

Digital nuclear shell model

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ABSTRACT

The subject of this thesis is a digital approach to the investigation of the digital basis of digital nuclear shell model. The shell model is partly analogous to the atomic shell model which describes the arrangement of electrons in an atom, in that a filled shell results in greater stability. When adding nucleons to a nucleus, there are certain points where the binding energy of the next nucleon is significantly less than the last one. Magic numbers of nucleons: 2, 8, 20, 28, 50, 82, 126 which are more tightly bound than the next higher number, is the origin of the shell model. "In a three-dimensional harmonic oscillator the total degeneracy at level n is $\frac{(n+1)(n+2)}{2}$. Due to the spin, the degeneracy is doubled and is $(n+1)(n+2)$. Thus the magic numbers would be $\sum_{n=0}^k (n+1)(n+2) = \frac{(k+1)(k+2)(k+3)}{3}$ for all integer k . This gives the following magic numbers: 2,8,20,40,70,112..., which agree with experiment only in the first three entries. These numbers are twice the tetrahedral numbers (1,4,10,20,35,56...) from the Pascal Triangle". http://en.wikipedia.org/wiki/Nuclear_shell_model. The digital mechanism of shell model have been analyzed by the application of cybernetic methods, information theory and system theory, respectively. This paper is to report that we discovered new methods for development of the new technologies in nuclear physics and chemistry. It is about the most advanced digital technology which is based on program, cybernetics and informational systems and laws. The results in practical application of the new technology could be useful in physics, chemistry, bioinformatics, and other natural sciences.

Keywords: Nuclear physics; shell model; magic number of nucleons; Pascal Triangle

1. INTRODUCTION

The sequencing of the nuclear shell model is determined not only by distinct chemical features, but also by cybernetic and information principles. For this reason, research in this field deals more with the quantitative rather than qualitative characteristics of chemical information and its chemical basis. For the purposes of this paper, specific physical and chemical factors have been selected in order to express the chemical information for shell model. Numerical values are then assigned to these factors, enabling them to be measured. In this way it is possible to determine if a connection really exists between the quantitative ratios in the process of transfer of chemical information and the qualitative appearance of the shell model. To select these factors, preference is given to classical physical and chemical

parameters, their analog values, and their frequencies. There is a large number of these parameters, and each of them gives important chemical information. Going through this process, it becomes clear that there is a mathematical relationship between quantitative ratios and the qualitative appearance of the digital nuclear shell model and that there is a measurement method that can be used to describe the chemistry of this model.

2. RESULTS

“In nuclear physics, a **magic number** is a number of nucleons (either protons or neutrons) such that they are arranged into complete shells within the atomic nucleus. The seven most widely recognized magic numbers as of 2007 are **2, 8, 20, 28, 50, 82, and 126** (sequence A018226 in OEIS). Atomic nuclei consisting of such a magic number of nucleons have a higher average binding energy per nucleon than one would expect based upon predictions such as the semi-empirical mass formula and are hence more stable against nuclear decay”. ([http://en.wikipedia.org/wiki/Magic_number_\(physics\)](http://en.wikipedia.org/wiki/Magic_number_(physics)))”.

Predicted magic numbers

Toeplitz matrix {2, 8, 20, 28, 50, 82, 126}

Input:

ToeplitzMatrix[{2, 8, 20, 28, 50, 82, 126}]

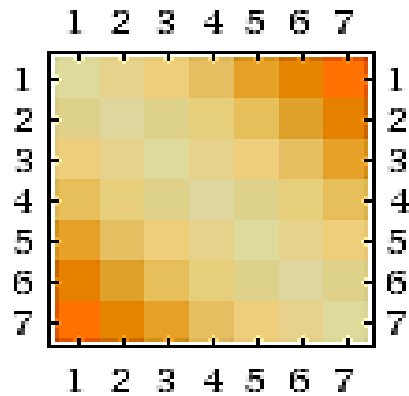
Result:

(2:2) = 1; (8:2) = 4; (20:2) = 10; etc.

$$2 \begin{pmatrix} 1 & 4 & 10 & 14 & 25 & 41 & 63 \\ 4 & 1 & 4 & 10 & 14 & 25 & 41 \\ 10 & 4 & 1 & 4 & 10 & 14 & 25 \\ 14 & 10 & 4 & 1 & 4 & 10 & 14 \\ 25 & 14 & 10 & 4 & 1 & 4 & 10 \\ 41 & 25 & 14 & 10 & 4 & 1 & 4 \\ 63 & 41 & 25 & 14 & 10 & 4 & 1 \end{pmatrix}$$

Dimensions

7 (rows) × 7 (columns)

Matrix plot:**Determinant:**

2151534827929600

$$2151534827929600 = 46384640^2;$$

symmetric

toeplitz

Determinant:

-724 375 168

Trace:

14

Characteristic polynomial:

$$-x^7 + 14x^6 + 42\,260x^5 + 1\,577\,672x^4 + 18\,080\,960x^3 + 46\,384\,640x^2 - 259\,459\,776x - 724\,375\,168$$

Eigenvalues:

$$\lambda_1 \approx 229.551$$

$$\lambda_2 \approx -177.333$$

$$\lambda_3 \approx -18.4947$$

$$\lambda_4 \approx -10.5847$$

$$\lambda_5 \approx -10.1218$$

$$\lambda_6 \approx 3.52885$$

$$\lambda_7 \approx -2.54493$$

Eigenvectors:

$$v_1 \approx (1., 0.645213, 0.464296, 0.392165, 0.464296, 0.645213, 1.)$$

$$v_2 \approx (-1., -0.613184, -0.265253, 0., 0.265253, 0.613184, 1.)$$

$$v_3 \approx (1., -0.346528, -1.19934, -1.11978, -1.19934, -0.346528, 1.)$$

$$v_4 \approx (1., -2.54939, -0.332316, 4.0758, -0.332316, -2.54939, 1.)$$

$$v_5 \approx (-1., 0.167502, 3.38277, 0., -3.38277, -0.167502, 1.)$$

$$v_6 \approx (1., -2.85677, 3.30504, -3.52554, 3.30504, -2.85677, 1.)$$

$$v_7 \approx (-1., 1.79721, -0.384607, 0., 0.384607, -1.79721, 1.)$$

Diagonalization:

$$M = S.J.S^{-1}$$

where

$$M = \begin{pmatrix} 2 & 8 & 20 & 28 & 50 & 82 & 126 \\ 8 & 2 & 8 & 20 & 28 & 50 & 82 \\ 20 & 8 & 2 & 8 & 20 & 28 & 50 \\ 28 & 20 & 8 & 2 & 8 & 20 & 28 \\ 50 & 28 & 20 & 8 & 2 & 8 & 20 \\ 82 & 50 & 28 & 20 & 8 & 2 & 8 \\ 126 & 82 & 50 & 28 & 20 & 8 & 2 \end{pmatrix}$$

$$S = \begin{pmatrix} -1 & -1 & -1 & 1 & 1 & 1 & 1 \\ -0.613184 & 0.167502 & 1.79721 & -0.346528 & -2.54939 & -2.85677 & 0.645213 \\ -0.265253 & 3.38277 & -0.384607 & -1.19934 & -0.332316 & 3.30504 & 0.464296 \\ 0 & 0 & 0 & -1.11978 & 4.0758 & -3.52554 & 0.392165 \\ 0.265253 & -3.38277 & 0.384607 & -1.19934 & -0.332316 & 3.30504 & 0.464296 \\ 0.613184 & -0.167502 & -1.79721 & -0.346528 & -2.54939 & -2.85677 & 0.645213 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$J = \begin{pmatrix} -177.333 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -10.1218 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2.54493 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -18.4947 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -10.5847 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3.52885 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 229.551 \end{pmatrix}$$

$$S^{-1} = \begin{pmatrix} -0.345697 & -0.211976 & -0.0916973 & -5.55112 \times 10^{-17} & 0.0916973 & 0.211976 & 0.345697 \\ -0.0400925 & 0.00671559 & 0.135624 & -6.93889 \times 10^{-18} & -0.135624 & -0.00671559 & 0.0400925 \\ -0.114211 & 0.20526 & -0.0439263 & 2.77556 \times 10^{-17} & 0.0439263 & -0.20526 & 0.114211 \\ 0.156964 & -0.0543926 & -0.188253 & -0.175765 & -0.188253 & -0.0543926 & 0.156964 \\ 0.0314151 & -0.0800893 & -0.0104397 & 0.128042 & -0.0104397 & -0.0800893 & 0.0314151 \\ 0.019012 & -0.054313 & 0.0628356 & -0.0670276 & 0.0628356 & -0.054313 & 0.019012 \\ 0.292608 & 0.188795 & 0.135857 & 0.114751 & 0.135857 & 0.188795 & 0.292608 \end{pmatrix}$$

Condition number:

111.705

DiagonalMatrix[{2, 8, 20, 28, 50, 82, 126}]

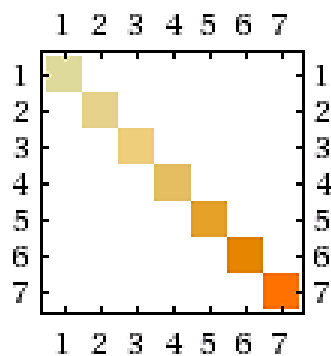
Result:

$$\begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 20 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 28 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 50 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 82 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 126 \end{pmatrix}$$

Dimensions:

7 (rows) \times 7 (columns)

Matrix plot:



Determinant:

4 628 736 000

Trace:

316

Characteristic polynomial:

$$-x^7 + 316x^6 - 36\,752x^5 + 1\,997\,872x^4 - 53\,507\,696x^3 + 679\,862\,656x^2 - 3\,475\,467\,520x + 4\,628\,736\,000$$

Eigenvalues:

$$\lambda_1 = 126$$

$$\lambda_2 = 82$$

$$\lambda_3 = 50$$

$$\lambda_4 = 28$$

$$\lambda_5 = 20$$

$$\lambda_6 = 8$$

$$\lambda_7 = 2$$

Diagonalization:

$$M = S.J.S^{-1}$$

where

$$M = \begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 20 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 28 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 50 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 82 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 126 \end{pmatrix}$$

$$S = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$J = \begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 20 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 28 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 50 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 82 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 126 \end{pmatrix}$$

$$S^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Condition number:

63

Inverse:

$$\frac{1}{516600} \begin{pmatrix} 258300 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 64575 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 25830 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 18450 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10332 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6300 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4100 \end{pmatrix}$$

The tetrahedral numbers from the Pascal Triangle

Magic numbers are twice the tetrahedral numbers (1,4,10,20,35,56...) from the Pascal Triangle”.

Example:

Row										
A	1	1	1	1	1	1	1	1	11
B	1	2	3	4	5	6	7	8	9N
C	1	3	6	10	15	21	28	36	45N
D	1	4	10	20	35	56	84	120	165N
E	1	5	15	35	70	126	210	330	495N
F	1	6	21	56	126	252	462	792	1287N
G	1	7	28	84	210	462	924	1716	3003N
H	1	8	36	120	330	792	1716	3432	6435N
J	1	9	45	165	495	1287	3003	6435	12870N

I	N	N	N	N	N	N	N	N	NN

The rows of Pascal's triangle are conventionally enumerated starting with row $n = 0$ at the top. The entries in each row are numbered from the left beginning with $k = 0$ and are usually staggered relative to the numbers in the adjacent rows. A simple construction of the triangle proceeds in the following manner. On row 0, write only the number 1. Then, to construct the elements of following rows, add the number above and to the left with the

number above and to the right to find the new value. If either the number to the right or left is not present, substitute a zero in its place. For example, the first number in the first row is $0 + 1 = 1$, whereas the numbers 1 and 3 in the third row are added to produce the number 4 in the fourth row.

Fourth diagonal

				1						
				1		1				
				1		2		1		
			1		3		3		1	
		1		4		6		4	1	
	1		5		10		10		5	1
	1	6		15		20		15	6	1
	1	7	21		35		35	21	7	1
1	8	28		56	70		56	28	8	1

				1						
				1		1				
				1		2		1		
			Sa		3		3		Sa	
		1		Sb		6		Sb	1	
	1		5		Sc		Sc		5	1
	1	6		15		Sd		15	6	1
	1	7	21		Se		Se	21	7	1
1	8	28		Sf	70		Sf	28	8	1

$$\begin{aligned}
 S_a &= S_1 = 1; \\
 S_b &= (S_1 + S_2) = (1 + 3) = 4; \\
 S_c &= (S_1 + S_2 + S_3) = (1 + 3 + 6) = 10; \\
 S_d &= (S_1 + S_2 + S_3 + S_4) = (1 + 3 + 6 + 10) = 20; \\
 S_e &= (S_1 + S_2 + S_3 + S_4 + S_5) = (1 + 3 + 6 + 10 + 15) = 35; \\
 S_f &= (S_1 + S_2 + S_3 + S_4 + S_5 + S_6) = (1 + 3 + 6 + 10 + 15 + 21) = 56; \\
 &\text{etc.}
 \end{aligned}$$

That expansion looks like

Diagonal 3	Groups numbers in diagonal 3	Tetrahedral numbers
1	1 →	1
3	(1+3) →	4
6	(1+3+6) →	10
10	(1+3+6+10) →	20
15	(1+3+6+10+15) →	35
21	(1+3+6+10+15+21) →	56
28	(1+3+6+10+15+21+28) →	84
36	(1+3+6+10+15+21+28+36) →	120

etc.

A Pentatope Numbers in the Pascal's triangle were developed with the expansion of the numbers from the diagonal 4:

That expansion looks like

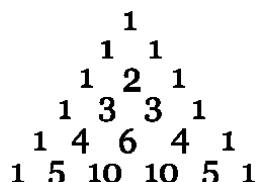
Diagonal 4	Groups numbers in diagonal 4	Pentatope numbers
1	1 →	1
4	(1+4) →	5
10	(1+4+10) →	15
20	(1+4+10+20) →	35
35	(1+4+10+20+35) →	70
56	(1+4+10+20+35+56) →	126
84	(1+4+10+20+35+56+84) →	210
120	(1+4+10+20+35+56+84+120) →	330

etc.

This construction is related to the binomial coefficients by [Pascal's rule](#), which says that if then for any nonnegative integer n and any integer k between 0 and n .

$$(x + y)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} y^k$$

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$



The first six rows of Pascal's triangle

In particular, the first six shells are:

- level 0: 2 states ($l = 0$) = 2.
- level 1: 6 states ($l = 1$) = 6.
- level 2: 2 states ($l = 0$) + 10 states ($l = 2$) = 12.
- level 3: 6 states ($l = 1$) + 14 states ($l = 3$) = 20.
- level 4: 2 states ($l = 0$) + 10 states ($l = 2$) + 18 states ($l = 4$) = 30.
- level 5: 6 states ($l = 1$) + 14 states ($l = 3$) + 22 states ($l = 5$) = 42.

where for every l there are $2l+1$ different values of m_l and 2 values of m_s , giving a total of $4l+2$ states for every specific level.

Example:

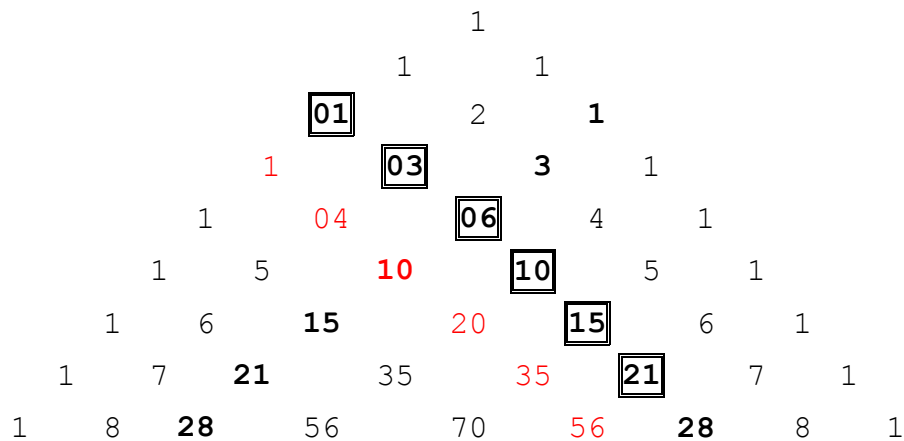
A binomial is a polynomial expression with two terms, such as:

Row Number	Binomial Expansion										Connection	Code	
A0	Â	Â	Â	Â	Â	01	Â	Â	Â	Â	1	101 ⁰	
A1	Â	Â	Â	Â	01	Â	01	Â	Â	Â	101	101 ¹	
A2	Â	Â	01	Â	02	Â	01	Â	Â	Â	10201	101 ²	
A3	Â	Â	01	Â	03	Â	03	Â	01	Â	1030301	101 ³	
A4	Â	01	Â	04	Â	06	Â	04	Â	01	104060401	101 ⁴	
A5	01	Â	05	Â	10	Â	10	Â	05	Â	01	10510100501	101 ⁵
etc.	"	"	"	"	"	"	"	"	"	"	etc.		

These numbers are twice the values of triangular numbers from the Pascal Triangle: 1,3,6,10,15,21...

Example :

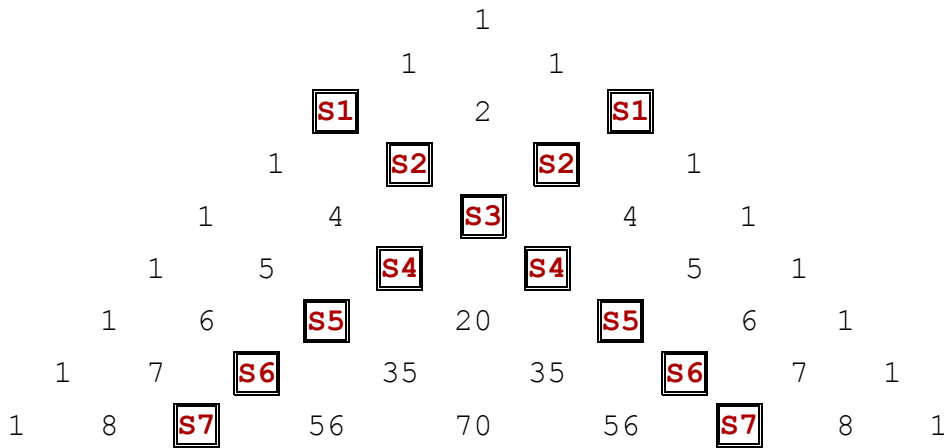
Row										
A	1	1	1	1	1	1	1	1	11
B	1	2	3	4	5	6	7	8	9N
C	1	3	6	10	15	21	28	36	45N
D	1	4	10	20	35	56	84	120	165N
E	1	5	15	35	70	126	210	330	495N
F	1	6	21	56	126	252	462	792	1287N
G	1	7	28	84	210	462	924	1716	3003N
H	1	8	36	120	330	792	1716	3432	6435N
J	1	9	45	165	495	1287	3003	6435	12870N
	·	·	·	·	·	·	·	·	·	·
	i	N	N	N	N	N	N	N	N	N



INPUT = Diagonal 1,03,06,10,15,21;
 OUTPUT = Diagonal 1,04,10,20,35,56;

TRIANGULAR NUMBER

A **triangular number** or **triangle number** numbers the objects that can form an equilateral triangle, as in the diagram on the right. The n th triangle number is the number of dots in a triangle with n dots on a side; it is the sum of the n natural numbers from 1 to n . The sequence of triangular numbers (sequence A000217 in OEIS), starting at the 0th triangular number, is: 0, 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, ...



$$\begin{aligned}
 S1 &= 1; \\
 S2 &= (1+2) = 3; \\
 S3 &= (1+2+3) = 6; \\
 S4 &= (1+2+3+4) = 10; \\
 S5 &= (1+2+3+4+5) = 15; \\
 S6 &= (1+2+3+4+5+6) = 21; \\
 S7 &= (1+2+3+4+5+6+7) = 28;
 \end{aligned}$$

Another way to predict magic (and semi-magic) numbers is by laying out the idealized filling order (with spin-orbit splitting but energy levels not overlapping). For consistency s is split into $j = 1/2$ and $j = -1/2$ components with 2 and 0 members respectively. Taking leftmost and rightmost total counts within sequences marked bounded by / here gives the magic and semi-magic numbers.

- $s(2,0)/p(4,2) > 2,2/6,8$, so (semi)magic numbers 2,2/6,8
- $d(6,4):s(2,0)/f(8,6):p(4,2) > 14,18:20,20/28,34:38,40$, so 14,20/28,40
- $g(10,8):d(6,4):s(2,0)/h(12,10):f(8,6):p(4,2) > 50,58,64,68,70,70/82,92,100,106,110,112$, so 50,70/82,112
- $i(14,12):g(10,8):d(6,4):s(2,0)/j(16,14):h(12,10):f(8,6):p(4,2) > 126,138,148,156,162,166,168,168/184,198,210,220,228,234,238,240$, so 126,168/184,240

The rightmost predicted magic numbers of each pair within the quartets bisected by / are double tetrahedral numbers from the Pascal Triangle: 2,8,20,40,70,112,168,240 are $2x$ 1,4,10,20,35,56,84,120..., and the leftmost members of the pairs differ from the rightmost by double triangular numbers: $2-2 = 0$, $8-6 = 2$, $20-14 = 6$, $40-28 = 12$, $70-50 = 20$, $112-82 = 30$, $168-126 = 42$, $240-184 = 56$, where 0,2,6,12,20,30,42,56... are $2x$ 0,1,3,6,10,15,21,28... http://en.wikipedia.org/wiki/Nuclear_shell_model.

3. CONCLUSIONS

These digital pictures reveal to us a whole new dimension to Digital nuclear shell model. They reveal to us that the cosmic process is strictly conditioned and determined by

programmatic, cybernetic and information principles. Now we have the exact scientific proofs that there is a Standard Model of Elementary Particles language that can be described by the theory of systems and cybernetics, and which functions in accordance with certain principles.

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