The Larmor angular velocity becomes,

$$\omega_L = \frac{d\phi}{dt} = \frac{\tau}{L \sin \alpha} = \frac{\mu_e B \sin \alpha}{L \sin \alpha} = \frac{\mu_e B}{L} \quad (18)$$

The magnetic moment of the circular current is calculated by Eq. 20:

$$\mu_s = iA = i(\pi r^2) \quad (19)$$

$$i = \frac{e\omega}{2\pi} \quad (20)$$

Electron angular momentum can be expressed by Eq. 22:

$$L = r \times p \quad (21)$$

$$|L| = mvr = mr^2\omega \quad (22)$$

By combining the previous equations, Eq. 23 is obtained:

$$\omega_L = \left(\frac{e\omega}{2\pi}\right) \left(\frac{\pi r^2}{mr^2\omega}\right) B = \frac{eB}{2m} \quad (23)$$

The equation suggests that the Larmor Frequency ω_L does not depend on the current loop direction and its overall effect is the rotation of the electronic structure around the applied magnetic field.

Optical rotation of polarized light passing through the electronic structure is known as Circular Birefringence. A linear polarized wave is the combination of two circular polarized waves and consists of two clockwise r and counterclockwise l components. Each component passes through different refractive index n, thus having different velocity. If we assume f is the frequency of light and it passes through transmission electron system and is rotating with frequency Larmor f_L , the frequency-dependent parameter will be determined by Eq. 24, 25 [7]:

$$n_l = n(f - f_l) \quad (24)$$

$$n_r = n(f + f_l) \quad (25)$$

By applying a magnetic field, linear polarization rotation is proportional to the difference between the refractive index n of the clockwise and counterclockwise light beams. Eq. 24, Eq. 25 shows the characteristics of the light passing through a magnetic field the same amount of light with a frequency of f in the field-free space. The optical path difference for the light beams l and r is $(n_r - n_l)d$. After passing through length d, Eq. 26-28 can be calculated [5, 6, 7]:

$$n_r - n_l = n(f + f_l) - n(f - f_l) = \left(n(f) + \frac{dn}{df} f_l\right) - \left(n(f) - \frac{dn}{df} f_l\right) = \frac{2f_l dn}{df} \quad (26)$$

$$\omega_L = 2\pi f_l \quad (27)$$

$$\left|\frac{dn}{df}\right| = \left(\frac{\lambda^2}{c}\right) dn/d\lambda \quad (28)$$

By combining Eq. 26-28, we have:

$$n_r - n_l = 2\left(\frac{\epsilon B}{4\pi m}\right)\left(\frac{\lambda^2}{c}\right)\left(\frac{dn}{d\lambda}\right) \quad (29)$$

The phase shift of each component is as follows:

$$\phi_r = \left(\frac{n_r d}{\lambda}\right) 2\pi \quad \text{and} \quad \phi_l = \left(\frac{n_l d}{\lambda}\right) 2\pi \quad (30)$$

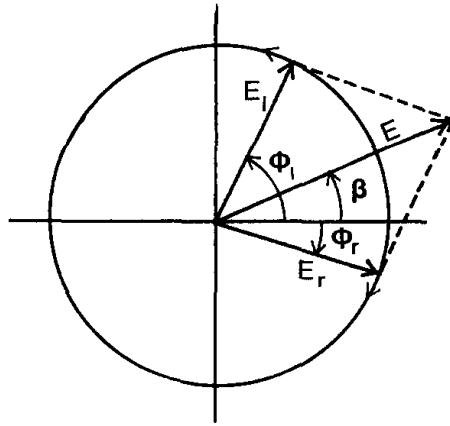


Figure 3. Position of left and right vectors with different velocities.

If the two phases ϕ_r and ϕ_l are considered with regard to $\phi = 0$ the resultant vector from two vectors E_l and E_r , is shown as E . if the angular difference between the resultant field and the reference is considered β , we have [7]:

$$\phi_l - \beta = \phi_r + \beta \quad (31)$$

Then, we have:

$$\beta = \frac{1}{2} \left(\frac{2\pi d}{\lambda}\right) (n_l - n_r) = \frac{\pi d}{\lambda} \left(\frac{\epsilon B}{2\pi m}\right) \left(\frac{\lambda^2}{c} \frac{dn}{d\lambda}\right) = \left(\frac{\pi}{\lambda} \frac{\epsilon}{2\pi m} \frac{\lambda^2}{c} \frac{dn}{d\lambda}\right) B d \quad (32)$$

According to the similarity of Eq. 13, Verdet Constant was obtained:

$$V = \frac{\epsilon}{2mc} \lambda \frac{dn}{d\lambda} \quad (33)$$

As we know, the refractive index decreases as wavelength increases. This behavior can be explained by the simple relation of Cauchy where the coefficients a and b are constant:

$$n \sim a + \frac{b}{\lambda^2} \quad (34)$$

By replacing in Verdet Constant equation:

$$V \approx \frac{be}{mc\lambda^2} \quad (35)$$

According to Eq. 35, as the wavelength increases, Verdet constant decreases by a power of two.

4. RESULTS AND DISCUSSIONS

A device called Faraday rotation was set up according to Figure 1 to determine Verdet constant. KBr and KCl and $MgCl_2$ solutions were placed in a cuvette with a width of 20 mm and between two nucleus poles.

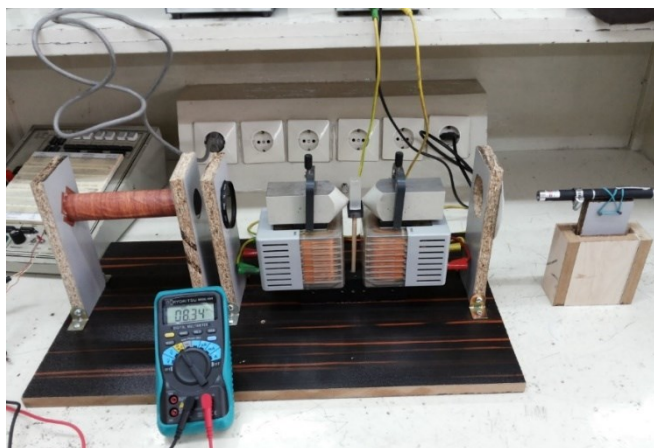


Figure 4. Final Faraday rotator with test material.

By establishing direct current in the coil and using the U-shaped core, the magnetic field was created. In this experiment, three lasers as light source according to Table 1 were used to measure the rotation of the polarization plane. For the current supply, two 6-amp parallel current sources have been used. U-shaped core (electromagnets) did not produce uniform magnetic field where cuvette is placed. To determine the magnitude of the magnetic field in the cell containing the solutions, the field is measured for 2 to 12 ampere currents.

Table 1. Wavelengths for different laser generator.

Laser	Wavelengths
Green Laser Pointer	515 nm
Blue Laser Pointer	450 nm
Violet Laser Pointer	405 nm

Accordingly, the relationship between the magnetic field and the current through the coil generator is obtained:

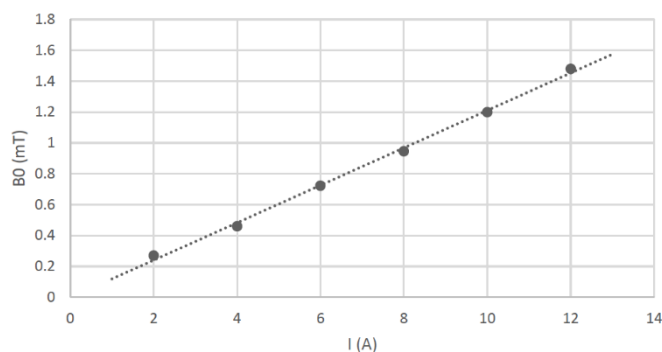


Figure 5. The magnetic field at the center of the poles by the current passing through coils.

After the magnetic field was measured at 9 points at intervals equal to the poles and for different currents, relative curve B/B_0 is drawn where, B is the field magnitude at the measurement point and B_0 is the field Magnitude at the center between the two poles. This curve was identical for all currents and the minimum magnitude of the field was in the center between the two poles. Thus, to calculate the mean magnetic field we obtain the integral of the area under the curve in distance by the cuvette length and then the result is divided by the cuvette length. Thus, the average magnetic field is estimated for each current.

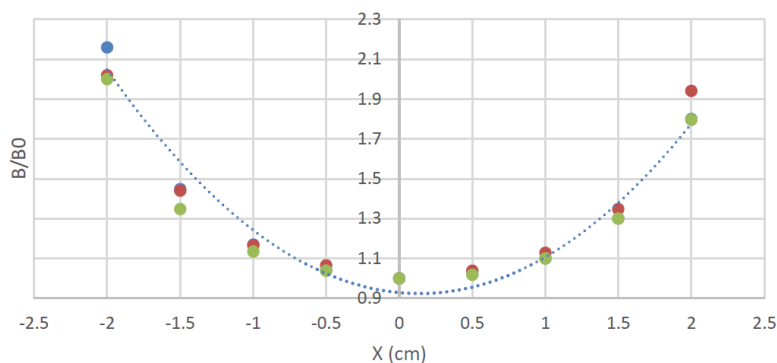


Figure 6. The magnetic field by distance.

Laser light is passed through poles and cuvette containing desired solution. Then polarizer and analyzer are placed at an angle of 90° relative to each. At any applied field, polarizer is rotated to the extent that the observed light intensity is minimized. Then the field direction is reversed. This will increase the intensity of light passing through the analyzer. Then the analyzer is rotated to the extent that the light intensity is re-minimized. $2\Delta\theta$ is the rotation amount of the analyzer.

$2\Delta\theta$ diagrams in terms of B for different solutions at a wavelength of 405 nm are shown in Figure 7:

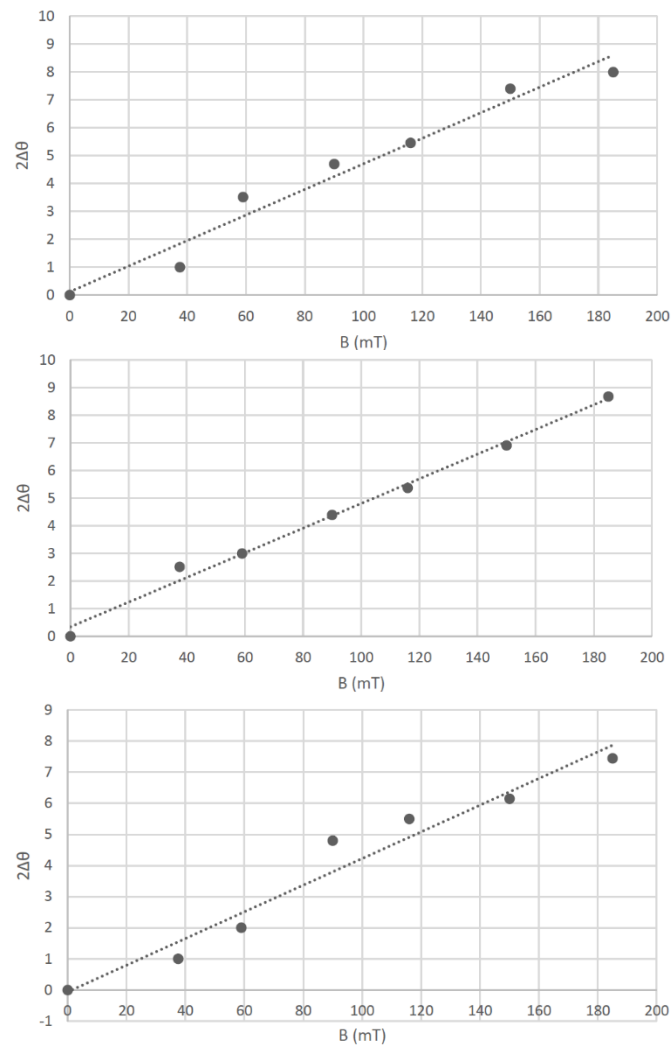
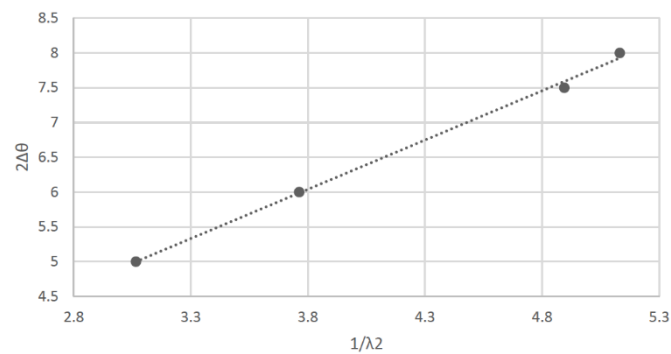


Figure 7. $2\Delta\theta$ by mean magnetic field for (a) KCl, (b) KBr, (c) MgCl₂.

As can be seen, as the light wavelength increases, the Faraday rotation is reduced and according to the Eq. 8 and Eq. 12 is changed by $1/\lambda^2$.



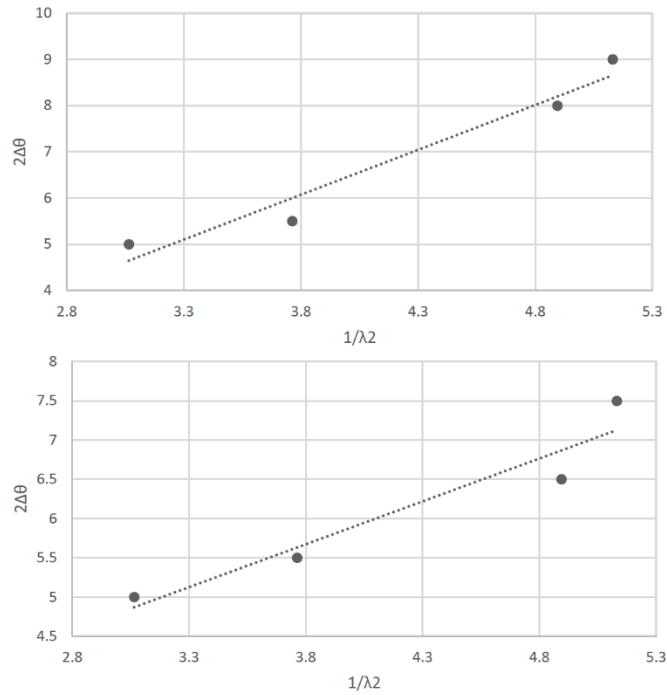


Figure 8. Measurements of the wavelength dependence of the Faraday rotation $2\Delta\theta$ for (a) KCl, (b) KBr, (c) $MgCl_2$.

It should be noted, $\frac{2\Delta\theta}{B}$ is the slope of the curves. This equation can be used to calculate Verdet Constant of any solution at different wavelengths in $\text{degree}\cdot\text{T}^{-1}\cdot\text{m}^{-1}$; the results are given in Table 2:

Table 2. Estimated Verdet constant at different wavelengths for KCl, KBr, $MgCl_2$ solutions.

515nm	450nm	405nm	V(degree/Tm)
382	565.25	602.25	KBr
429.125	494.125	595.5	KCl
311.625	449.25	705	MgCl₂

4. CONCLUSION

Applying a magnetic field to the solutions KCl and KBr and $MgCl_2$, their Verdet Constant was measured at different wavelengths. On the other hand, as angular field intensity increases polarization rotation increases. Given the Verdet constant, by selecting different samples of solutions, it can be used to measure DC currents in cables, so the solution is used as the core of an optical fiber.

References

- [1] Optics and Laser Physics Laboratory Linear Polarization of Light, *Miami University*, 2014.
- [2] Physics Experiment, *University of Michigan*, 341(4), 2010, 1-18.
- [3] Physics 231 Polarization, *Rice University*, 2014, pp. 1-10.
- [4] Polarization and Polarization Control, *NEW FOCUS Inc.*, 1993.
- [5] R. M. Besancon, Encyclopedia of Physics, *Van Nostrand*, New York, 1958.
- [6] J. E Thompson, L. H. Luessen, Fast Electrical and Optical Measurements, *Springer Science & Business Media*, 1, 1986.
- [7] F.L. Pedrotti and P. Bandettini, Faraday rotation in the undergraduate advanced laboratory, *Am. J. Phys.* 58, 1990, 542-544.

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