Diffractive electroproduction of $\rho$ mesons

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ABSTRACT
A tow pomeron model is used to analyze the distribution of the $\rho$ mesons electroproduction cross section with energy for different $Q^2$. It is found that the steepness in the energy distribution can be attributed to the increase of the respective weight of the hard pomeron to the soft pomeron with increasing $Q^2$. Using some suitable functions to fit the variations of the weights with $Q^2$, a remarkable agreement with the experimental data is achieved.

Keywords: vector mesons; diffractive electroproduction

1. INTRODUCTION
In Regge phenomenology, hadrons interactions are dominated by soft pomeron exchange [1]. The pomeron trajectory is linear in $t$

$$\alpha_p(t) = \alpha_p(0) + \alpha_p' t$$

(1)

The energy behavior of the total cross section is given as

$$\sigma_T \sim (w)^\epsilon$$

(2)

with $\epsilon = 2(\alpha_p(0) - 1)$. The measured value of $\alpha_p(0)$ is around 1.08 which indicates a smooth increase of $\sigma_T$ with $w$. The value of $\alpha_p'$ is 0.25 for the soft interaction. Processes of real photon interactions with hadrons are also dominated by soft pomeron exchange [2]. Measurements of the elastic photoproduction cross section for $\rho$ [3,4], $\omega$ [5] and $\varphi$ [6] indicate that the cross sections behave like $w^{0.22}$ which is consistent with the soft pomeron exchange. By contrast the electoproduction of vector mesons [7-9] has a steep increase with energy. The rate of this steep increase is growing with increasing the virtual photon mass ($Q^2$). This means that the parameter $\epsilon$ is increasing with $Q^2$. The phenomena is called (hardening). The steep increase of the cross section with energy is also observed in the photoproduction and electroproduction of high mass vector mesons. In this case the parameter $\epsilon$ is almost constant with $Q^2$. Analysis [10-12] of the $J/\psi$ cross section shows that an intercept of around 1.4 is assumed for the pomeron trajectory. This phenomenon [13] has been explained as the presence of two different pomerons a soft pomeron and a hard pomeron.
The interactions at high \( Q^2 \) or high vector meson mass \( (m_v) \) are usually analyzed in terms of perturbative Quantum Chromo Dynamics (pQCD) [14-17]. The exclusive production of vector meson (VM) has been postulated to proceed through three steps. These are the fluctuation of the photon into q\( \bar{q} \) system, the interactions of this system with the proton through two gluon exchanges and the recombination of the q\( \bar{q} \) system into VM in the final state. The pQCD models are affected by large uncertainties related to the VM wave function, the parameterization of the gluon distribution in the proton, the overall normalization due to the higher order corrections, as well as the scale of transition to the hard region. As the pQCD models work in the region of high \( m_v \) and/or high \( Q^2 \), it is not expected to get a convincing result for the low mass vector mesons in the region of low and intermediate values of \( Q^2 \). In the present work the electroproduction of \( \rho \) mesons is analyzed. Regge theory with two pomeron model is adopted. The \( Q^2 \) dependence is included through the vertex of the coupling of the pomeron to the interacting particles.

2. THE MODEL

It is known that Regge theory is formulated on mass shell. To use the theory for a virtual photon, one has to deal with the photon virtuality. Models have introduced the photon virtuality in different ways. Some models [18,19] introduced it in the parameters of Regge pole, particularly in the pomeron intercept. Others introduced in the photon- vector meson vertex using a phenomenological \( Q^2 \) dependence [20]. The total cross section is then written as

\[
\sigma_{\gamma^* p \rightarrow \rho p}(Q^2, w) = \sigma(Q^2) \sigma_{\gamma^* p \rightarrow \rho p}(w) \tag{3}
\]

where \( \sigma(Q^2) \) involves the \( Q^2 \) dependence of the cross section, while the energy behavior of the vector meson electroproduction cross section \( \sigma_{\gamma^* p \rightarrow \rho p}(w) \) is given by. \( \sigma(Q^2) \) can be either fixed by the experiment or using a vertex function given. Therefore, the main task is to find \( \sigma_{\gamma^* p \rightarrow \rho p}(w) \). To do so, a two pomeron model is adopted for the photoproduction cross section \( (\sigma_{\gamma p \rightarrow \rho p}(w)) \). A generalization to the electroproduction case will follow. It has been found [21,22] that the inclusion of a hard pomeron in the soft data is possible and it is a necessary ingredient to obtain a good fit for all soft data. However, it is possible to assume [23] that the hard singularities manifest themselves only in the photon scattering, because the photon includes a non hadronic phase in addition to the hadronic phase. This non hadronic phase is believed to be responsible for the hard singularity. Donnachie and landshoff [24,25] indicated that a better fit for the \( \gamma p \rightarrow \rho p \) data is obtained if a hard pomeron term is included. The amplitude is given as

\[
A(s, t) = \sum_i X_i F(t)G(t)\left(\frac{s}{s_i}\right)^{\alpha_i(t)-1} e^{-\frac{1}{2}t\alpha_i(t)} \tag{4}
\]

The sum is over the exchanges of the reggeons, soft and hard pomerons. The parameters \( X_0, X_1 \) and \( X_2 \) are the couplings of the hard, soft pomerons and reggeons with interacting particles, where

\[
X_0 = 0.036 \quad X_1 = 6 \quad \text{and} \quad X_2 = 15.6 \tag{5}
\]
The parameters are in micro barn. The form factor $F(t)$ is the proton Dirac form factor

$$F(t) = \frac{4m_p^2 - 2.79t}{4m_p^2 - t} \frac{1}{(1-t/0.71)^2}$$  \hspace{1cm} (6)$$

while $G(t)$ is the $\gamma \rightarrow \rho$ transition form factor given by

$$G(t) = \frac{1}{1-t/M^2}$$  \hspace{1cm} (7)$$

with $M^2 = 0.71 \text{ GeV}^2$. The hard pomeron trajectory is given by

$$\alpha_0(t) = 1.45 + 0.1t$$  \hspace{1cm} (8)$$

The parameters $s_i$ in Eq. (4) are related to the slopes of the trajectories.

3. **THE $\sigma(Q^2)$ CROSS SECTION**

According to the experimental data, the parameterization of the cross section is given as

$$\sigma_v(Q^2) \propto \frac{1}{(Q^2+m_\rho^2)^n}$$  \hspace{1cm} (9)$$

with $n_\rho = 2.32$ for ZEUS 95 data [8] at $Q^2 > 5 \text{ GeV}^2$, $n_\rho = 2.24$ for H1 96 data and $Q^2 > 5 \text{ GeV}^2$ [26]. Therefore the value of $n_\rho$ rises steadily from 2 in the soft region to 2.6 at the highest $Q^2$. The calculation of the vertex of coupling of the pomeron to the virtual photon-vector meson vertex is given by [27]

$$f(Q^2, 0, m_\rho^2) = \frac{m_\rho^2}{Q^2+m_\rho^2} \left(1 + \frac{1}{n^2-\ln^2 \left(\frac{1+\beta_2}{1-\beta_2} \right)} \ln^2 \left(\frac{\beta_1+1}{\beta_1-1} \right) \right)$$  \hspace{1cm} (10)$$

with

$$\beta_2 = \left(1 - \frac{4m_q^2}{m_\rho^2} \right)^{1/2}$$  \hspace{1cm} (11)$$

and

$$\beta_1 = \left(1 + \frac{4m_q^2}{Q^2} \right)^{1/2}$$  \hspace{1cm} (12)$$

Then the cross section is given as

$$\sigma(Q^2) = \left[ \frac{m_\rho^2}{Q^2+m_\rho^2} f(Q^2, 0, m_\rho^2) \right]^2$$  \hspace{1cm} (13)$$
4. THE ENERGY DISTRIBUTION

The two pomeron model used in Eq.(4) produces the variation of the photoproduction cross section with energy. This behavior is clearly inconsistent with that observed [28-30] for $\rho$ meson electroproduction. The electroproduction cross section increases steeply with energy and the rate of the increase is growing with $Q^2$ (the hardening) as shown in Fig.(1). The amplitude should reflect that behavior instead of the soft behavior given by Eq. (4).

In the present case, it is assumed that the weights of the hard and the soft pomeron are functions of $Q^2$. Therefore, the steep increase with energy is attributed to the increase in the hard pomeron weight accompanied by a decrease in the soft pomeron weight as $Q^2$ increases. To calculate a such variation with $Q^2$ the following procedure is adopted:

The hard and the soft pomeron terms in Eq.(4) are multiplied by the factors $c_1$ and $c_2$ respectively. The reggeon term is kept as it is. Using this new amplitude and Eq.(13) to calculate the total cross section. At each value of $Q^2$ in Fig.(1), the parameters $c_1$ and $c_2$ are varied until a good fit for the data points is obtained. The variation of the parameters is inspired by the fact that the hard pomeron should dominate the cross section at high $Q^2$. Of course, the values at $Q^2 = 0$ are $c_1 = 1$ and $c_2 = 1$.

![Fig. 1. Energy distribution of $\rho$ mesons electroproduction cross section at different values of $Q^2$. The solid curves represent the prediction from the model using the forms in Eqs.(14,15).](image)

The data points are explained in Ref. [30]. The results of variation of the pomeron weights with $Q^2$ are tabulated in Table (1).
Table 1. Hard pomeron weight (c1) and soft pomeron weight (c2) as a function of $Q^2$.

<table>
<thead>
<tr>
<th>$Q^2$</th>
<th>0</th>
<th>9.5</th>
<th>14.5</th>
<th>22</th>
<th>35</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>1</td>
<td>2.7</td>
<td>3.7</td>
<td>4.8</td>
<td>6.3</td>
</tr>
<tr>
<td>C2</td>
<td>1</td>
<td>0.96</td>
<td>0.9</td>
<td>0.75</td>
<td>.53</td>
</tr>
</tbody>
</table>

The values of c1 and c2 are plotted against $Q^2$ and represented by the data points in Fig.(2) and Fig.(3).

![Graph showing the hard pomeron weight as a function of $Q^2$](image)

**Fig. 2.** The hard pomeron weight as a function of $Q^2$. The solid curve represents the fit from Eq.(14).

It is clear from these figures that the contribution from the hard pomeron increases with $Q^2$ and that of the soft pomeron decreases. The best fit for the data in Fig.(2) is given as by the equation

$$1 + n1(Q^2)^{n2}$$  \hspace{1cm} (14)

with $n1 = 0.36$ and $n2 = 0.75$. On the other hand, the fit for the data in
Fig. 3. The soft pomeron weight as a function of $Q^2$. The solid curve represents the fit from Eq.(15).

Fig. 4. $\rho$ meson electroproduction total cross section as a function of $Q^2$. The solid curve is the prediction from the model using Eq.(14,15). The data points are explained in Ref. [30].
Fig.(3) is given by the equation
\[
\frac{1}{1+y^2} \quad (15)
\]
with \( y = \frac{Q^2}{Q_0} \) and \( Q_0 = 37 \text{ GeV}^2 \). We notice that \( y < 1 \). In terms of Eq. (14,15), the total cross sections for each \( Q^2 \) are represented by the curves in Fig.(1). A remarkable agreement with the data is achieved. The distribution with \( Q^2 \) is also shown in Fig.(4) using Eq.(14,15). Again a reasonable agreement with the data is achieved.

5. DISCUSSION AND CONCLUSION

A two pomeron model is used to analyze the distribution of the \( \rho \) mesons electroproduction with energy for different \( Q^2 \). Regge theory relates the energy dependence of the amplitude to the position of the pole in the complex angular momentum plane. The position is independent of \( Q^2 \) [31]. Therefore, it is hard to believe that one pomeron model in which the pomeron parameters are function of \( Q^2 \) is suitable. The two pomeron model with the pomeron weights are function of \( Q^2 \) is more acceptable. Accurate forms of these functions are necessary to reproduce the variation of the cross section with energy at different \( Q^2 \).

References


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