ABSTRACT

In inertial system, co-ordinate transformation from one frame to another is possible by using Lorentz transformation matrix. But in non-inertial or rotating system it is not applicable by using Lorentz transformation matrix. In this paper, co-ordinate transformation from one frame to another in three simultaneous superimposed rotating systems has been introduced. This also leads to assume a picture of space-time geometry of same system.

Keywords: non-inertial system; co-ordinate transformation; space-time geometry

1. THREE SIMULTANEOUS SUPERIMPOSED ROTATIONAL MOTION

According to [1, 2] A particle or an event may possess two simultaneous superimposed motions (i.e. either linear or rotational). Similarly a particle or an event may possess three simultaneous superimposed rotational motions. For clarity of three simultaneous superimposed rotational motions as well as three simultaneous superimposed spins, following [3], it is stated that frames $S$ and $S_1$ have both their $X$ axes aligned and $S_1$ is moving at an angular velocity $\omega_1$ about $X_1$ axis as observed by $S$.

The frame $S_1$ has another co-ordinate reference frame $S_2$, where $X_2$ axis of $S_2$, are rotated by an angle $\theta$ counter clockwise with respect to $S_1$ on $X_1Y_1$ plane. Frames $S_2$ and $S_3$ have both their $X$ axes aligned and $S_3$ is moving at an angular velocity $\omega_2$ about $X_3$ axis as observed by $S_2$. Similarly $S_3$ has another co-ordinate reference frame $S_4$, where $X_3$ axis of $S_3$, are rotated by an angle $\psi$ counter clockwise with respect to $S_3$ on $X_3Y_3$ plane. Frames $S_4$ and $S_5$ have both their $X$ axes aligned and $S_5$ is moving at an angular velocity $\omega_3$ about $X_5$ axis as observed by $S_4$.

For the case when the origin of frames are same with respect to $S$ and the particle be at the origin of $S_3$, then it possesses three simultaneous superimposed spins with respect to frame $S$. Coordinate transformation matrix from $S$ to $S_5$ would be $P_{ij} = R_{jz(\omega_3)} R_{x\varphi(\psi)} R_{y\varphi(\omega_2)} R_{x\theta(\theta)} R_{y\varphi(\omega_1)}$.
and \( R_{yz(\omega t)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \omega t & \sin \omega t & 0 \\ 0 & -\sin \omega t & \cos \omega t & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \), where, \( i = 1, 2, 3 \)

Hence, we get the relations between co-ordinates of different frames \( S, S_1, S_2, S_3, S_4 \) and \( S_5 \) as stated below

\[
\begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ t_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \omega t & \sin \omega t & 0 \\ 0 & -\sin \omega t & \cos \omega t & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix},
\]

\[
\begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ t_2 \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta & 0 & 0 \\ -\sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ t_1 \end{bmatrix},
\]

\[
\begin{bmatrix} x_3 \\ y_3 \\ z_3 \\ t_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \omega_2 t & \sin \omega_2 t & 0 \\ 0 & -\sin \omega_2 t & \cos \omega_2 t & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ t_2 \end{bmatrix},
\]

\[
\begin{bmatrix} x_4 \\ y_4 \\ z_4 \\ t_4 \end{bmatrix} = \begin{bmatrix} \cos \psi & 0 & \sin \psi & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \psi & 0 & \cos \psi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_3 \\ y_3 \\ z_3 \\ t_3 \end{bmatrix},
\]

Therefore, the transformation matrix for coordinates of an event from \( S \) to \( S_5 \) would be as shown below

\[
P_{ij} = R_{yz(\omega t)} R_{xz(\phi t)} R_{yz(\omega t)} R_{xy(\theta t)} R_{yz(\omega t)} =
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \phi t & \sin \phi t & 0 \\
0 & -\sin \phi t & \cos \phi t & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \omega t & \sin \omega t & 0 \\
0 & -\sin \omega t & \cos \omega t & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \theta t & \sin \theta t & 0 \\
0 & -\sin \theta t & \cos \theta t & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \psi t & \sin \psi t & 0 \\
0 & -\sin \psi t & \cos \psi t & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \omega t & \sin \omega t & 0 \\
0 & -\sin \omega t & \cos \omega t & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\[
\begin{align*}
\begin{pmatrix}
\cos \theta \cos \psi \\
+ \sin \theta \sin \psi \sin \omega_t \\
\sin \theta \cos \psi \cos \omega_t \\
- \cos \theta \sin \psi \sin \omega_t \\
- \sin \theta \sin \psi \cos \omega_t \\
- \cos \theta \cos \psi \sin \omega_t \\
\end{pmatrix}
&= \\
\begin{pmatrix}
\cos \theta \cos \psi \cos \omega_t \\
- \sin \theta \sin \psi \cos \omega_t \\
- \cos \theta \sin \psi \sin \omega_t \\
+ \sin \theta \cos \psi \cos \omega_t \\
\end{pmatrix}
\begin{pmatrix}
\sin \theta \cos \psi \sin \omega_t \\
+ \sin \theta \sin \psi \cos \omega_t \\
\cos \theta \sin \psi \cos \omega_t \\
- \cos \theta \cos \psi \cos \omega_t \\
\end{pmatrix}
\begin{pmatrix}
\sin \theta \cos \psi \sin \omega_t \\
+ \sin \theta \sin \psi \cos \omega_t \\
\cos \theta \sin \psi \cos \omega_t \\
- \cos \theta \cos \psi \cos \omega_t \\
\end{pmatrix}
\begin{pmatrix}
\sin \theta \cos \psi \sin \omega_t \\
+ \sin \theta \sin \psi \cos \omega_t \\
\cos \theta \sin \psi \cos \omega_t \\
- \cos \theta \cos \psi \cos \omega_t \\
\end{pmatrix}
\begin{pmatrix}
0 \\
0 \\
0 \\
1 \\
\end{pmatrix}
\end{align*}
\]

Similarly, the coordinate transformation matrix of an event from \( S_3 \) to \( S \) would be

\[
\bar{P}_{ij} = R_{yz(-\omega_t)} R_{xy(-\theta)} R_{yz(-\omega_t)} R_{xz(-\psi)} R_{yz(-\omega_t)} =
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & \cos \omega_t & - \sin \omega_t & 0 & 0 \\
0 & \sin \omega_t & \cos \omega_t & 0 & 0 \\
0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 1 & 0 \\
\end{pmatrix}
\begin{pmatrix}
\cos \theta & - \sin \theta & 0 & 0 & 0 \\
\sin \theta & \cos \theta & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 \\
\end{pmatrix}
\begin{pmatrix}
\cos \psi & 0 & - \sin \psi & 0 \\
0 & \cos \omega_t & - \sin \omega_t & 0 \\
\sin \psi & 0 & \cos \omega_t & 0 \\
0 & \sin \omega_t & \cos \omega_t & 0 \\
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & \cos \omega_t & - \sin \omega_t & 0 & 0 \\
0 & \sin \omega_t & \cos \omega_t & 0 & 0 \\
0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 1 & 0 \\
\end{pmatrix}
\begin{pmatrix}
\cos \theta \cos \psi \cos \omega_t \\
- \sin \theta \sin \psi \cos \omega_t \\
- \cos \theta \sin \psi \sin \omega_t \\
+ \sin \theta \cos \psi \cos \omega_t \\
\end{pmatrix}
\begin{pmatrix}
\sin \theta \cos \psi \sin \omega_t \\
+ \sin \theta \sin \psi \cos \omega_t \\
\cos \theta \sin \psi \cos \omega_t \\
- \cos \theta \cos \psi \cos \omega_t \\
\end{pmatrix}
\begin{pmatrix}
\sin \theta \cos \psi \sin \omega_t \\
+ \sin \theta \sin \psi \cos \omega_t \\
\cos \theta \sin \psi \cos \omega_t \\
- \cos \theta \cos \psi \cos \omega_t \\
\end{pmatrix}
\begin{pmatrix}
\sin \theta \cos \psi \sin \omega_t \\
+ \sin \theta \sin \psi \cos \omega_t \\
\cos \theta \sin \psi \cos \omega_t \\
- \cos \theta \cos \psi \cos \omega_t \\
\end{pmatrix}
\begin{pmatrix}
0 \\
0 \\
0 \\
1 \\
\end{pmatrix}
\]

\[
\begin{align*}
\begin{pmatrix}
\cos \theta \cos \psi \\
+ \sin \theta \sin \psi \sin \omega_t \\
\sin \theta \cos \psi \cos \omega_t \\
- \cos \theta \sin \psi \sin \omega_t \\
- \sin \theta \sin \psi \cos \omega_t \\
- \cos \theta \cos \psi \sin \omega_t \\
\end{pmatrix}
&= \\
\begin{pmatrix}
\cos \theta \cos \psi \cos \omega_t \\
- \sin \theta \sin \psi \cos \omega_t \\
- \cos \theta \sin \psi \sin \omega_t \\
+ \sin \theta \cos \psi \cos \omega_t \\
\end{pmatrix}
\begin{pmatrix}
\sin \theta \cos \psi \sin \omega_t \\
+ \sin \theta \sin \psi \cos \omega_t \\
\cos \theta \sin \psi \cos \omega_t \\
- \cos \theta \cos \psi \cos \omega_t \\
\end{pmatrix}
\begin{pmatrix}
\sin \theta \cos \psi \sin \omega_t \\
+ \sin \theta \sin \psi \cos \omega_t \\
\cos \theta \sin \psi \cos \omega_t \\
- \cos \theta \cos \psi \cos \omega_t \\
\end{pmatrix}
\begin{pmatrix}
\sin \theta \cos \psi \sin \omega_t \\
+ \sin \theta \sin \psi \cos \omega_t \\
\cos \theta \sin \psi \cos \omega_t \\
- \cos \theta \cos \psi \cos \omega_t \\
\end{pmatrix}
\begin{pmatrix}
0 \\
0 \\
0 \\
1 \\
\end{pmatrix}
\end{align*}
\]
Now using (1) we get the relation between co-ordinates of frames $S$ and $S_5$ as shown below

$$X_5(x_5, y_5, z_5, t_5) = P_5 P_0 X(x, y, z, t)$$

(3)

Similarly using (2) we obtain the relation between co-ordinates of frames $S$ and $S_5$

$$X(x, y, z, t) = P_5 P_0 X_5(x_5, y_5, z_5, t_5)$$

(4)

where, $X_5(x_5, y_5, z_5, t_5) = \begin{pmatrix} x_5 \\ y_5 \\ z_5 \\ t_5 \end{pmatrix}$ and $X(x, y, z, t) = \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}$

2. SPACE-TIME GEOMETRY

From (4) we obtain

$$dx^2 = (P_{11}dx + P_{12}dy + P_{13}dz + P_{14}dt')^2$$

$$dy^2 = (P_{21}dx + P_{22}dy + P_{23}dz + P_{24}dt')^2$$

$$dz^2 = (P_{31}dx + P_{32}dy + P_{33}dz + P_{34}dt')^2$$

$$dt'^2 = (P_{41}dx + P_{42}dy + P_{43}dz + P_{44}dt')^2$$

(5)

Now, we have Cartesian Co-ordinate geometry in flat space-time

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

(6)

Using (5) we obtain from (6), the space-time geometry of three simultaneous superimposed rotating system

$$ds^2 = A_1 dx^2 + A_2 dy^2 + A_3 dz^2 + A_4 dt'^2 + 2(B_1 dx'dy' + B_2 dx'dz' + B_3 dy'dt' + B_4 dz'dt')$$

(7)
where,

\[ A_1 = p_{11}^2 + p_{21}^2 + p_{31}^2 - p_{41}^2, \quad A_2 = p_{12}^2 + p_{22}^2 + p_{32}^2 - p_{42}^2 \]

\[ A_3 = p_{13}^2 + p_{23}^2 + p_{33}^2 - p_{43}^2, \quad A_4 = p_{14}^2 + p_{24}^2 + p_{34}^2 - p_{44}^2 \]

\[ B_1 = p_{11}p_{12} + p_{21}p_{22} + p_{31}p_{32} - p_{41}p_{42}, \quad B_2 = p_{11}p_{13} + p_{21}p_{23} + p_{31}p_{33} - p_{41}p_{43} \]

\[ B_3 = p_{11}p_{14} + p_{21}p_{24} + p_{31}p_{34} - p_{41}p_{44}, \quad B_4 = p_{12}p_{13} + p_{22}p_{23} + p_{32}p_{33} - p_{42}p_{43} \]

\[ B_5 = p_{12}p_{14} + p_{22}p_{24} + p_{32}p_{34} - p_{42}p_{44}, \quad B_6 = p_{13}p_{14} + p_{23}p_{24} + p_{33}p_{34} - p_{43}p_{44} \]

3. CONCLUSION

In this way any number of transformations or rotations may be considered. Also the process leads to conclude that a particle may possess more than one simultaneous superimposed spins with its mutual effects. Co-ordinate transformation in non-inertial systems would be done following the process as in the text.

References