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Space-Time Geometry of Electromagnetic Field in the System of Photon

Mukul Chandra Das[1,2,*], Rampada Misra[3]

[1]Singhania University, Pacheri Bari - 333515, Jhunjhunu, Rajasthan, India

[2]Satmile High School, Satmile - 721452, Contai, West Bengal, India

[3]Department of Electronics, Vidyasagar University, Midnapore-721102, West Bengal, India

*E-mail address: mukuldas.100@gmail.com

ABSTRACT

In the concept of general relativity gravity is the space-time geometry. Again, a relation between electromagnetic field and gravitational field is expected. In this paper, space-time geometry of electromagnetic field in the system of photon has been introduced to unify electromagnetic field and gravitational field in flat and curvature space-time.

***Keywords*:** space-time geometry; unified field; electromagnetic field

1. INTRODUCTION

In physics, a unified field theory is a type that allows all fundamental forces and elementary particles to be written in terms of a single field.

The term was proposed by Einstein, who attempted to unify the general theory of relativity with electromagnetism. According to Einstein's general relativity [2,3], gravity is the space-time geometry. Also, he suggested [4] the field equation for the gravity of an electromagnetic wave as $G_{\alpha\beta} = -KT(E)_{ab}$ where, $G_{\alpha\beta}$ is the Einstein tensor, and K is the coupling constant. But, the problem of the unification of fundamental fields into a single theory has not been solved until now in a satisfactory manner, although, in different time, a lot of papers have been published which attempt to unify the fundamental fields.

Recently, in [1], a relation between electromagnetic field and gravitational field has been introduced by considering a super system in photon. In this paper a trial has been made to introduce a geometrical relation between electromagnetic field and gravitational field.

2. SPACE-TIME GEOMETRY OF SYSTEMS

In [1], to clarify two simultaneous superimposed motion (either linear or rotational), three types of system has been assumed which are L-L system, S-S system and S-L system;

depending upon the S-L system SSP picture of photon has been considered; also, using this picture (SSP) a connection between electro-magnetic field ($\psi_\alpha(r,t)$) and gravitational field ($G'_\alpha(r',t')$) has been introduced by the relation

$$\psi_{\alpha}(r,t) = \Upsilon_1 \, \bar{Z}_{ij} G'_{\alpha}(r',t') \tag{1}$$

where, $\bar{Z}_{ij}$ are transformation matrix in the picture of SSP.
It is also pointed out that to clarify L-L or S-S or S-L system, four reference frames (S, S_1, S_2, S_3) has been considered in a simultaneous superimposed form.

Relation for co-ordinate transformation from S_3 to S in S-L system [1] is

$$X(x,y,z,t) = \bar{Z}_{i\,j} X'(x',y',z',t') \tag{2}$$

where, $\bar{Z}_{ij}$ is co-ordinate transformation matrix and the co-ordinates of an event in S_3 be

$X'(x',y',z',t') = \begin{pmatrix} x' \\ y' \\ z' \\ t' \end{pmatrix}$ which would be $X(x,y,z,t) = \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}$ with respectively in S.

Now, following the space-time geometry as in [5], one can introduced the Space-time geometry of the said system as stated below

From (2) we obtain

$$\begin{aligned} dx^2 &= (\bar{Z}_{11}dx' + \bar{Z}_{12}dy' + \bar{Z}_{13}dz' + \bar{Z}_{14}dt')^2 \\ dy^2 &= (\bar{Z}_{21}dx' + \bar{Z}_{22}dy' + \bar{Z}_{23}dz' + \bar{Z}_{24}dt')^2 \\ dz^2 &= (\bar{Z}_{31}dx' + \bar{Z}_{32}dy' + \bar{Z}_{33}dz' + \bar{Z}_{34}dt')^2 \\ dt^2 &= (\bar{Z}_{41}dx' + \bar{Z}_{42}dy' + \bar{Z}_{43}dz' + \bar{Z}_{44}dt')^2 \end{aligned} \tag{3}$$

Now, we have Cartesian Co-ordinate geometry in flat space-time

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 \tag{4}$$

Using (3) we obtain from (4) the space-time geometry in S-L system

$$\begin{aligned} ds^2 = P_1 dx'^2 + P_2 dy'^2 + P_3 dz'^2 + P_4 dt'^2 + 2(\mathrm{Q}_1 dx'dy' + \mathrm{Q}_2 dx'dz' \\ + \mathrm{Q}_3 dx'dt' + \mathrm{Q}_4 dy'dz' + \mathrm{Q}_5 dy'dt' + \mathrm{Q}_6 dz'dt') \end{aligned} \tag{5}$$

where,

$$P_1 = \bar{Z}_{11}{}^2 + \bar{Z}_{21}{}^2 + \bar{Z}_{31}{}^2 - \bar{Z}_{41}{}^2, \qquad P_2 = \bar{Z}_{12}{}^2 + \bar{Z}_{22}{}^2 + \bar{Z}_{32}{}^2 - \bar{Z}_{42}{}^2$$

$$P_3 = \bar{Z}_{13}^{\,2} + \bar{Z}_{23}^{\,2} + \bar{Z}_{33}^{\,2} - \bar{Z}_{43}^{\,2}, \qquad P_4 = \bar{Z}_{14}^{\,2} + \bar{Z}_{24}^{\,2} + \bar{Z}_{34}^{\,2} - \bar{Z}_{44}^{\,2}$$
$$\mathrm{Q}_1 = \bar{Z}_{11}\bar{Z}_{12} + \bar{Z}_{21}\bar{Z}_{22} + \bar{Z}_{31}\bar{Z}_{32} - \bar{Z}_{41}\bar{Z}_{42}, \qquad \mathrm{Q}_2 = \bar{Z}_{11}\bar{Z}_{13} + \bar{Z}_{21}\bar{Z}_{23} + \bar{Z}_{31}\bar{Z}_{33} - \bar{Z}_{41}\bar{Z}_{43}$$
$$\mathrm{Q}_3 = \bar{Z}_{11}\bar{Z}_{14} + \bar{Z}_{21}\bar{Z}_{24} + \bar{Z}_{31}\bar{Z}_{34} - \bar{Z}_{41}\bar{Z}_{44}, \qquad \mathrm{Q}_4 = \bar{Z}_{12}\bar{Z}_{13} + \bar{Z}_{22}\bar{Z}_{23} + \bar{Z}_{32}\bar{Z}_{33} - \bar{Z}_{42}\bar{Z}_{43}$$
$$\mathrm{Q}_5 = \bar{Z}_{12}\bar{Z}_{14} + \bar{Z}_{22}\bar{Z}_{24} + \bar{Z}_{32}\bar{Z}_{34} - \bar{Z}_{42}\bar{Z}_{44}, \qquad \mathrm{Q}_6 = \bar{Z}_{13}\bar{Z}_{14} + \bar{Z}_{23}\bar{Z}_{24} + \bar{Z}_{33}\bar{Z}_{34} - \bar{Z}_{43}\bar{Z}_{44}$$

Again, relation for co-ordinate transformation from S_3 to S in S-S system [1] is

$$X(x, y, z, t) = \bar{S}_{i\,j} X'(x', y', z', t') \tag{6}$$

where, $\bar{S}_{i\,j}$ is co-ordinate transformation matrix.

From (6) we obtain

$$\begin{aligned} dx^2 &= (\bar{S}_{11}dx' + \bar{S}_{12}dy' + \bar{S}_{13}dz' + \bar{S}_{14}dt')^2 \\ dy^2 &= (\bar{S}_{21}dx' + \bar{S}_{22}dy' + \bar{S}_{23}dz' + \bar{S}_{24}dt')^2 \\ dz^2 &= (\bar{S}_{31}dx' + \bar{S}_{32}dy' + \bar{S}_{33}dz' + \bar{S}_{34}dt')^2 \\ dt^2 &= (\bar{S}_{41}dx' + \bar{S}_{42}dy' + \bar{S}_{43}dz' + \bar{S}_{44}dt')^2 \end{aligned} \tag{7}$$

Using (4) and (7) we obtain the space-time geometry in S-S system as in (5) where,

$$P_1 = \bar{S}_{11}^{\,2} + \bar{S}_{21}^{\,2} + \bar{S}_{31}^{\,2} - \bar{S}_{41}^{\,2}, \qquad P_2 = \bar{S}_{12}^{\,2} + \bar{S}_{22}^{\,2} + \bar{S}_{32}^{\,2} - \bar{S}_{42}^{\,2}$$
$$P_3 = \bar{S}_{13}^{\,2} + \bar{S}_{23}^{\,2} + \bar{S}_{33}^{\,2} - \bar{S}_{43}^{\,2}, \qquad P_4 = \bar{S}_{14}^{\,2} + \bar{S}_{24}^{\,2} + \bar{S}_{34}^{\,2} - \bar{S}_{44}^{\,2}$$
$$\mathrm{Q}_1 = \bar{S}_{11}\bar{S}_{12} + \bar{S}_{21}\bar{S}_{22} + \bar{S}_{31}\bar{S}_{32} - \bar{S}_{41}\bar{S}_{42}, \qquad \mathrm{Q}_2 = \bar{S}_{11}\bar{S}_{13} + \bar{S}_{21}\bar{S}_{23} + \bar{S}_{31}\bar{S}_{33} - \bar{S}_{41}\bar{S}_{43}$$
$$\mathrm{Q}_3 = \bar{S}_{11}\bar{S}_{14} + \bar{S}_{21}\bar{S}_{24} + \bar{S}_{31}\bar{S}_{34} - \bar{S}_{41}\bar{S}_{44}, \qquad \mathrm{Q}_4 = \bar{S}_{12}\bar{S}_{13} + \bar{S}_{22}\bar{S}_{23} + \bar{S}_{32}\bar{S}_{33} - \bar{S}_{42}\bar{S}_{43}$$
$$\mathrm{Q}_5 = \bar{S}_{12}\bar{S}_{14} + \bar{S}_{22}\bar{S}_{24} + \bar{S}_{32}\bar{S}_{34} - \bar{S}_{42}\bar{S}_{44}, \qquad \mathrm{Q}_6 = \bar{S}_{13}\bar{S}_{14} + \bar{S}_{23}\bar{S}_{24} + \bar{S}_{33}\bar{S}_{34} - \bar{S}_{43}\bar{S}_{44}$$

3. SPACE-TIME GEOMETRY OF ELECTROMAGNETIC FIELD IN PHOTON

Since picture of SSP depends upon the S-L system so, following (1) and using $dx' = dx^g$, $dy' = dy^g$, $dz' = dz^g$, $dt' = dt^g$ we obtain from (5), the space-time geometry of electromagnetic field in the SSP

$$\begin{aligned} (ds^{em})^2 = {} & P_1(dx^g)^2 + P_2(dy^g)^2 + P_3(dz^g)^2 + P_4(dt^g)^2 + 2(\mathrm{Q}_1 dx^g dy^g \\ & + \mathrm{Q}_2 dx^g dz^g + \mathrm{Q}_3 dx^g dt^g + \mathrm{Q}_4 dy^g dz^g + \mathrm{Q}_5 dy^g dt^g + \mathrm{Q}_6 dz^g dt^g) \end{aligned} \tag{8}$$

where, superscript ‘g’ represents the gravitational system and superscript *em* represents electromagnetic system.

Following the convention as in (1), one may assume a relation between electromagnetic field and gravitational field as

$$\psi_\alpha(r,t) = \overline{\Upsilon}_1 \, \overline{S}_{ij} G'_\alpha(r',t') \tag{9}$$

where, $\overline{\Upsilon}_1$ is a constant and $\overline{S}_{ij}$ is transformation matrix in S-S system.

This means that, in S-S system, gravitational field of frame S_3 would be electromagnetic field with respect to frame S. For this system space-time geometry of electromagnetic field would also be as in (8) where, transformation matrix would be $\overline{S}_{ij}$

However, equation (8) would be the space-time geometry of electromagnetic field connecting gravitational field and electromagnetic field in the system of photon.

4. CONCLUSION

Equation (8) represents a picture of space-time geometry of the electromagnetic field in the system of photon. This implies that a geometrical relation is existed in between electromagnetic field and gravitational field.

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