Adequate Theory of Oscillator: A Prelude to Verification of Classical Mechanics
Part 3

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“The essence of scientific discovery relies on the fact that one looks at the same what everyone sees and notices what nobody has seen.”

L. Pauling

ABSTRACT

In the paper, the adequate theory of oscillator is presented, being a sort of prelude to verification of the classical theory of mechanics. The developed theory is based on a properly understood notion of energy, quantum value changes of its determined measures (potentials), as well as of such changes types of sites of full energetic states which presents the essence of the true principle of the energy conservation. In the first part of the paper the principle of energy conservation was considered. Then the energetic aspects of the oscillator motion, with an exemplary real system motion, were presented in the second Part. This part of the paper is devoted to the kinetics of a body in harmonic motion and verification of the adequate theory of oscillator. At the end, the determination of the gravity acceleration by means of mathematical pendulum is performed to confirm the presented findings.

Keywords

Oscillator; Energy; Principle of energy conservation; Space-time; Potential field; Gravitation

1. INTRODUCTION

First two parts of the paper served to introduce readers into the problems of verification of the classical mechanics [1, 2]. General characteristics of the classical mechanics was presented based on the references. The exemplification of artifacts of the classical theory of oscillator was developed. Part 2 of the paper was to consider the energetic aspects of oscillator motion, by description of an exemplary real system motion in reference to the existent differential equation of oscillator motion. Part 3 covers kinetics of a body in harmonic motion and verification of the adequate theory of oscillator.
2. KINETICS OF A BODY IN HARMONIC MOTION

Kinetics is a part of body dynamics (also kinematics) which does not take into account its mass and neither inertia force by focusing the attention only on such magnitudes like the length of path/way, velocity, acceleration, impulse (properly understood as the derivative of acceleration or the third derivative of path length). These frames of considerations are sufficient to achieve an ultimate goal which was assumed by the Authors of this work. This goal is a verification of the value of acceleration of gravity just using this new adequate theory of oscillator and the determined experimental studies with its application.

Further on these mentioned magnitudes will be considered in function of time. All they are variables in the space-time with the exception of acceleration of gravity, with the value being constant.

Coming out of the very source one should write first this initial/source differential equation which was recently used also in other Authors’ works [3–6]. This may give the beginning of all description of the phenomenon of the variable motion. Its solution will be the dependence of the path/way length on time. The next step is simply to determine consecutive derivatives of the way length in view of getting the functional courses of velocity, acceleration, impulse, and further still not named (and used herewith) the kinetic magnitudes. Anyway their collection is in fact unlimited.

Firstly, here is this mentioned source differential equation:

\[ dx = \pm \frac{\partial x}{\partial t} dt \]  \hspace{1cm} (17)

Particular symbols denote here: \( dx \) – total differential of the path/way length, \( \partial x \) – partial differential of this magnitude, \( \partial t \) – partial differential of time, \( dt \) – total differential of time. Signs (+), (−) are operators, with (+) possessing a formal meaning and confirming only the physical sense of determined description of a phenomenon, and (−) attributing such a sense to this description. One should admit that, in reference to the accelerated variable motion, such operation is not indispensable. Description of the retarded motion requires a sort of intervention.

The solution of this equation (17) has been presented in [3] concerned with the fundamentals of surface smoothing by elastic grinding wheels. Just this property of tool brings about that the dynamics of abrasive grains may be considered in the machining zone. Here also the need to use the considered equation appears to describe properly in-depth the variable motion of abrasive grains.

For the accelerated motion of a body more detailed equation possesses the following form:

\[ x = x_0 \left( e^{\frac{t}{T_0}} - 1 \right) \]  \hspace{1cm} (18)

Retarded variable motion is characterized by the dependence of type:
In these equations (18), (19), the symbol $x$ denotes the path length, $x_o$ is the length of space-time, that is the distance between the neighboring potential fields, and $T$ is a time constant. The last magnitude is interpreted as the time of body transition into the neighboring potential field with a constant initial velocity.

For analyzed here the harmonic motion, the symbol $x_o$ is simultaneously the amplitude of this phenomenon. This motion, proceeding all the time with a constant amplitude, begins in fact from the lower position of the body. Let us bring it to that point of unstable static potential field by means of a determined external stimulus. The action force of this stimulus $F$, the elasticity force $S$, inertia force $B$, and the gravity force $Q$ – all these forces are in this system the measures of interaction of the particular its elements (Fig. 9). The unstable static equilibrium of the system is reflected by the following equation: $S_1 + B_1 = Q + F_1$.

![Diagram of oscillator](image)

**Fig. 9.** Unstable time equilibrium of oscillator referred to its lowest energetic position

If that external stimulus stops, then the acting force $F_1$ will disappear and the body will begin a harmonic free motion by a non-uniform displacement in the determined space-times between their potential fields.
The first space-time will be crossed by the body with the accelerated variable motion, the next one by a retarded motion. Afterwards this cycle will be repeated again and again. Courses of the path lengths (Fig. 10) are described by the following equations (18), (19).

As it is apparent, the plot of path length $x$ possesses the form of shifted progressive exponential function. This course finds its end on the neighboring stable potential field (denoted by asterisk) after a time equaling $t_o = T\ln2$. This dependence results from the formula (8) [2], after substituting $x = x_o$.

Therefore this motion is an accelerated motion.

In the second space-time, where a retarded motion takes place, the course of the path length corresponds with the shifted degressive exponential function. This dependence, in turn, results from the formula (19). It is worth noting that the time constant of this course is at the height of apparent field which overlaps (for consecutive motion, this time accelerating motion) with a stable potential field.

One may state that there is a real/proper space for the retarded motion, and further improper one where the curve of the path length (denoted with a dashed line) tends asymptotically to the mentioned apparent field.
Now the following derivatives of the path length may be determined. Thus for the accelerated motion the following relationships have been obtained:

\[ v = \dot{x} = \frac{dx}{dt} = \frac{x_0}{T} e^{\frac{t}{T}} = v_0 e^{\frac{t}{T}} \]  
(20)

\[ a = \ddot{x} = \frac{d^2x}{dt^2} = \frac{x_0}{T^2} e^{\frac{t}{T}} = \frac{v_0}{T} e^{\frac{t}{T}} = a_0 e^{\frac{t}{T}} \]  
(21)

\[ i = \dddot{x} = \frac{d^3x}{dt^3} = \frac{x_0}{T^3} e^{\frac{t}{T}} = \frac{v_0}{T^2} e^{\frac{t}{T}} = \frac{a_0}{T} e^{\frac{t}{T}} = i_0 e^{\frac{t}{T}} \]  
(22)

where \( v \) denotes the velocity, \( a \) denotes the acceleration, and \( i \) denotes just the impulse, discussed earlier, at the beginning of this chapter.

For the retarded motion, the following set of formulae concerning these physical magnitudes are obtained:

\[ v = \ddot{x} = \frac{dx}{dt} = \frac{2x_0}{T} e^{-\frac{t}{T}} = 2v_0 e^{-\frac{t}{T}} \]  
(23)

\[ a = -\ddot{x} = -\frac{d^2x}{dt^2} = \frac{2x_0}{T^2} e^{-\frac{t}{T}} = \frac{2v_0}{T} e^{-\frac{t}{T}} = 2a_0 e^{-\frac{t}{T}} \]  
(24)

\[ i = -\dddot{x} = -\frac{d^3x}{dt^3} = \frac{2x_0}{T^3} e^{-\frac{t}{T}} = \frac{2v_0}{T^2} e^{-\frac{t}{T}} = \frac{2a_0}{T} e^{-\frac{t}{T}} = 2i_0 e^{-\frac{t}{T}} \]  
(25)

Just here the algebraic operator \((-\)), enabling to obtain the formulae on the acceleration and impulse, possesses a physical sense.

In the further considerations the graphical illustrations have been omitted, concentrating the attention on huge discrepancies between them and those presented in the classical mechanics [7]. The evidence has been revealed on the example of accelerated variable motion (Fig. 11). It has been commonly known how these formulae on the path length, velocity, and acceleration were obtained in the classical theory. (Further magnitudes could not be emerged because the acceleration was the last one; a constant acceleration with the derivative equaling zero).

One begins from the linear dependence of the velocity on time and then gets to the description of the path length by integrating the former one, whereas the description of acceleration is the result of differentiating the formula on velocity.
As the result (Fig. 11a), a parabola of the path length appears, spanned over the vertexes of a triangle of the time constant and the space-time length (dashed area). This parabola and other classical courses have been denoted by a letter K in Fig. 11, as they form an aftermath of the classical imagination. The classical course of velocity (Fig. 11b) is linear and it connects the beginning of the coordinate system with final value of the velocity. This corresponds with the usual real linearization of a non-linear of such type of the system characteristics.
Shocking results of such linearization were obtained in reference to the acceleration course (Fig. 11c). Here the real characteristics of the system were replaced by a straight line, parallel to the time axis and passing through one, a final point of the characteristics. That way the acceleration in the variable motion (here the accelerated motion) is constant and positive according to the classical mechanics. To say more, in the uniform motion the acceleration is zero, and in the retarded motion it takes a constant negative value. (How far from the truth about reality!)

3. VERIFICATION OF THE ADEQUATE THEORY OF OSCILLATOR

To verify that new presented here the theory of oscillator, a spring oscillator has been designed and manufactured (Fig. 12). Thereafter the oscillator was imposed with a harmonic motion and some determined parameters were measured. That made it possible to measure the constant magnitudes which appear in the formulae describing particular characteristics of the system. A real value of the gravity acceleration was also determined by means of the mentioned oscillator.

The harmonic oscillator (Fig. 12) has a very simple structure. Its main elements are dead weight 1 and pull spring 6. This spring is caught down to the sleeve 4 linked with a weight. At the top, this spring is mounted on a bolt 5 connected with a grip 2 which in turn is embedded in an arm 3 and fixed by means of a nut 7. On one of the ends of the sleeve 4 there is a rubber connection clip 8 which grips a pen 9 seated in the hole of this sleeve.

![Fig. 12. Structure of harmonic oscillator (notations in text)](image-url)
At the beginning a series of experiments were performed with the use of a pen by recording the trajectory on the uniformly moving paper tape. It was stated that the formed traces surely correspond with the exponential curves; a progressive curve for the accelerated motion and a degressive one for the retarded variable motion. (One should admit that additional confirmation of this finding are the results of tests of vertical free cutting of material by a single abrasive grain [3]. The similar forms of traces were obtained in one machining cycle.) Undoubtedly the real trajectories differ in their shape from the sinusoid which is still used to approximate them (Fig. 13). Only three common points possess real and approximative courses. They are just on the potential fields. In the space-times an evident differentiation of both characteristics occurs.

Fig. 13. Classical sinusoidal approximation of real course of the path length of harmonically vibrating a material body

One may connect these characteristic points (only these points) with the motion of a point on the periphery of an imaginative rotating circle with the sinusoidal course in time of the motion in the projection presented on the direction of motion of the oscillator.

It is enough to compare the time of one rotation $T_o$ of this circle with the time $4t_o$ of transition of the vibrating body through four consecutive space-times (this is the period of oscillator vibration $T^* = 4t_o$).
Thus we have

\[ 4t_0 = \frac{2\pi}{\omega_0} \]  

(26)

where the right side of the equation results from the known definition of the angular velocity \( \omega_o = \frac{2\pi}{T_o} \).

Assuming further that time \( t_o = T\ln2 \), one may learn the angular velocity of rotation of that virtual circle, that is:

\[ \omega_0 = \frac{2\pi}{4T\ln2} = \frac{\pi}{T\ln2} \]  

(27)

Further on, by comparison the definition of angular velocity and the formula (27), one obtains a connection between the period of this sinusoid \( T_o \) (at the same time with the vibration period \( T' \) of oscillator) and the time constant \( T \), that is:

\[ T_0 = T' = 4T\ln2 \]  

(28)

The course of length of this sinusoidal path in time, in the system connected with the beginning of motion phenomenon, is described by the following relationship:

\[ x = x_0\left(1 - \cos \omega_0 t\right) \]  

(29)

As can be seen in Fig. 13, this curve is lying below the real characteristics for the accelerated motion and for the retarded motion it appears over that characteristics. One may state, an approximation hysteresis takes place.

Therefore the shadow of that vibrating sphere/nodule, as presented earlier in the handbook [7], will not be moving in the shadowed projection on the screen identically as the shadow of sphere rotating on the circle.

There will be the shadows of two spheres visible on the screen. As the result of speculative creativity an approximation aberration appears.

This and other consequences of unjustified approximation creations (Fig. 14) are the proof of some kind of the escapism.

That escape from the reality, as can be seen clearly in Fig. 14, leads to a strange notion of reality, far away from the truth of the considered phenomenon of the harmonic motion.
Fig. 14. Real and classical courses of length $x$, velocity $v$, and acceleration $a$ for the first cycle of variable motion of oscillator

\[ x = x_0 \left( e^t - 1 \right) \]

\[ v = v_0 e^{\frac{t}{T}} \]

\[ a = a_0 e^{\frac{t}{T}} \]

\[ x = x_0 \left( 1 - \cos \omega_0 t \right) \]

\[ v = 2v_0 e^{-\frac{t}{T}} \]

\[ a = 2a_0 e^{-\frac{t}{T}} \]

\[ a = x_0 \omega_0^2 \cos \omega_0 t \]
The middle column of the Fig. 14 presents the graphical illustration of real and approximated courses of the path length $x$, velocity $v$, and acceleration $a$, as well as the analytical records referred to these, rather fictitious, courses.

The utmost strips describe the real courses of the mentioned magnitudes, on the left side for the accelerated motion, and on the right side for a retarded motion, respectively.

The presented material clearly shows an unbelievable lack of sense of the classical descriptions of the harmonic motion. The real and fictitious courses possess only three common points on the path length. Such points do not appear at all and the areas of their changes are separated in reference to the velocity.

This issue is even worse in case of the acceleration. Here only the initial point is common and further fragments of the classical curve spread on considerably larger, that is twice as large, the area reaching even negative values.

It is time now to check, by means of the oscillator and a new subordinated theory, the value of the acceleration of gravity which is still assumed and used in science. To determine the acceleration of gravity it is enough to take one piece of the theory.

It is that one which connects the gravity $g$ with the initial inertia acceleration. It results that these accelerations describe the energetic states of the material body.

Now it is enough to take advantage of the proper formula (21) where the time of a body transition through the space-time, i.e. $t_o$, should be presented, connected with the time constant by the following relationship: $t_o = T\ln 2$. Then the existent equality $a(t_o) = g$ should be written, leading to:

$$a(t_0) = \frac{x_0}{T^2} e^{\frac{T \ln 2}{T}} = \frac{2x_0}{T^2} = g$$

(30)

that is:

$$g = \frac{2x_0}{T^2}$$

(31)

and after introducing the dependence $T = t_o/\ln 2$, to the formula (31) finally one obtains the following formula on the acceleration of gravity:

$$g = \frac{2x_0 (\ln 2)^2}{t_0^2}$$

(32)
This formula may be recorded in the form much simpler by assuming all constant values as one value \( C = 2(\ln 2)^2 = 0.961 \). Therefore:

\[
g = 0.961 \frac{x_0}{t_0^2} \quad (33)
\]

The initial static elongation of spring was \( x_s = 0.040 \) m, and the amplitude of the harmonic motion \( x_o = 0.0277 \) m. The time corresponding to one hundred periods of vibration of the oscillator motion was measured and then it was averaged, obtaining \( t_o = 0.12 \) s. Finally:

\[
g = 0.961 \frac{0.0277}{(0.12)^2} = 3.85 \text{ m} \cdot \text{s}^{-2}
\]

Now one may determine also the initial elasticity force \( S_o \), and then the coefficient of elasticity \( k \) of the spring. The mentioned force may be determined in accordance with the rule of instantaneous energy conservation (their measures, potentials) presented by the equation (10). By introducing the relation \( S_o = qQ \), into the principle structure one may write it as follows:

\[
\left( \frac{3}{2}Q + qQ \right)x_s = 3Qx_0
\]

that after its solving, due to the coefficient \( q \), gives the result:

\[
q = 3\frac{x_0}{x_s} - \frac{3}{2} \quad (34)
\]

and after introducing the variable values \( x_o, x_s - q = 0.5775 \), and \( S_o = 0.5775Q \), so finally \( S_o = 0.5775 \text{ mg} \). The weight mass \( m \) was 1 kg, so \( Q = 1 \cdot 3.85 \text{ kg} \cdot \text{m} \cdot \text{s}^{-2} = 3.85 \text{ N} \), and finally \( S_o = 0.5775 \cdot 3.85 \text{ N} = 2.223 \text{ N} \).

Now one may determine also the coefficient of elasticity \( k \), then:

\[
k = \frac{\Delta S}{x_s} = \frac{S_1 - S_0}{x_s} = \frac{Q - S_0}{x_s} \quad (36)
\]

and after introducing the above data:

\[
k = \frac{3.85 - 2.223}{0.04} = 40.675 \text{ N} \cdot \text{m}^{-1}
\]
4. CONCLUSION

In conclusion to this Part 3 of the paper it is worth stating that a verification of the adequate theory of oscillator was presented. A structure of the harmonic oscillator was revealed. It was used to determine the true acceleration of gravitation which, as it appears, is different than that commonly accepted in the literature.

The last Part 4 of the paper will be provided to reveal another method of calculation of the acceleration of gravity in view of confirming the finding presented in Part 3. There also some general conclusions will be presented.

REFERENCES


