

Space-Time Geometry of Electromagnetic Field in the System of Photon

Mukul Chandra Das^{1,2,*}, Rampada Misra³

¹Singhania University, Pacheri Bari - 333515, Jhunjhunu, Rajasthan, India

²Satmile High School, Satmile - 721452, Contai, West Bengal, India

³Department of Electronics, Vidyasagar University, Midnapore-721102, West Bengal, India

*E-mail address: mukuldas.100@gmail.com

ABSTRACT

In the concept of general relativity gravity is the space-time geometry. Again, a relation between electromagnetic field and gravitational field is expected. In this paper, space-time geometry of electromagnetic field in the system of photon has been introduced to unify electromagnetic field and gravitational field in flat and curvature space-time.

Keywords: space-time geometry; unified field; electromagnetic field

1. INTRODUCTION

In physics, a unified field theory is a type that allows all fundamental forces and elementary particles to be written in terms of a single field.

The term was proposed by Einstein, who attempted to unify the general theory of relativity with electromagnetism. According to Einstein's general relativity [2,3], gravity is the space-time geometry. Also, he suggested [4] the field equation for the gravity of an electromagnetic wave as $G_{\alpha\beta} = -KT(E)_{ab}$ where, $G_{\alpha\beta}$ is the Einstein tensor, and K is the coupling constant. But, the problem of the unification of fundamental fields into a single theory has not been solved until now in a satisfactory manner, although, in different time, a lot of papers have been published which attempt to unify the fundamental fields.

Recently, in [1], a relation between electromagnetic field and gravitational field has been introduced by considering a super system in photon. In this paper a trial has been made to introduce a geometrical relation between electromagnetic field and gravitational field.

2. SPACE-TIME GEOMETRY OF SYSTEMS

In [1], to clarify two simultaneous superimposed motion (either linear or rotational), three types of system has been assumed which are L-L system, S-S system and S-L system; depending upon the S-L system SSP picture of photon has been considered; also, using this picture (SSP) a connection between electro-magnetic field ($\psi_{\alpha}(r,t)$) and gravitational field ($G'_{\alpha}(r',t')$) has been introduced by the relation

$$\psi_\alpha(r, t) = \Upsilon_1 \bar{Z}_{ij} G'_\alpha(r', t') \quad (1)$$

where, \bar{Z}_{ij} are transformation matrix in the picture of SSP.

It is also pointed out that to clarify L-L or S-S or S-L system, four reference frames (S, S_1, S_2, S_3) has been considered in a simultaneous superimposed form.

Relation for co-ordinate transformation from S_3 to S in S-L system [1] is

$$X(x, y, z, t) = \bar{Z}_{ij} X'(x', y', z', t') \quad (2)$$

where, \bar{Z}_{ij} is co-ordinate transformation matrix and the co-ordinates of an event in S_3 be

$$X'(x', y', z', t') = \begin{pmatrix} x' \\ y' \\ z' \\ t' \end{pmatrix} \text{ which would be } X(x, y, z, t) = \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} \text{ with respectively in } S.$$

Now, following the space-time geometry as in [5], one can introduced the Space-time geometry of the said system as stated below

From (2) we obtain

$$\begin{aligned} dx^2 &= (\bar{Z}_{11} dx' + \bar{Z}_{12} dy' + \bar{Z}_{13} dz' + \bar{Z}_{14} dt')^2 \\ dy^2 &= (\bar{Z}_{21} dx' + \bar{Z}_{22} dy' + \bar{Z}_{23} dz' + \bar{Z}_{24} dt')^2 \\ dz^2 &= (\bar{Z}_{31} dx' + \bar{Z}_{32} dy' + \bar{Z}_{33} dz' + \bar{Z}_{34} dt')^2 \\ dt^2 &= (\bar{Z}_{41} dx' + \bar{Z}_{42} dy' + \bar{Z}_{43} dz' + \bar{Z}_{44} dt')^2 \end{aligned} \quad (3)$$

Now, we have Cartesian Co-ordinate geometry in flat space-time

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 \quad (4)$$

Using (3) we obtain from (4) the space-time geometry in S-L system

$$\begin{aligned} ds^2 &= P_1 dx'^2 + P_2 dy'^2 + P_3 dz'^2 + P_4 dt'^2 + 2(Q_1 dx' dy' + Q_2 dx' dz' \\ &\quad + Q_3 dx' dt' + Q_4 dy' dz' + Q_5 dy' dt' + Q_6 dz' dt') \end{aligned} \quad (5)$$

where,

$$P_1 = \bar{Z}_{11}^2 + \bar{Z}_{21}^2 + \bar{Z}_{31}^2 - \bar{Z}_{41}^2, \quad P_2 = \bar{Z}_{12}^2 + \bar{Z}_{22}^2 + \bar{Z}_{32}^2 - \bar{Z}_{42}^2$$

$$\begin{aligned}
P_3 &= \bar{Z}_{13}^2 + \bar{Z}_{23}^2 + \bar{Z}_{33}^2 - \bar{Z}_{43}^2, & P_4 &= \bar{Z}_{14}^2 + \bar{Z}_{24}^2 + \bar{Z}_{34}^2 - \bar{Z}_{44}^2 \\
Q_1 &= \bar{Z}_{11}\bar{Z}_{12} + \bar{Z}_{21}\bar{Z}_{22} + \bar{Z}_{31}\bar{Z}_{32} - \bar{Z}_{41}\bar{Z}_{42}, & Q_2 &= \bar{Z}_{11}\bar{Z}_{13} + \bar{Z}_{21}\bar{Z}_{23} + \bar{Z}_{31}\bar{Z}_{33} - \bar{Z}_{41}\bar{Z}_{43} \\
Q_3 &= \bar{Z}_{11}\bar{Z}_{14} + \bar{Z}_{21}\bar{Z}_{24} + \bar{Z}_{31}\bar{Z}_{34} - \bar{Z}_{41}\bar{Z}_{44}, & Q_4 &= \bar{Z}_{12}\bar{Z}_{13} + \bar{Z}_{22}\bar{Z}_{23} + \bar{Z}_{32}\bar{Z}_{33} - \bar{Z}_{42}\bar{Z}_{43} \\
Q_5 &= \bar{Z}_{12}\bar{Z}_{14} + \bar{Z}_{22}\bar{Z}_{24} + \bar{Z}_{32}\bar{Z}_{34} - \bar{Z}_{42}\bar{Z}_{44}, & Q_6 &= \bar{Z}_{13}\bar{Z}_{14} + \bar{Z}_{23}\bar{Z}_{24} + \bar{Z}_{33}\bar{Z}_{34} - \bar{Z}_{43}\bar{Z}_{44}
\end{aligned}$$

Again, relation for co-ordinate transformation from S_3 to S in S-S system [1] is

$$X(x, y, z, t) = \bar{S}_{ij}X'(x', y', z', t') \quad (6)$$

where, \bar{S}_{ij} is co-ordinate transformation matrix.

From (6) we obtain

$$\begin{aligned}
dx^2 &= (\bar{S}_{11}dx' + \bar{S}_{12}dy' + \bar{S}_{13}dz' + \bar{S}_{14}dt')^2 \\
dy^2 &= (\bar{S}_{21}dx' + \bar{S}_{22}dy' + \bar{S}_{23}dz' + \bar{S}_{24}dt')^2 \\
dz^2 &= (\bar{S}_{31}dx' + \bar{S}_{32}dy' + \bar{S}_{33}dz' + \bar{S}_{34}dt')^2 \\
dt^2 &= (\bar{S}_{41}dx' + \bar{S}_{42}dy' + \bar{S}_{43}dz' + \bar{S}_{44}dt')^2
\end{aligned} \quad (7)$$

Using (4) and (7) we obtain the space-time geometry in S-S system as in (5) where,

$$\begin{aligned}
P_1 &= \bar{S}_{11}^2 + \bar{S}_{21}^2 + \bar{S}_{31}^2 - \bar{S}_{41}^2, & P_2 &= \bar{S}_{12}^2 + \bar{S}_{22}^2 + \bar{S}_{32}^2 - \bar{S}_{42}^2 \\
P_3 &= \bar{S}_{13}^2 + \bar{S}_{23}^2 + \bar{S}_{33}^2 - \bar{S}_{43}^2, & P_4 &= \bar{S}_{14}^2 + \bar{S}_{24}^2 + \bar{S}_{34}^2 - \bar{S}_{44}^2 \\
Q_1 &= \bar{S}_{11}\bar{S}_{12} + \bar{S}_{21}\bar{S}_{22} + \bar{S}_{31}\bar{S}_{32} - \bar{S}_{41}\bar{S}_{42}, & Q_2 &= \bar{S}_{11}\bar{S}_{13} + \bar{S}_{21}\bar{S}_{23} + \bar{S}_{31}\bar{S}_{33} - \bar{S}_{41}\bar{S}_{43} \\
Q_3 &= \bar{S}_{11}\bar{S}_{14} + \bar{S}_{21}\bar{S}_{24} + \bar{S}_{31}\bar{S}_{34} - \bar{S}_{41}\bar{S}_{44}, & Q_4 &= \bar{S}_{12}\bar{S}_{13} + \bar{S}_{22}\bar{S}_{23} + \bar{S}_{32}\bar{S}_{33} - \bar{S}_{42}\bar{S}_{43} \\
Q_5 &= \bar{S}_{12}\bar{S}_{14} + \bar{S}_{22}\bar{S}_{24} + \bar{S}_{32}\bar{S}_{34} - \bar{S}_{42}\bar{S}_{44}, & Q_6 &= \bar{S}_{13}\bar{S}_{14} + \bar{S}_{23}\bar{S}_{24} + \bar{S}_{33}\bar{S}_{34} - \bar{S}_{43}\bar{S}_{44}
\end{aligned}$$

3. SPACE-TIME GEOMETRY OF ELECTROMAGNETIC FIELD IN PHOTON

Since picture of SSP depends upon the S-L system so, following (1) and using $dx' = dx^g$, $dy' = dy^g$, $dz' = dz^g$, $dt' = dt^g$ we obtain from (5), the space-time geometry of electromagnetic field in the SSP

$$\begin{aligned}
(ds^{em})^2 &= P_1(dx^g)^2 + P_2(dy^g)^2 + P_3(dz^g)^2 + P_4(dt^g)^2 + 2(Q_1dx^gdy^g \\
&+ Q_2dx^g dz^g + Q_3dx^g dt^g + Q_4dy^g dz^g + Q_5dy^g dt^g + Q_6dz^g dt^g)
\end{aligned} \quad (8)$$

where, superscript 'g' represents the gravitational system and superscript *em* represents electromagnetic system.

Following the convention as in (1), one may assume a relation between electromagnetic field and gravitational field as

$$\psi_{\alpha}(r, t) = \bar{Y}_1 \bar{S}_{ij} G'_{\alpha}(r', t') \quad (9)$$

where, \bar{Y}_1 is a constant and \bar{S}_{ij} is transformation matrix in S-S system.

This means that, in S-S system, gravitational field of frame S_3 would be electromagnetic field with respect to frame S . For this system space-time geometry of electromagnetic field would also be as in (8) where, transformation matrix would be \bar{S}_{ij}

However, equation (8) would be the space-time geometry of electromagnetic field connecting gravitational field and electromagnetic field in the system of photon.

4. CONCLUSION

Equation (8) represents a picture of space-time geometry of the electromagnetic field in the system of photon. This implies that a geometrical relation is existed in between electromagnetic field and gravitational field.

References

- [1] M. C. Das, R. Misra, *International Letters of Chemistry, Physics and Astronomy* 7(2) (2013) 73-84.
- [2] J. B. Hartle, "Gravity: An Introduction to Einstein's General Relativity," 3rd Edition, Pearson Education Inc, India, 2009,
- [3] A. Einstein, *Annalen der Physik* 354(7) (1916) 769-822, doi:10.1002/andp.19163540702.
- [4] Einstein A., *Physics and relativity (1936) in Ideas and Opinions*, Crown, New York 1954, p. 311.
- [5] M. C. Das, R. Misra, *International Letters of Chemistry, Physics and Astronomy* 8(3) (2013) 215-219.

(Received 15 April 2013; accepted 18 April 2013)