Three Simultaneous Superimposed Rotating System

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ABSTRACT

In inertial system, co-ordinate transformation from one frame to another is possible by using Lorentz transformation matrix. But in non-inertial or rotating system it is not applicable by using Lorentz transformation matrix. In this paper, co-ordinate transformation from one frame to another in three simultaneous superimposed rotating systems has been introduced. This also leads to assume a picture of space-time geometry of same system.

Keywords: non-inertial system; co-ordinate transformation; space-time geometry

1. THREE SIMULTANEOUS SUPERIMPOSED ROTATIONAL MOTION

According to [1,2] A particle or an event may possess two simultaneous superimposed motions (i.e. either linear or rotational). Similarly a particle or an event may possess three simultaneous superimposed rotational motions. For clarity of three simultaneous superimposed rotational motions as well as three simultaneous superimposed spins, following [3], it is stated that frames \( S \) and \( S_1 \) have both their \( X \) axes aligned and \( S_1 \) is moving at an angular velocity \( \omega_1 \) about \( X_1 \) axis as observed by \( S \).

The frame \( S_2 \) has another co-ordinate reference frame \( S_2 \), where \( X_2 \) axis of \( S_2 \), are rotated by an angle \( \theta \) counter clockwise with respect to \( S_1 \) on \( X_1Y_1 \) plane. Frames \( S_2 \) and \( S_3 \) have both their \( X \) axes aligned and \( S_3 \) is moving at an angular velocity \( \omega_2 \) about \( X_3 \) axis as observed by \( S_2 \). Similarly \( S_3 \) has another co-ordinate reference frame \( S_4 \), where \( X_3 \) axis of \( S_3 \), are rotated by an angle \( \psi \) counter clockwise with respect to \( S_1 \) on \( X_1Y_1 \) plane. Frames \( S_4 \) and \( S_5 \) have both their \( X \) axes aligned and \( S_5 \) is moving at an angular velocity \( \omega_3 \) about \( X_5 \) axis as observed by \( S_4 \).

For the case when the origin of frames are same with respect to \( S \) and the particle be at the origin of \( S_3 \), then it possesses three simultaneous superimposed spins with respect to frame \( S \). Coordinate transformation matrix from \( S \) to \( S_3 \) would be:

\[
P_{ij} = R_{y(z(\omega_3))}R_{x(z(\psi))}R_{y(z(\omega_2))}R_{x(y(\theta))}R_{y(z(\omega_1))}
\]
\[ R_{yz(\omega t)} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \omega t & \sin \omega t & 0 \\
0 & -\sin \omega t & \cos \omega t & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}, \quad \text{where, } i = 1, 2, 3 \]

Hence, we get the relations between co-ordinates of different frames \(S, S_1, S_2, S_3, S_4\) and \(S_5\) as stated below

\[
\begin{align*}
(x_1) & = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \omega_1 t & \sin \omega_1 t & 0 \\ 0 & -\sin \omega_1 t & \cos \omega_1 t & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} (x) = \begin{pmatrix} \cos \theta & \sin \theta & 0 & 0 \\ -\sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} (x_1) \\
y_1 & = \begin{pmatrix} \cos \omega_2 t & \sin \omega_2 t & 0 & 0 \\ -\sin \omega_2 t & \cos \omega_2 t & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} (y) = \begin{pmatrix} \cos \psi & 0 & \sin \psi & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \psi & 0 & \cos \psi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} (y_1) \\
z_1 & = \begin{pmatrix} \cos \omega_3 t & \sin \omega_3 t & 0 & 0 \\ -\sin \omega_3 t & \cos \omega_3 t & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} (z) = \begin{pmatrix} \cos \theta & \sin \theta & 0 & 0 \\ -\sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} (z_1) \\
t_1 & = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \omega_1 t & \sin \omega_1 t & 0 \\ 0 & -\sin \omega_1 t & \cos \omega_1 t & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} (t) \\
\end{align*}
\]

Therefore, the transformation matrix for coordinates of an event from \(S\) to \(S_5\) would be as shown below

\[
P_{ij} = R_{yz(\omega_1 t)} R_{xz(\omega_2 t)} R_{xy(\theta)} R_{yz(\omega_3 t)} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \omega_1 t & \sin \omega_1 t & 0 \\
0 & -\sin \omega_1 t & \cos \omega_1 t & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix} \begin{pmatrix}
\cos \psi & 0 & \sin \psi & 0 \\
0 & 1 & 0 & 0 \\
-\sin \psi & 0 & \cos \psi & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix} \begin{pmatrix}
\cos \theta & \sin \theta & 0 & 0 \\
-\sin \theta & \cos \theta & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & -\sin \omega_3 t & \cos \omega_3 t & 0 \\
\end{pmatrix} \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \omega_1 t & \sin \omega_1 t & 0 \\
0 & -\sin \omega_1 t & \cos \omega_1 t & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix} \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \omega_2 t & \sin \omega_2 t & 0 \\
0 & -\sin \omega_2 t & \cos \omega_2 t & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix} \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \omega_3 t & \sin \omega_3 t & 0 \\
0 & -\sin \omega_3 t & \cos \omega_3 t & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix} \]
Similarly, coordinate transformation matrix of an event from $S_2$ to $S$ would be

$$
\tilde{P}_{ij} = R_{y2(-\omega t)} R_{x1(-\psi)} R_{x2(-\omega t)} R_{x2(-\psi)} R_{y2(-\omega t)} =
$$

$$
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \omega t & -\sin \omega t & 0 \\
0 & \sin \omega t & \cos \omega t & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\cos \theta & -\sin \theta & 0 & 0 \\
-\sin \theta & \cos \theta & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\cos \psi & 0 & -\sin \psi & 0 \\
0 & \cos \psi & 0 & 0 \\
-\sin \psi & 0 & \cos \psi & 0 \\
0 & \sin \omega t & \cos \omega t & 0
\end{bmatrix}
$$

(1)
Now using (1) we get the relation between co-ordinates of frames $S$ and $S_5$ as shown below

$$X_S(x_5, y_5, z_5, t_5) = P_{ij} X(x, y, z, t)$$

(3)

Similarly using (2) we obtain the relation between co-ordinates of frames $S$ and $S_5$

$$X(x, y, z, t) = \overline{P}_{ij} X_S(x_5, y_5, z_5, t_5)$$

(4)

where, $X_S(x_5, y_5, z_5, t_5) = \begin{pmatrix} x_5 \\ y_5 \\ z_5 \\ t_5 \end{pmatrix}$ and $X(x, y, z, t) = \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}$

2. SPACE-TIME GEOMETRY

From (4) we obtain

\[
dx^2 = (\overline{P}_{11} dx' + \overline{P}_{12} dy' + \overline{P}_{13} dz' + \overline{P}_{14} dt')^2 \\
dy^2 = (\overline{P}_{21} dx' + \overline{P}_{22} dy' + \overline{P}_{23} dz' + \overline{P}_{24} dt')^2 \\
dz^2 = (\overline{P}_{31} dx' + \overline{P}_{32} dy' + \overline{P}_{33} dz' + \overline{P}_{34} dt')^2 \\
dt^2 = (\overline{P}_{41} dx' + \overline{P}_{42} dy' + \overline{P}_{43} dz' + \overline{P}_{44} dt')^2
\]

(5)

Now, we have Cartesian Co-ordinate geometry in flat space-time

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

(6)

Using (5) we obtain from (6), the space-time geometry of three simultaneous superimposed rotating system

\[
ds^2 = A_1 dx'^2 + A_2 dy'^2 + A_3 dz'^2 + A_4 dt'^2 + 2(B_1 dx' dy' + B_2 dx' dz' + B_3 dy' dz' + B_4 dy' dt' + B_5 dz' dt')
\]

(7)
where,

\[ A_1 = \vec{P}_{11}^2 + \vec{P}_{21}^2 + \vec{P}_{31}^2 - \vec{P}_{41}^2, \quad A_2 = \vec{P}_{12}^2 + \vec{P}_{22}^2 + \vec{P}_{32}^2 - \vec{P}_{42}^2 \]

\[ A_3 = \vec{P}_{13}^2 + \vec{P}_{23}^2 + \vec{P}_{33}^2 - \vec{P}_{43}^2, \quad A_4 = \vec{P}_{14}^2 + \vec{P}_{24}^2 + \vec{P}_{34}^2 - \vec{P}_{44}^2 \]

\[ B_1 = \vec{P}_{11} \vec{P}_{12} + \vec{P}_{21} \vec{P}_{22} + \vec{P}_{31} \vec{P}_{32} - \vec{P}_{41} \vec{P}_{42}, \quad B_2 = \vec{P}_{11} \vec{P}_{13} + \vec{P}_{21} \vec{P}_{23} + \vec{P}_{31} \vec{P}_{33} - \vec{P}_{41} \vec{P}_{43} \]

\[ B_3 = \vec{P}_{11} \vec{P}_{14} + \vec{P}_{21} \vec{P}_{24} + \vec{P}_{31} \vec{P}_{34} - \vec{P}_{41} \vec{P}_{44}, \quad B_4 = \vec{P}_{12} \vec{P}_{13} + \vec{P}_{22} \vec{P}_{23} + \vec{P}_{32} \vec{P}_{33} - \vec{P}_{42} \vec{P}_{43} \]

\[ B_5 = \vec{P}_{12} \vec{P}_{14} + \vec{P}_{22} \vec{P}_{24} + \vec{P}_{32} \vec{P}_{34} - \vec{P}_{42} \vec{P}_{44}, \quad B_6 = \vec{P}_{13} \vec{P}_{14} + \vec{P}_{23} \vec{P}_{24} + \vec{P}_{33} \vec{P}_{34} - \vec{P}_{43} \vec{P}_{44} \]

3. **CONCLUSION**

In this way any number of transformations or rotations may be considered. Also the process leads to conclude that a particle may possess more than one simultaneous superimposed spins with its mutual effects. Co-ordinate transformation in non-inertial systems would be done following the process as in the text.

**References**

