International Letters of Chemistry, Physics and Astronomy *Online: 2013-05-03*
ISSN: 2299-3843, Vol. 13, pp 215-219
doi:10.18052/www.scipress.com/ILCPA.13.215
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Three Simultaneous Superimposed Rotating System

Mukul Chandra Das[1, 2, *], Rampada Misra[3]

[1]Singhania University, Pacheri Bari - 333515, Jhunjhunu, Rajasthan, India

[2]Satmile High School, Satmile - 721452, Contai, West Bengal, India

[3]Department of Electronics, Vidyasagar University, Midnapore - 721102, West Bengal, India

*Email address: mukuldas.100@gmail.com

ABSTRACT

In inertial system, co-ordinate transformation from one frame to another is possible by using Lorentz transformation matrix. But in non-inertial or rotating system it is not applicable by using Lorentz transformation matrix. In this paper, co-ordinate transformation from one frame to another in three simultaneous superimposed rotating systems has been introduced. This also leads to assume a picture of space-time geometry of same system.

***Keywords*:** non-inertial system; co-ordinate transformation; space-time geometry

1. THREE SIMULTANEOUS SUPERIMPOSED ROTATIONAL MOTION

According to [1,2] A particle or an event may possess two simultaneous superimposed motions (i.e. either linear or rotational). Similarly a particle or an event may possess three simultaneous superimposed rotational motions. For clarity of three simultaneous superimposed rotational motions as well as three simultaneous superimposed spins, following [3], it is stated that frames S and S_1 have both their X axes aligned and S_1 is moving at an angular velocity ω_1 about X_1 axis as observed by S.

The frame S_1 has another co-ordinate reference frame S_2, where X_2 axis of S_2, are rotated by an angle θ counter clockwise with respect to S_1 on X_1Y_1 plane. Frames S_2 and S_3 have both their X axes aligned and S_3 is moving at an angular velocity ω_2 about X_3 axis as observed by S_2. Similarly S_3 has another co-ordinate reference frame S_4, where X_3 axis of S_3, are rotated by an angle ψ counter clockwise with respect to S_3 on X_3Y_3 plane. Frames S_4 and S_5 have both their X axes aligned and S_5 is moving at an angular velocity ω_3 about X_5 axis as observed by S_4.

For the case when the origin of frames are same with respect to S and the particle be at the origin of S_3, then it possesses three simultaneous superimposed spins with respect to frame S. Coordinate transformation matrix from S to S_5 would be $P_{ij} = R_{yz(\omega_3 t)} R_{xz(\psi)} R_{yz(\omega_2 t)} R_{xy(\theta)} R_{yz(\omega_1 t)}$

and $R_{yz(\omega_i t)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\omega_i t & \sin\omega_i t & 0 \\ 0 & -\sin\omega_i t & \cos\omega_i t & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$, where, $i = 1,\ 2,\ 3$

Hence, we get the relations between co-ordinates of different frames S, S_1, S_2, S_3, S_4 and S_5 as stated below

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \\ t_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\omega_1 t & \sin\omega_1 t & 0 \\ 0 & -\sin\omega_1 t & \cos\omega_1 t & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}, \quad \begin{pmatrix} x_2 \\ y_2 \\ z_2 \\ t_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \\ z_1 \\ t_1 \end{pmatrix}$$

$$\begin{pmatrix} x_3 \\ y_3 \\ z_3 \\ t_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\omega_2 t & \sin\omega_2 t & 0 \\ 0 & -\sin\omega_2 t & \cos\omega_2 t & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \\ z_2 \\ t_2 \end{pmatrix}, \quad \begin{pmatrix} x_4 \\ y_4 \\ z_4 \\ t_4 \end{pmatrix} = \begin{pmatrix} \cos\psi & 0 & \sin\psi & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\psi & 0 & \cos\psi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_3 \\ y_3 \\ z_3 \\ t_3 \end{pmatrix},$$

$$\begin{pmatrix} x_5 \\ y_5 \\ z_5 \\ t_5 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\omega_3 t & \sin\omega_3 t & 0 \\ 0 & -\sin\omega_3 t & \cos\omega_3 t & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_4 \\ y_4 \\ z_4 \\ t_4 \end{pmatrix}$$

Therefore, the transformation matrix for coordinates of an event from S to S_5 would be as shown below

$$P_{ij} = R_{yz(\omega_3 t)} R_{xz(\psi)} R_{yz(\omega_2 t)} R_{xy(\theta)} R_{yz(\omega_1 t)} =$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\omega_3 t & \sin\omega_3 t & 0 \\ 0 & -\sin\omega_3 t & \cos\omega_3 t & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\psi & 0 & \sin\psi & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\psi & 0 & \cos\psi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\omega_2 t & \sin\omega_2 t & 0 \\ 0 & -\sin\omega_2 t & \cos\omega_2 t & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\omega_1 t & \sin\omega_1 t & 0 \\ 0 & -\sin\omega_1 t & \cos\omega_1 t & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$
= \begin{pmatrix}
\begin{array}{l}(\cos\theta\cos\psi \\ +\sin\theta\sin\psi\sin\omega_2 t)\end{array} & \begin{array}{l}(\sin\theta\cos\psi\cos\omega_1 t \\ -\cos\theta\sin\psi\sin\omega_2 t\cos\omega_1 t \\ -\sin\omega_1 t\sin\psi\cos\omega_2 t)\end{array} & \begin{array}{l}(\sin\theta\cos\psi\sin\omega_1 t \\ +\sin\psi\cos\omega_1 t\cos\omega_2 t \\ -\sin\psi\cos\theta\sin\omega_1 t\sin\omega_2 t\end{array} & 0 \\
\begin{array}{l}(\sin\theta\cos\psi\sin\omega_2 t\sin\omega_3 t \\ -\sin\theta\cos\omega_2 t\cos\omega_3 t \\ -\sin\psi\cos\theta\sin\omega_3 t)\end{array} & \begin{array}{l}(\cos\theta\cos\omega_1 t\cos\omega_2 t\cos\omega_3 t \\ -\sin\omega_1 t\sin\omega_2 t\cos\omega_3 t \\ -\sin\psi\sin\theta\cos\omega_1 t\sin\omega_3 t \\ -\cos\psi\cos\theta\cos\omega_1 t\sin\omega_2 t\cos\omega_3 t \\ -\cos\psi\sin\omega_1 t\sin\omega_3 t\cos\omega_2 t)\end{array} & \begin{array}{l}(-\sin\omega_1 t\sin\theta\sin\psi\sin\omega_3 t \\ +\sin\omega_1 t\cos\theta\cos\omega_2 t\cos\omega_3 t \\ -\sin\omega_1 t\cos\theta\cos\psi\sin\omega_2 t\cos\omega_3 t \\ +\cos\omega_1 t\sin\omega_2 t\cos\omega_3 t \\ +\cos\psi\cos\omega_1 t\sin\omega_2 t\sin\omega_3 t)\end{array} & 0 \\
\begin{array}{l}(\cos\psi\sin\theta\sin\omega_2 t\cos\omega_3 t \\ +\sin\theta\sin\omega_3 t\cos\omega_2 t \\ -\cos\theta\sin\psi\cos\omega_3 t)\end{array} & \begin{array}{l}(-\cos\psi\cos\omega_2 t\cos\omega_3 t\sin\omega_1 t \\ -\cos\theta\sin\omega_3 t\cos\omega_1 t\cos\omega_2 t \\ -\sin\theta\sin\psi\cos\omega_1 t\cos\omega_3 t \\ -\cos\psi\cos\theta\cos\omega_1 t\sin\omega_2 t\cos\omega_3 t \\ +\sin\omega_1 t\sin\omega_2 t\sin\omega_3 t)\end{array} & \begin{array}{l}(-\sin\theta\sin\psi\sin\omega_1 t\cos\omega_3 t \\ -\cos\theta\sin\omega_1 t\sin\omega_3 t\cos\omega_2 t \\ -\sin\omega_1 t\sin\omega_2 t\cos\omega_3 t\cos\psi\cos\theta \\ -\cos\omega_1 t\sin\omega_2 t\sin\omega_3 t \\ +\cos\psi\cos\omega_1 t\cos\omega_2 t\cos\omega_3 t)\end{array} & 0 \\
0 & 0 & 0 & 1
\end{pmatrix} \tag{1}
$$

Similarly, coordinate transformation matrix of an event from S_5 to S would be

$$
\bar{P}_{ij} = R_{yz(-\omega_1 t)} R_{xy(-\theta)} R_{yz(-\omega_2 t)} R_{xz(-\psi)} R_{yz(-\omega_3 t)} =
$$

$$
\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\omega_1 t & -\sin\omega_1 t & 0 \\ 0 & \sin\omega_1 t & \cos\omega_1 t & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
\begin{pmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\omega_2 t & -\sin\omega_2 t & 0 \\ 0 & \sin\omega_2 t & \cos\omega_2 t & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
\begin{pmatrix} \cos\psi & 0 & -\sin\psi & 0 \\ 0 & 1 & 0 & 0 \\ \sin\psi & 0 & \cos\psi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\omega_3 t & -\sin\omega_3 t & 0 \\ 0 & \sin\omega_3 t & \cos\omega_3 t & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
= \begin{pmatrix}
\begin{array}{l}(\cos\theta\cos\psi \\ +\sin\theta\sin\psi\sin\omega_2 t)\end{array} & \begin{array}{l}(-\sin\theta\cos\omega_2 t\cos\omega_3 t \\ -\cos\theta\sin\psi\sin\omega_3 t \\ +\sin\theta\sin\omega_2 t\sin\omega_3 t\cos\psi)\end{array} & \begin{array}{l}(\sin\theta\cos\omega_2 t\sin\omega_3 t \\ -\cos\theta\sin\psi\cos\omega_3 t \\ +\sin\theta\sin\omega_2 t\cos\omega_3 t\cos\psi\end{array} & 0 \\
\begin{array}{l}(\sin\theta\cos\psi\cos\omega_1 t - \\ \sin\psi\sin\omega_2 t\cos\theta\cos\omega_1 t \\ -\sin\omega_1 t\sin\psi\cos\omega_2 t)\end{array} & \begin{array}{l}(\cos\theta\cos\omega_1 t\cos\omega_2 t\cos\omega_3 t \\ -\sin\omega_1 t\sin\omega_2 t\cos\omega_3 t \\ -\sin\psi\sin\theta\cos\omega_1 t\sin\omega_3 t \\ -\cos\psi\cos\theta\cos\omega_1 t\sin\omega_2 t\sin\omega_3 t \\ -\cos\psi\sin\omega_1 t\sin\omega_3 t\cos\omega_2 t)\end{array} & \begin{array}{l}(-\sin\omega_3 t\cos\theta\cos\omega_1 t\cos\omega_2 t \\ +\sin\omega_1 t\sin\omega_2 t\sin\omega_3 t \\ -\sin\psi\sin\theta\cos\omega_1 t\cos\omega_3 t \\ -\cos\theta\cos\psi\cos\omega_1 t\cos\omega_3 t\sin\omega_2 t \\ -\cos\psi\sin\omega_1 t\cos\omega_2 t\cos\omega_3 t)\end{array} & 0 \\
\begin{array}{l}(\cos\psi\sin\theta\sin\omega_1 t - \\ \sin\psi\sin\omega_1 t\sin\omega_2 t\cos\theta \\ +\sin\psi\cos\omega_1 t\cos\omega_2 t)\end{array} & \begin{array}{l}(\cos\theta\cos\omega_2 t\cos\omega_3 t\sin\omega_1 t \\ +\sin\omega_2 t\cos\omega_1 t\cos\omega_3 t \\ -\sin\theta\sin\psi\sin\omega_1 t\sin\omega_3 t \\ -\cos\psi\cos\theta\sin\omega_1 t\sin\omega_2 t\sin\omega_3 t \\ +\cos\omega_1 t\cos\omega_2 t\sin\omega_3 t\cos\psi)\end{array} & \begin{array}{l}(-\cos\theta\cos\omega_2 t\sin\omega_1 t\sin\omega_3 t \\ -\cos\omega_1 t\sin\omega_2 t\sin\omega_3 t \\ -\sin\psi\sin\theta\sin\omega_1 t\cos\omega_3 t \\ -\cos\theta\cos\psi\sin\omega_1 t\sin\omega_2 t\cos\omega_2 t \\ +\cos\psi\cos\omega_1 t\cos\omega_3 t\cos\omega_3 t)\end{array} & 0 \\
0 & 0 & 0 & 1
\end{pmatrix} \tag{2}
$$

Now using (1) we get the relation between co-ordinates of frames S and S_5 as shown below

$$X_5(x_5, y_5, z_5, t_5) = P_{ij} X(x, y, z, t) \tag{3}$$

Similarly using (2) we obtain the relation between co-ordinates of frames S and S_5

$$X(x, y, z, t) = \overline{P}_{ij} X_5(x_5, y_5, z_5, t_5) \tag{4}$$

where, $X_5(x_5, y_5, z_5, t_5) = \begin{pmatrix} x_5 \\ y_5 \\ z_5 \\ t_5 \end{pmatrix}$ and $X(x, y, z, t) = \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}$

2. SPACE-TIME GEOMETRY

From (4) we obtain

$$\begin{aligned} dx^2 &= (\overline{P}_{11}dx' + \overline{P}_{12}dy' + \overline{P}_{13}dz' + \overline{P}_{14}dt')^2 \\ dy^2 &= (\overline{P}_{21}dx' + \overline{P}_{22}dy' + \overline{P}_{23}dz' + \overline{P}_{24}dt')^2 \\ dz^2 &= (\overline{P}_{31}dx' + \overline{P}_{32}dy' + \overline{P}_{33}dz' + \overline{P}_{34}dt')^2 \\ dt^2 &= (\overline{P}_{41}dx' + \overline{P}_{42}dy' + \overline{P}_{43}dz' + \overline{P}_{44}dt')^2 \end{aligned} \tag{5}$$

Now, we have Cartesian Co-ordinate geometry in flat space-time

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 \tag{6}$$

Using (5) we obtain from (6), the space-time geometry of three simultaneous superimposed rotating system

$$\begin{aligned} ds^2 = A_1 dx'^2 + A_2 dy'^2 + A_3 dz'^2 + A_4 dt'^2 + 2(\mathrm{B}_1 dx'dy' + B_2 dx'dz' \\ + B_3 dx'dt' + B_4 dy'dz' + B_5 dy'dt' + B_6 dz'dt') \end{aligned} \tag{7}$$

where,

$$A_1 = \overline{P}_{11}^{\,2} + \overline{P}_{21}^{\,2} + \overline{P}_{31}^{\,2} - \overline{P}_{41}^{\,2}, \qquad A_2 = \overline{P}_{12}^{\,2} + \overline{P}_{22}^{\,2} + \overline{P}_{32}^{\,2} - \overline{P}_{42}^{\,2}$$

$$A_3 = \overline{P}_{13}^{\,2} + \overline{P}_{23}^{\,2} + \overline{P}_{33}^{\,2} - \overline{P}_{43}^{\,2}, \qquad A_4 = \overline{P}_{14}^{\,2} + \overline{P}_{24}^{\,2} + \overline{P}_{34}^{\,2} - \overline{P}_{44}^{\,2}$$

$$B_1 = \overline{P}_{11}\overline{P}_{12} + \overline{P}_{21}\overline{P}_{22} + \overline{P}_{31}\overline{P}_{32} - \overline{P}_{41}\overline{P}_{42}, \qquad B_2 = \overline{P}_{11}\overline{P}_{13} + \overline{P}_{21}\overline{P}_{23} + \overline{P}_{31}\overline{P}_{33} - \overline{P}_{41}\overline{P}_{43}$$

$$\mathrm{B}_3 = \overline{P}_{11}\overline{P}_{14} + \overline{P}_{21}\overline{P}_{24} + \overline{P}_{31}\overline{P}_{34} - \overline{P}_{41}\overline{P}_{44}, \qquad B_4 = \overline{P}_{12}\overline{P}_{13} + \overline{P}_{22}\overline{P}_{23} + \overline{P}_{32}\overline{P}_{33} - \overline{P}_{42}\overline{P}_{43}$$

$$\mathrm{B}_5 = \overline{P}_{12}\overline{P}_{14} + \overline{P}_{22}\overline{P}_{24} + \overline{P}_{32}\overline{P}_{34} - \overline{P}_{42}\overline{P}_{44}, \qquad \mathrm{B}_6 = \overline{P}_{13}\overline{P}_{14} + \overline{P}_{23}\overline{P}_{24} + \overline{P}_{33}\overline{P}_{34} - \overline{P}_{43}\overline{P}_{44}$$

3. CONCLUSION

In this way any number of transformations or rotations may be considered. Also the process leads to conclude that a particle may possess more then one simultaneous superimposed spins with its mutual effects. Co-ordinate transformation in non-inertial systems would be done following the process as in the text.

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