

Three Simultaneous Superimposed Rotating System

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ABSTRACT

In inertial system, co-ordinate transformation from one frame to another is possible by using Lorentz transformation matrix. But in non-inertial or rotating system it is not applicable by using Lorentz transformation matrix. In this paper, co-ordinate transformation from one frame to another in three simultaneous superimposed rotating systems has been introduced. This also leads to assume a picture of space-time geometry of same system.

Keywords: non-inertial system; co-ordinate transformation; space-time geometry

1. THREE SIMULTANEOUS SUPERIMPOSED ROTATIONAL MOTION

According to [1,2] A particle or an event may possess two simultaneous superimposed motions (i.e. either linear or rotational). Similarly a particle or an event may possess three simultaneous superimposed rotational motions. For clarity of three simultaneous superimposed rotational motions as well as three simultaneous superimposed spins, following [3], it is stated that frames S and S_1 have both their X axes aligned and S_1 is moving at an angular velocity ω_1 about X_1 axis as observed by S .

The frame S_1 has another co-ordinate reference frame S_2 , where X_2 axis of S_2 , are rotated by an angle θ counter clockwise with respect to S_1 on X_1Y_1 plane. Frames S_2 and S_3 have both their X axes aligned and S_3 is moving at an angular velocity ω_2 about X_3 axis as observed by S_2 . Similarly S_3 has another co-ordinate reference frame S_4 , where X_3 axis of S_3 , are rotated by an angle ψ counter clockwise with respect to S_3 on X_3Y_3 plane. Frames S_4 and S_5 have both their X axes aligned and S_5 is moving at an angular velocity ω_3 about X_5 axis as observed by S_4 .

For the case when the origin of frames are same with respect to S and the particle be at the origin of S_3 , then it possesses three simultaneous superimposed spins with respect to frame S . Coordinate transformation matrix from S to S_5 would be

$$P_{ij} = R_{yz(\omega_3 t)} R_{xz(\psi)} R_{yz(\omega_2 t)} R_{xy(\theta)} R_{yz(\omega_1 t)}$$

$$\text{and } R_{yz(\omega_i t)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \omega_i t & \sin \omega_i t & 0 \\ 0 & -\sin \omega_i t & \cos \omega_i t & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \text{ where, } i = 1, 2, 3$$

Hence, we get the relations between co-ordinates of different frames S, S_1, S_2, S_3, S_4 and S_5 as stated below

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \\ t_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \omega_1 t & \sin \omega_1 t & 0 \\ 0 & -\sin \omega_1 t & \cos \omega_1 t & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}, \quad \begin{pmatrix} x_2 \\ y_2 \\ z_2 \\ t_2 \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta & 0 & 0 \\ -\sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \\ z_1 \\ t_1 \end{pmatrix}$$

$$\begin{pmatrix} x_3 \\ y_3 \\ z_3 \\ t_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \omega_2 t & \sin \omega_2 t & 0 \\ 0 & -\sin \omega_2 t & \cos \omega_2 t & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \\ z_2 \\ t_2 \end{pmatrix}, \quad \begin{pmatrix} x_4 \\ y_4 \\ z_4 \\ t_4 \end{pmatrix} = \begin{pmatrix} \cos \psi & 0 & \sin \psi & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \psi & 0 & \cos \psi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_3 \\ y_3 \\ z_3 \\ t_3 \end{pmatrix},$$

$$\begin{pmatrix} x_5 \\ y_5 \\ z_5 \\ t_5 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \omega_3 t & \sin \omega_3 t & 0 \\ 0 & -\sin \omega_3 t & \cos \omega_3 t & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_4 \\ y_4 \\ z_4 \\ t_4 \end{pmatrix}$$

Therefore, the transformation matrix for coordinates of an event from S to S_5 would be as shown below

$$P_{ij} = R_{yz(\omega_3 t)} R_{xz(\psi)} R_{yz(\omega_2 t)} R_{xy(\theta)} R_{yz(\omega_1 t)} =$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \omega_3 t & \sin \omega_3 t & 0 \\ 0 & -\sin \omega_3 t & \cos \omega_3 t & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos \psi & 0 & \sin \psi & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \psi & 0 & \cos \psi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \omega_2 t & \sin \omega_2 t & 0 \\ 0 & -\sin \omega_2 t & \cos \omega_2 t & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos \theta & \sin \theta & 0 & 0 \\ -\sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \omega_1 t & \sin \omega_1 t & 0 \\ 0 & -\sin \omega_1 t & \cos \omega_1 t & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix}
 \begin{matrix}
 (\cos \theta \cos \psi \\
 + \sin \theta \sin \psi \sin \omega_2 t)
 \end{matrix} &
 \begin{matrix}
 (\sin \theta \cos \psi \cos \omega_1 t \\
 - \cos \theta \sin \psi \sin \omega_2 t \cos \omega_1 t \\
 - \sin \omega_1 t \sin \psi \cos \omega_2 t)
 \end{matrix} &
 \begin{matrix}
 (\sin \theta \cos \psi \sin \omega_1 t \\
 + \sin \psi \cos \omega_1 t \cos \omega_2 t \\
 - \sin \psi \cos \theta \sin \omega_1 t \sin \omega_2 t)
 \end{matrix} &
 0 \\
 \begin{matrix}
 (\sin \theta \cos \psi \sin \omega_2 t \sin \omega_3 t \\
 - \sin \theta \cos \omega_2 t \cos \omega_3 t \\
 - \sin \psi \cos \theta \sin \omega_3 t)
 \end{matrix} &
 \begin{matrix}
 (\cos \theta \cos \omega_1 t \cos \omega_2 t \cos \omega_3 t \\
 - \sin \omega_1 t \sin \omega_2 t \cos \omega_3 t \\
 - \sin \psi \sin \theta \cos \omega_1 t \sin \omega_3 t \\
 - \cos \psi \cos \theta \cos \omega_1 t \sin \omega_2 t \cos \omega_3 t \\
 - \cos \psi \sin \omega_1 t \sin \omega_3 t \cos \omega_2 t)
 \end{matrix} &
 \begin{matrix}
 (- \sin \omega_1 t \sin \theta \sin \psi \sin \omega_3 t \\
 + \sin \omega_1 t \cos \theta \cos \omega_2 t \cos \omega_3 t \\
 - \sin \omega_1 t \cos \theta \cos \psi \sin \omega_2 t \cos \omega_3 t \\
 + \cos \omega_1 t \sin \omega_2 t \cos \omega_3 t \\
 + \cos \psi \cos \omega_1 t \sin \omega_2 t \sin \omega_3 t)
 \end{matrix} &
 0 \\
 \begin{matrix}
 (\cos \psi \sin \theta \sin \omega_2 t \cos \omega_3 t \\
 + \sin \theta \sin \omega_3 t \cos \omega_2 t \\
 - \cos \theta \sin \psi \cos \omega_3 t)
 \end{matrix} &
 \begin{matrix}
 (- \cos \psi \cos \omega_2 t \cos \omega_3 t \sin \omega_1 t \\
 - \cos \theta \sin \omega_3 t \cos \omega_1 t \cos \omega_2 t \\
 - \sin \theta \sin \psi \cos \omega_1 t \cos \omega_3 t \\
 - \cos \psi \cos \theta \cos \omega_1 t \sin \omega_2 t \cos \omega_3 t \\
 + \sin \omega_1 t \sin \omega_2 t \sin \omega_3 t)
 \end{matrix} &
 \begin{matrix}
 (- \sin \theta \sin \psi \sin \omega_1 t \cos \omega_2 t \\
 - \cos \theta \sin \omega_1 t \sin \omega_3 t \cos \omega_2 t \\
 - \sin \omega_1 t \sin \omega_2 t \cos \omega_3 t \cos \psi \cos \theta \\
 - \cos \omega_1 t \sin \omega_2 t \sin \omega_3 t \\
 + \cos \psi \cos \omega_1 t \cos \omega_2 t \cos \omega_3 t)
 \end{matrix} &
 0 \\
 0 & 0 & 0 & 1
 \end{pmatrix} \quad (1)$$

Similarly, coordinate transformation matrix of an event from S_5 to S would be

$$\bar{P}_{ij} = R_{yz(-\omega_1 t)} R_{xy(-\theta)} R_{yz(-\omega_2 t)} R_{xz(-\psi)} R_{yz(-\omega_3 t)} =$$

$$\begin{pmatrix}
 1 & 0 & 0 & 0 \\
 0 & \cos \omega_1 t & -\sin \omega_1 t & 0 \\
 0 & \sin \omega_1 t & \cos \omega_1 t & 0 \\
 0 & 0 & 0 & 1
 \end{pmatrix}
 \begin{pmatrix}
 \cos \theta & -\sin \theta & 0 & 0 \\
 \sin \theta & \cos \theta & 0 & 0 \\
 0 & 0 & 1 & 0 \\
 0 & 0 & 0 & 1
 \end{pmatrix}
 \begin{pmatrix}
 1 & 0 & 0 & 0 \\
 0 & \cos \omega_2 t & -\sin \omega_2 t & 0 \\
 0 & \sin \omega_2 t & \cos \omega_2 t & 0 \\
 0 & 0 & 0 & 1
 \end{pmatrix}
 \begin{pmatrix}
 \cos \psi & 0 & -\sin \psi & 0 \\
 0 & 1 & 0 & 0 \\
 \sin \psi & 0 & \cos \psi & 0 \\
 0 & 0 & 0 & 1
 \end{pmatrix}
 \begin{pmatrix}
 1 & 0 & 0 & 0 \\
 0 & \cos \omega_3 t & -\sin \omega_3 t & 0 \\
 0 & \sin \omega_3 t & \cos \omega_3 t & 0 \\
 0 & 0 & 0 & 1
 \end{pmatrix}$$

$$= \begin{pmatrix}
 \begin{matrix}
 (\cos \theta \cos \psi \\
 + \sin \theta \sin \psi \sin \omega_2 t)
 \end{matrix} &
 \begin{matrix}
 (- \sin \theta \cos \omega_2 t \cos \omega_3 t \\
 - \cos \theta \sin \psi \sin \omega_3 t \\
 + \sin \theta \sin \omega_2 t \sin \omega_3 t \cos \psi)
 \end{matrix} &
 \begin{matrix}
 (\sin \theta \cos \omega_2 t \sin \omega_3 t \\
 - \cos \theta \sin \psi \cos \omega_3 t \\
 + \sin \theta \sin \omega_2 t \cos \omega_3 t \cos \psi)
 \end{matrix} &
 0 \\
 \begin{matrix}
 (\sin \theta \cos \psi \cos \omega_1 t - \\
 \sin \psi \sin \omega_2 t \cos \theta \cos \omega_1 t \\
 - \sin \omega_1 t \sin \psi \cos \omega_2 t)
 \end{matrix} &
 \begin{matrix}
 (\cos \theta \cos \omega_1 t \cos \omega_2 t \cos \omega_3 t \\
 - \sin \omega_1 t \sin \omega_2 t \cos \omega_3 t \\
 - \sin \psi \sin \theta \cos \omega_1 t \sin \omega_3 t \\
 - \cos \psi \cos \theta \cos \omega_1 t \sin \omega_2 t \sin \omega_3 t \\
 - \cos \psi \sin \omega_1 t \sin \omega_3 t \cos \omega_2 t)
 \end{matrix} &
 \begin{matrix}
 (- \sin \omega_3 t \cos \theta \cos \omega_1 t \cos \omega_2 t \\
 + \sin \omega_1 t \sin \omega_2 t \sin \omega_3 t \\
 - \sin \psi \sin \theta \cos \omega_1 t \cos \omega_3 t \\
 - \cos \theta \cos \psi \cos \omega_1 t \cos \omega_3 t \sin \omega_2 t \\
 - \cos \psi \sin \omega_1 t \cos \omega_2 t \cos \omega_3 t)
 \end{matrix} &
 0 \\
 \begin{matrix}
 (\cos \psi \sin \theta \sin \omega_1 t - \\
 \sin \psi \sin \omega_1 t \sin \omega_2 t \cos \theta \\
 + \sin \psi \cos \omega_1 t \cos \omega_2 t)
 \end{matrix} &
 \begin{matrix}
 (\cos \theta \cos \omega_2 t \cos \omega_3 t \sin \omega_1 t \\
 + \sin \omega_2 t \cos \omega_1 t \cos \omega_3 t \\
 - \sin \theta \sin \psi \sin \omega_1 t \sin \omega_3 t \\
 - \cos \psi \cos \theta \sin \omega_1 t \sin \omega_2 t \sin \omega_3 t \\
 + \cos \omega_1 t \cos \omega_2 t \sin \omega_3 t \cos \psi)
 \end{matrix} &
 \begin{matrix}
 (- \cos \theta \cos \omega_2 t \sin \omega_1 t \sin \omega_3 t \\
 - \cos \omega_1 t \sin \omega_2 t \sin \omega_3 t \\
 - \sin \psi \sin \theta \sin \omega_1 t \cos \omega_3 t \\
 - \cos \theta \cos \psi \sin \omega_1 t \sin \omega_2 t \cos \omega_2 t \\
 + \cos \psi \cos \omega_1 t \cos \omega_3 t \cos \omega_3 t)
 \end{matrix} &
 0 \\
 0 & 0 & 0 & 1
 \end{pmatrix} \quad (2)$$

Now using (1) we get the relation between co-ordinates of frames S and S_5 as shown below

$$X_5(x_5, y_5, z_5, t_5) = P_{ij}X(x, y, z, t) \quad (3)$$

Similarly using (2) we obtain the relation between co-ordinates of frames S and S_5

$$X(x, y, z, t) = \bar{P}_{ij}X_5(x_5, y_5, z_5, t_5) \quad (4)$$

$$\text{where, } X_5(x_5, y_5, z_5, t_5) = \begin{pmatrix} x_5 \\ y_5 \\ z_5 \\ t_5 \end{pmatrix} \quad \text{and} \quad X(x, y, z, t) = \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}$$

2. SPACE-TIME GEOMETRY

From (4) we obtain

$$\begin{aligned} dx^2 &= (\bar{P}_{11}dx' + \bar{P}_{12}dy' + \bar{P}_{13}dz' + \bar{P}_{14}dt')^2 \\ dy^2 &= (\bar{P}_{21}dx' + \bar{P}_{22}dy' + \bar{P}_{23}dz' + \bar{P}_{24}dt')^2 \\ dz^2 &= (\bar{P}_{31}dx' + \bar{P}_{32}dy' + \bar{P}_{33}dz' + \bar{P}_{34}dt')^2 \\ dt^2 &= (\bar{P}_{41}dx' + \bar{P}_{42}dy' + \bar{P}_{43}dz' + \bar{P}_{44}dt')^2 \end{aligned} \quad (5)$$

Now, we have Cartesian Co-ordinate geometry in flat space-time

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 \quad (6)$$

Using (5) we obtain from (6), the space-time geometry of three simultaneous superimposed rotating system

$$\begin{aligned} ds^2 &= A_1dx'^2 + A_2dy'^2 + A_3dz'^2 + A_4dt'^2 + 2(B_1dx'dy' + B_2dx'dz' \\ &+ B_3dx'dt' + B_4dy'dz' + B_5dy'dt' + B_6dz'dt') \end{aligned} \quad (7)$$

where,

$$A_1 = \bar{P}_{11}^2 + \bar{P}_{21}^2 + \bar{P}_{31}^2 - \bar{P}_{41}^2,$$

$$A_2 = \bar{P}_{12}^2 + \bar{P}_{22}^2 + \bar{P}_{32}^2 - \bar{P}_{42}^2$$

$$A_3 = \bar{P}_{13}^2 + \bar{P}_{23}^2 + \bar{P}_{33}^2 - \bar{P}_{43}^2,$$

$$A_4 = \bar{P}_{14}^2 + \bar{P}_{24}^2 + \bar{P}_{34}^2 - \bar{P}_{44}^2$$

$$B_1 = \bar{P}_{11}\bar{P}_{12} + \bar{P}_{21}\bar{P}_{22} + \bar{P}_{31}\bar{P}_{32} - \bar{P}_{41}\bar{P}_{42},$$

$$B_2 = \bar{P}_{11}\bar{P}_{13} + \bar{P}_{21}\bar{P}_{23} + \bar{P}_{31}\bar{P}_{33} - \bar{P}_{41}\bar{P}_{43}$$

$$B_3 = \bar{P}_{11}\bar{P}_{14} + \bar{P}_{21}\bar{P}_{24} + \bar{P}_{31}\bar{P}_{34} - \bar{P}_{41}\bar{P}_{44},$$

$$B_4 = \bar{P}_{12}\bar{P}_{13} + \bar{P}_{22}\bar{P}_{23} + \bar{P}_{32}\bar{P}_{33} - \bar{P}_{42}\bar{P}_{43}$$

$$B_5 = \bar{P}_{12}\bar{P}_{14} + \bar{P}_{22}\bar{P}_{24} + \bar{P}_{32}\bar{P}_{34} - \bar{P}_{42}\bar{P}_{44},$$

$$B_6 = \bar{P}_{13}\bar{P}_{14} + \bar{P}_{23}\bar{P}_{24} + \bar{P}_{33}\bar{P}_{34} - \bar{P}_{43}\bar{P}_{44}$$

3. CONCLUSION

In this way any number of transformations or rotations may be considered. Also the process leads to conclude that a particle may possess more than one simultaneous superimposed spins with its mutual effects. Co-ordinate transformation in non-inertial systems would be done following the process as in the text.

References

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