Space-Time Geometry of Electromagnetic Field in the System of Photon

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ABSTRACT

In the concept of general relativity gravity is the space-time geometry. Again, a relation between electromagnetic field and gravitational field is expected. In this paper, space-time geometry of electromagnetic field in the system of photon has been introduced to unify electromagnetic field and gravitational field in flat and curvature space-time.

Keywords: space-time geometry; unified field; electromagnetic field

1. INTRODUCTION

In physics, a unified field theory is a type that allows all fundamental forces and elementary particles to be written in terms of a single field.

The term was proposed by Einstein, who attempted to unify the general theory of relativity with electromagnetism. According to Einstein’s general relativity [2,3], gravity is the space-time geometry. Also, he suggested [4] the field equation for the gravity of an electromagnetic wave as \( G_{\alpha\beta} = -K T(\E)_{\alpha\beta} \) where, \( G_{\alpha\beta} \) is the Einstein tensor, and \( K \) is the coupling constant. But, the problem of the unification of fundamental fields into a single theory has not been solved until now in a satisfactory manner, although, in different time, a lot of papers have been published which attempt to unify the fundamental fields.

Recently, in [1], a relation between electromagnetic field and gravitational field has been introduced by considering a super system in photon. In this paper a trial has been made to introduce a geometrical relation between electromagnetic field and gravitational field.

2. SPACE-TIME GEOMETRY OF SYSTEMS

In [1], to clarify two simultaneous superimposed motion (either linear or rotational), three types of system has been assumed which are L-L system, S-S system and S-L system; depending upon the S-L system SSP picture of photon has been considered; also, using this picture (SSP) a connection between electro-magnetic field \((\psi_\alpha(r,t))\) and gravitational field \((G'_\alpha(r',t'))\) has been introduced by the relation
where, $\mathbf{Z}_{ij}$ are transformation matrix in the picture of SSP.  

It is also pointed out that to clarify L-L or S-S or S-L system, four reference frames $(S, S_1, S_2, S_3)$ has been considered in a simultaneous superimposed form.

Relation for co-ordinate transformation from $S_3$ to $S$ in S-L system $[1]$ is

$$X(x, y, z, t) = \mathbf{Z}_{ij} X'(x', y', z', t')$$

where, $\mathbf{Z}_{ij}$ is co-ordinate transformation matrix and the co-ordinates of an event in $S_3$ be

$$X'(x', y', z', t') = \begin{pmatrix} x' \\ y' \\ z' \\ t' \end{pmatrix}$$

which would be $X(x, y, z, t) = \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}$ with respectively in $S$.

Now, following the space-time geometry as in $[5]$, one can introduced the Space-time geometry of the said system as stated below

From (2) we obtain

$$dx^2 = (\mathbf{Z}_{11} dx' + \mathbf{Z}_{12} dy' + \mathbf{Z}_{13} dz' + \mathbf{Z}_{14} dt')^2$$

$$dy^2 = (\mathbf{Z}_{21} dx' + \mathbf{Z}_{22} dy' + \mathbf{Z}_{23} dz' + \mathbf{Z}_{24} dt')^2$$

$$dz^2 = (\mathbf{Z}_{31} dx' + \mathbf{Z}_{32} dy' + \mathbf{Z}_{33} dz' + \mathbf{Z}_{34} dt')^2$$

$$dt^2 = (\mathbf{Z}_{41} dx' + \mathbf{Z}_{42} dy' + \mathbf{Z}_{43} dz' + \mathbf{Z}_{44} dt')^2$$

Now, we have Cartesian Co-ordinate geometry in flat space-time

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

Using (3) we obtain from (4) the space-time geometry in S-L system

$$ds^2 = P_1 dx'^2 + P_2 dy'^2 + P_3 dz'^2 + P_4 dt'^2 + 2(Q_1 dx' dy' + Q_2 dx' dz') + Q_3 dx' dt' + Q_4 dy' dz' + Q_5 dy' dt' + Q_6 dz' dt')$$

where,

$$P_1 = \mathbf{Z}_{11}^2 + \mathbf{Z}_{22}^2 + \mathbf{Z}_{32}^2 - \mathbf{Z}_{41}^2,$$  

$$P_2 = \mathbf{Z}_{12}^2 + \mathbf{Z}_{22}^2 + \mathbf{Z}_{32}^2 - \mathbf{Z}_{42}^2.$$
Again, relation for co-ordinate transformation from \( S_3 \) to \( S \) in S-S system \([1]\) is

\[
X(x, y, z, t) = \tilde{S}_{ij}X'(x', y', z', t')
\]

where, \( \tilde{S}_{ij} \) is co-ordinate transformation matrix.

From (6) we obtain

\[
\begin{align*}
&dx^2 = \left( \tilde{S}_{11}dx' + \tilde{S}_{12}dy' + \tilde{S}_{13}dz' + \tilde{S}_{14}dt' \right)^2 \\
&dy^2 = \left( \tilde{S}_{21}dx' + \tilde{S}_{22}dy' + \tilde{S}_{23}dz' + \tilde{S}_{24}dt' \right)^2 \\
&dz^2 = \left( \tilde{S}_{31}dx' + \tilde{S}_{32}dy' + \tilde{S}_{33}dz' + \tilde{S}_{34}dt' \right)^2 \\
&dt^2 = \left( \tilde{S}_{41}dx' + \tilde{S}_{42}dy' + \tilde{S}_{43}dz' + \tilde{S}_{44}dt' \right)^2
\end{align*}
\]

Using (4) and (7) we obtain the space-time geometry in S-S system as in (5) where,

\[
\begin{align*}
P_1 &= \tilde{S}_{11}^2 + \tilde{S}_{21}^2 + \tilde{S}_{31}^2 - \tilde{S}_{41}^2, & P_2 &= \tilde{S}_{12}^2 + \tilde{S}_{22}^2 + \tilde{S}_{32}^2 - \tilde{S}_{42}^2, \\
P_3 &= \tilde{S}_{13}^2 + \tilde{S}_{23}^2 + \tilde{S}_{33}^2 - \tilde{S}_{43}^2, & P_4 &= \tilde{S}_{14}^2 + \tilde{S}_{24}^2 + \tilde{S}_{34}^2 - \tilde{S}_{44}^2, \\
Q_1 &= \tilde{S}_{11}\tilde{S}_{12} + \tilde{S}_{21}\tilde{S}_{22} + \tilde{S}_{31}\tilde{S}_{32} - \tilde{S}_{41}\tilde{S}_{42}, & Q_2 &= \tilde{S}_{11}\tilde{S}_{13} + \tilde{S}_{21}\tilde{S}_{23} + \tilde{S}_{31}\tilde{S}_{33} - \tilde{S}_{41}\tilde{S}_{43}, \\
Q_3 &= \tilde{S}_{11}\tilde{S}_{14} + \tilde{S}_{21}\tilde{S}_{24} + \tilde{S}_{31}\tilde{S}_{34} - \tilde{S}_{41}\tilde{S}_{44}, & Q_4 &= \tilde{S}_{12}\tilde{S}_{13} + \tilde{S}_{22}\tilde{S}_{23} + \tilde{S}_{32}\tilde{S}_{33} - \tilde{S}_{42}\tilde{S}_{43}, \\
Q_5 &= \tilde{S}_{12}\tilde{S}_{14} + \tilde{S}_{22}\tilde{S}_{24} + \tilde{S}_{32}\tilde{S}_{34} - \tilde{S}_{42}\tilde{S}_{44}, & Q_6 &= \tilde{S}_{13}\tilde{S}_{14} + \tilde{S}_{23}\tilde{S}_{24} + \tilde{S}_{33}\tilde{S}_{34} - \tilde{S}_{43}\tilde{S}_{44},
\end{align*}
\]

3. SPACE-TIME GEOMETRY OF ELECTROMAGNETIC FIELD IN PHOTON

Since picture of SSP depends upon the S-L system so, following (1) and using \( dx' = dx^g \), \( dy' = dy^g \), \( dz' = dz^g \), \( dt' = dt^g \) we obtain from (5), the space-time geometry of electromagnetic field in the SSP

\[
(d\text{s}^\text{em})^2 = P_1(dx^g)^2 + P_2(dy^g)^2 + P_3(dz^g)^2 + P_4(dt^g)^2 + 2(Q_1dx^gdy^g + Q_2dx^gdz^g + Q_3dx^gdz^g + Q_4dy^gdz^g + Q_5dy^gdt^g + Q_6dz^gdt^g)
\]

where, superscript ‘g’ represents the gravitational system and superscript \( \text{em} \) represents electromagnetic system.
Following the convention as in (1), one may assume a relation between electromagnetic field and gravitational field as

$$\psi_a(r,t) = \bar{\gamma}_i \bar{S}_y G'_a(r',t')$$

(9)

where, $\bar{\gamma}_i$ is a constant and $\bar{S}_y$ is transformation matrix in S-S system.

This means that, in S-S system, gravitational field of frame $S_3$ would be electromagnetic field with respect to frame $S$. For this system space-time geometry of electromagnetic field would also be as in (8) where, transformation matrix would be $\bar{S}_y$.

However, equation (8) would be the space-time geometry of electromagnetic field connecting gravitational field and electromagnetic field in the system of photon.

4. CONCLUSION

Equation (8) represents a picture of space-time geometry of the electromagnetic field in the system of photon. This implies that a geometrical relation is existed in between electromagnetic field and gravitational field.

References


