

# Critics of Existent Theory of Mathematical Pendulum Part 2

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## ABSTRACT

In the second part of the paper, the thesis is proved to state that the existent theory describes simply a shadow of the rotating apparent mathematical pendulum. Hence, it appears, even that existent description is not sufficiently adequate. Finally, all defects of the theory, which resulted in so inadequate description of the oscillation motion of the simple mathematical pendulum, have been revealed. The necessity to re-build the existent theory has been indicated in the conclusion. Return to the source is to be the first, essential step on the new path of the cognitive action.

**Keywords:** Mathematical pendulum; Vibration period; Amplitude; Angular velocity; Force of gravity; Force of inertia

## 1. INTRODUCTION

In this part of the paper, the thesis is developed that the existent theory of mathematical pendulum is not correct and it describes rather a shadow of the rotating apparent mathematical pendulum. The insufficiencies and faults disqualifying the existent theory of mathematical pendulum will be presented. By returning to the source, an essential advance on the path of the cognitive action to re-build the theory is performed.

## 2. APPARENT ROTATIONAL MATHEMATICAL PENDULUM

It is quite obvious that a real simple mathematical pendulum moves with the harmonic oscillatory motion but only in one plane, just vertical plane. One has not prove that fact. The definition of such pendulum itself clearly explains that. One cannot assume any rotary motion here, because such a motion is not performed by the simple mathematical pendulum [1].

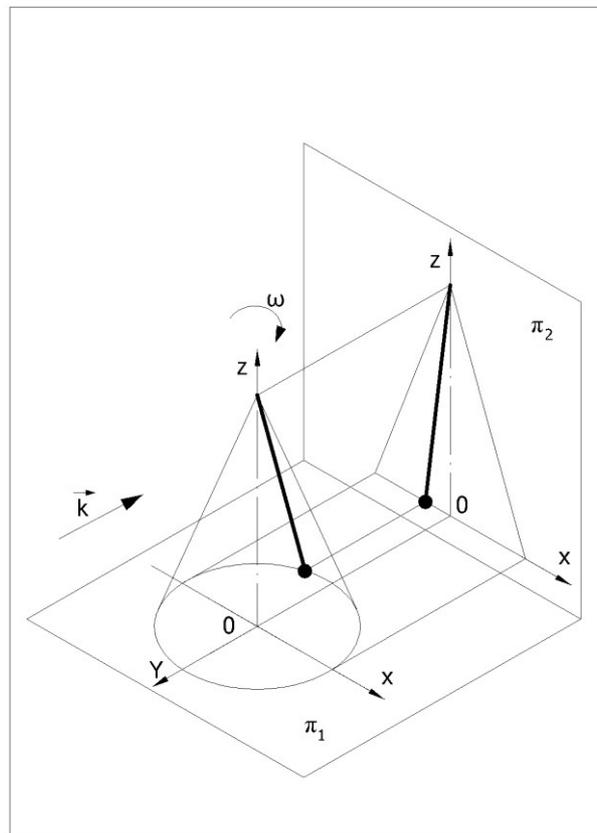
Thus a link which is characteristic with a rotary motion has been introduced to describe the motion of simple mathematical pendulum. The quest is about the angular velocity, expressed by the formula (1)

$$\omega = \frac{2\pi}{T} \quad (1)$$

where  $T$  is the vibration period of mathematical pendulum (period of harmonic motion) [1]. That is not a general definition of angular velocity; that dependence clearly refers to a detailed case when the length of angular way/path ( $2\pi$ ) corresponds with one cycle of motion of one full revolution of the rotating system.

It would be enough that the absurdity could be the proof that existent theory of the considered pendulum has a mystification nature. The Authors of this work are going to present a series of other, equally extreme/flagrant defects of this theory. Now, however, a reasoning which could accompany this cognitive process [2-5] is to be presented.

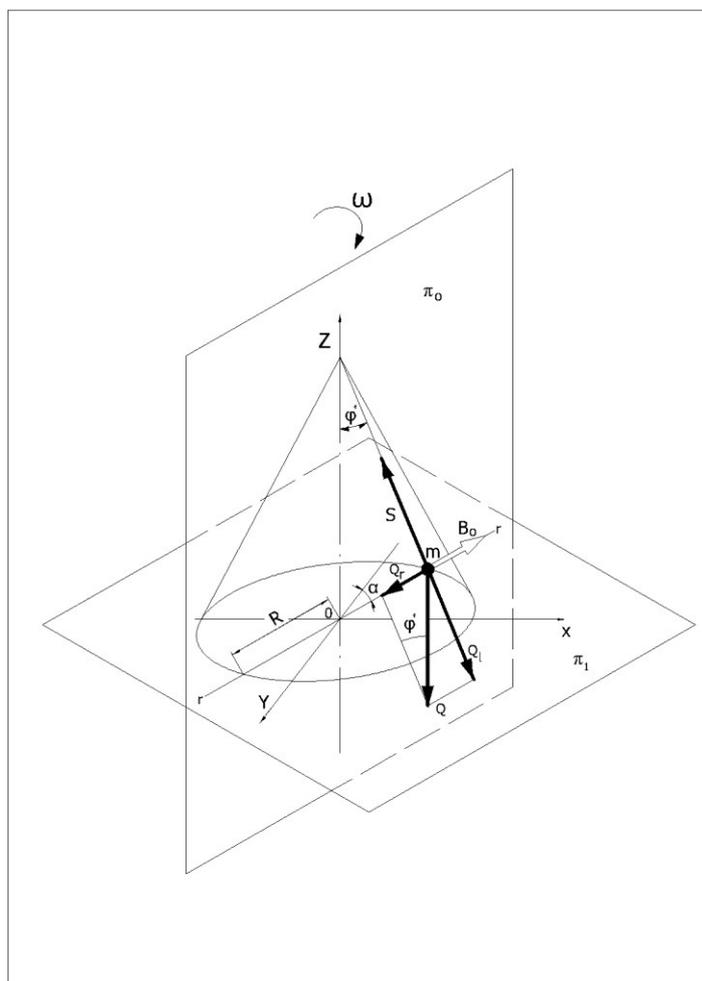
One could think an apparent rotary mathematical pendulum was analysed/considered, whereas the motion of its shadow on the vertical plane (projection plane) was described (**Fig. 1**). The weight rotates in the plane parallel to horizontal projection plane  $\pi_1$ , and its shadow (visible on vertical projection plane  $\pi_2$ ) performs the oscillatory motion. The shadow projected on the plane in direction  $\vec{k}$  behaves similar as a real simple mathematical pendulum. This, however, is far from the reality.



**Fig. 1.** Rotary mathematical pendulum and its shadow on the vertical projection plane.

In the axial plane  $\pi_o$  of such a rotary mathematical pendulum (**Fig. 2**) there appears a constellation of vectors of: gravity force  $Q$ , radial component  $Q_r$  of the gravity force, forming component  $Q_l$  of the gravity force (component which is situated on the direction of forming the apparent conical surface, created by rotating the thread of pendulum around the vertical axis), centrifugal force of inertia  $B_o$  (in Figure 2 it has been denoted by a contour vector to

underline that the force has a fictional character). It is worth noting that the fictional nature of centrifugal force of inertia is strongly accented, especially by references [1, 6].



**Fig. 2.** Vectors of forces in the moving axial plane of the rotary mathematical pendulum.

In a radial direction  $r - r$  forces  $Q_r$  and  $B_o$  are in equilibrium, then

$$Q_r = B_o \quad (2)$$

That equilibrium takes place in a horizontal plane  $\pi_1$  and in this plane the rotary motion of pendulum weight is performed (**Fig. 3a**). Projections of these components of forces on the direction  $x - x$  will prove of the force equilibrium of a rotating body just in this direction. Therefore

$$B_o \sin \alpha = Q_r \sin \alpha \quad (3)$$

or

$$B_o \sin \omega t = Q_r \sin \omega t \quad (4)$$

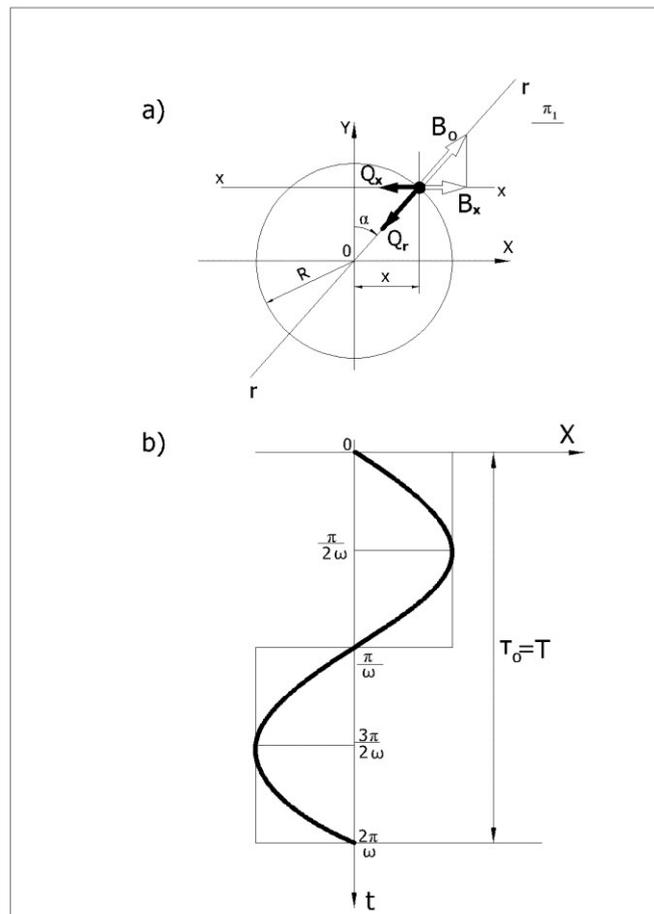
that results from the definition of angular velocity

$$\omega = \frac{\alpha}{t} \quad (5)$$

From that, also geometrically illustrated equilibrium (Fig. 3a), it results the dependence of coordinate of the weight position (in the direction  $x - x$ ) on time  $t$  of its peripheral motion. The analytical presentation of function  $x = f(t)$  has the following form:

$$x = R \sin \alpha = R \sin \omega t \quad (6)$$

Geometric picture of the former dependence (**Fig. 3b**) covers one whole phase of the weight motion in the peripheral direction, that is period  $T_o$  of the motion. That period quantitatively corresponds with the period  $T$  of vibrations of the weight shadow in the vertical plane  $\pi_2$  (see Fig. 1).



**Fig. 3.** Vectors of forces in the horizontal plane of the weight rotation (a) and dependence (b) of its position/location (in  $x - x$  direction) on time.

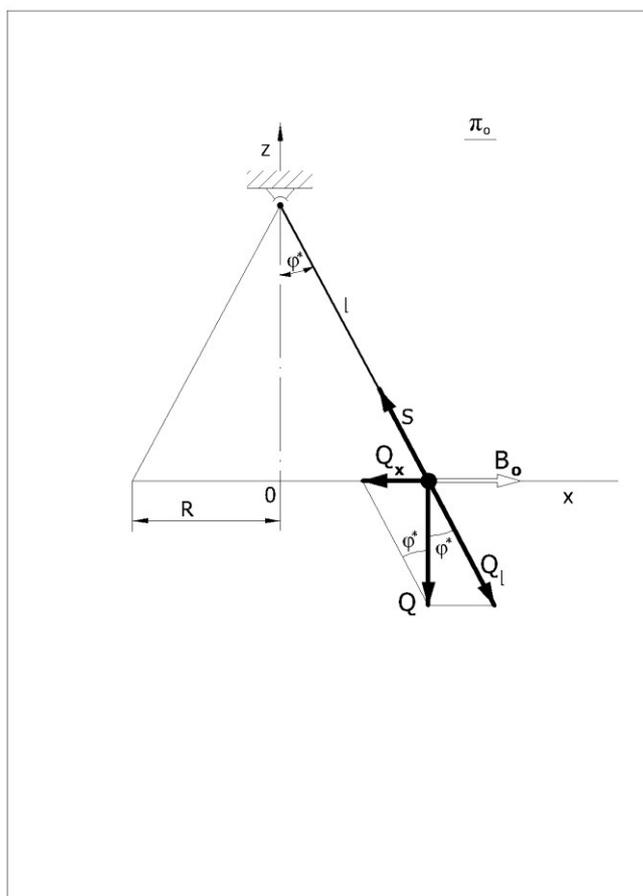
The centrifugal force of inertia is described by the formula of type

$$B_o = m\omega^2 R \quad (7)$$

and  $Q_r$  is connected with the force  $Q$  by the following dependence:

$$Q_r = Q \sin \varphi^* \quad (8)$$

that results from the system of forces in the axial plane of weight motion (**Fig. 4**).



**Fig. 4.** System of force vectors in the axial plane of weight motion.

By regarding (7) and (8) in the formula (4), one obtains the equilibrium equation of the following form:

$$m\omega^2 R \sin \alpha t = mg \sin \varphi^* \quad (9)$$

and after substituting (7) and (8) to (2) one obtains respectively developed the equation of equilibrium in the radial direction, that is

$$m\omega^2 R = mg \sin \varphi^* \quad (10)$$

The second power of angular velocity, described by formula (1), is expressed by the formula

$$\omega^2 = \left(\frac{2\pi}{T}\right)^2 = \frac{4\pi^2}{T^2} \quad (11)$$

Then, by regarding (11) in the formula (10), after performing simple algebraic operations, one obtains formula on the period of circulation motion of the pendulum weight, that is

$$T_o = T = 2\pi \sqrt{\frac{R}{g \sin \varphi^*}} \quad (12)$$

Taking into account the dependence

$$\sin \varphi^* = \frac{R}{l} \quad (13)$$

resulting from Fig. 4, one obtains the final form of the formula on time  $T_o$  of circulation of the weight around vertical axis, corresponding with the vibrations period  $T$  of shadow of that apparent rotary mathematical pendulum, that is

$$T_o = T = 2\pi \sqrt{\frac{l}{g}} \quad (14)$$

It is worth noting that the radius  $R$  and periods  $T$  and  $T_o$  are the overall-dimension (extreme, final) parameters of motion of the apparent pendulum. Both, for the pendulum itself as well as for its shadow, the parameters are the same. However, in the range of time parameters, i.e. the mentioned periods, there are highly differentiated courses of the determined magnitudes. Let us illustrate them, for instance, by two parameters of motion in the peripheral and axial directions (**Fig. 5**). The courses of all these magnitudes have been denoted here, as can be seen, in the first quarter-periods of particular motions.

In the peripheral direction (Fig. 5a) they are the path length  $s$  and peripheral velocity  $v$ , with

$$s = v \cdot t \quad (15)$$

and

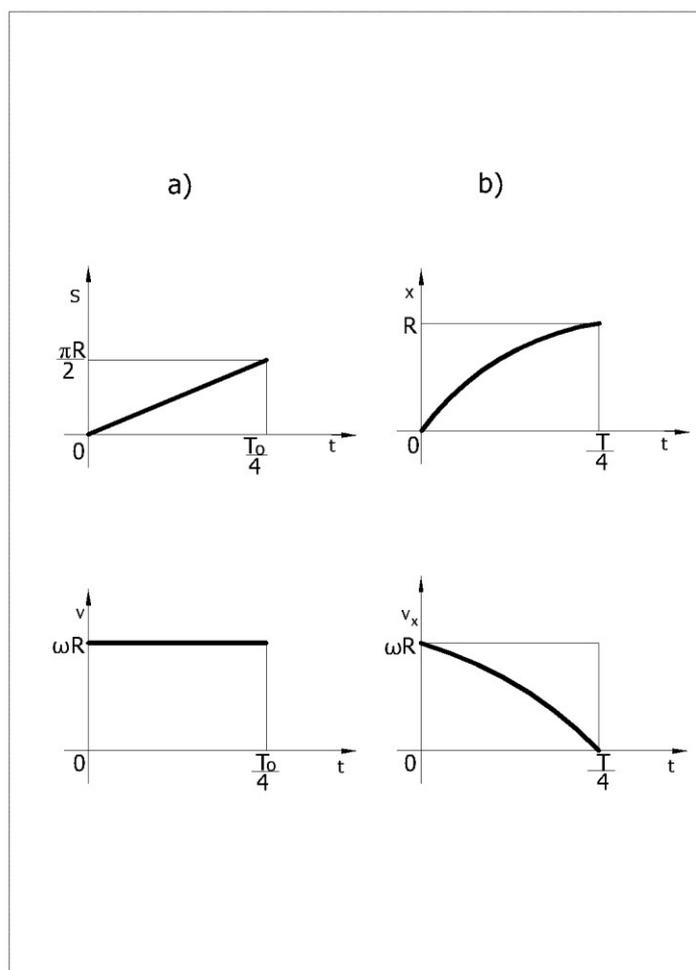
$$v = \omega \cdot R = \text{const.} \quad (16)$$

In the axial direction  $x-x$ , that is in the direction of shadow of the rotating pendulum (Fig. 5b), the axial motion is described by the following parameters: coordinate  $x$  and projection of the peripheral velocity on that direction, that is  $v_x$ . Here the plot of dependence  $x = f(t)$  results from the formula (6), whereas geometric picture of the dependence  $v_x = f(t)$  corresponds with the first derivative of dependence of coordinate  $x$  with time  $t$ , that is

$$v_x = \frac{dx}{dt} = \frac{d(R \sin \omega t)}{dt} = \omega R \cos \omega t \quad (17)$$

In the context of two presented descriptions of the mathematical pendulum motion it is worth analyzing the differential equation (23) [1] which reflects this motion. It presents simply a differential identity, then

$$\frac{d^2x}{dt^2} + \omega^2x \equiv 0 \quad (18)$$



**Fig. 5.** Kinetic characteristics of the weight motions in the peripheral (a) and axial (b) directions.

This thesis may be proved quite easily. For that purpose firstly the second derivative of function described by the formula (24) [1] should be determined. That derivate has the following configuration:

$$\frac{d^2x}{dt^2} = -A\omega^2 \sin \omega t \quad (19)$$

Now, substituting (24) [1], that is  $x = A \sin \omega t$ , and (19) to the formula (18), one obtains

$$-A\omega^2 \sin \omega t + \omega^2 A \sin \omega t \equiv 0 \quad (20)$$

By analogy, one may prove that the former equation (11) [1] is also the differential identity, that is

$$\frac{d^2\varphi}{dt^2} + \omega^2\varphi \equiv 0 \quad (44)$$

It appears, all these actions, relying on the way of description of motion of the mathematical pendulum have been performed by these type of equations, known as the differential identities. Therefore that confirms the earlier thesis of mystification character of the existent theory of the mathematical pendulum. There are several reasons which resulted in obtaining such a descriptive form. This Chapter was just to indicate them; the broader treatment of the faults will be presented in the next Chapter of the work.

### 3. FAULTS DISQUALIFYING THE EXISTENT THEORY OF MATHEMATICAL PENDULUM

The list of reasoning faults, which contributed in creating of this mystification theory of mathematical pendulum should be opened with the first one, visible on the motion schemes of this type of pendulum (see Fig. 1 and Fig. 3 [1]). Why the pendulum position has been determined by the deflection of thread from the vertical position? That angle does not refer to the path length travelled by pendulum from the turning point, in which it began its variable free motion. Naturally, at the beginning it is a free accelerated motion. From the lowest point of weight the reversal type of the free variable motion takes place which is that type of retarded motion. That parameter of the pendulum motion characteristics is not fully univocal and clearly marks the cognitive tendencies which aim to create and describe an apparent reality. One may notice here the aiming to consider the rotary motion of mathematical pendulum, and then the motion of shadow of such an apparent pendulum. Thus, at the very beginning of the problem analysis, one may feel augury/presage of erroneous, inadequate description of the simple mathematical pendulum motion.

It is quite obvious that the simple mathematical pendulum moves in one plane and its angular path between the neighbouring turning points is rather small. It is contained in the definition of the pendulum itself. Meanwhile the known descriptions of motion of this object reveal the existence of a round angle, i.e.  $2\pi$  in its structure. Thus the pendulum is to rotate?

In a determined place of the existent theory of pendulum (see equation (8) [1]) the quotient  $g:l$  was named the square/second power of the angular velocity, and further on by treating it quite seriously. The consequences of that fraction-related operation is alas pejorative. The simple mathematical pendulum is considered as a rotary pendulum of this type and then – its rotating shadow is as well. That is true, a unit of that quotient corresponds with the unit of square angular velocity (angular velocity in power two). That is not enough argument, however, to ascribe this unit to the square physical magnitude. That argument may fulfill only the role of a necessary indispensable condition of rightness of such assignment.

The necessary condition is not enough; the sufficient condition has to be fulfilled as well. However, fulfilling the second condition is not possible because the considered phenomenon does not relates with the rotating motion. The simple mathematical pendulum does not rotate around the point of its fixation. It only swings/oscillates around the equilibrium position. It does not perform even one revolution. How can this specific motion be described by the angular velocity which is related with a rotary velocity (a number of

revolutions per minute)? Unfortunately, it appears, such a procedure should be excluded from any methodic approach.

That considerable transgression of the existent theory of mathematical pendulum is a strange, quite erroneous approach to the gravitation phenomenon. A sort of bifurcation is noticed and those components of the gravity force prove of that imaginary interpretation.

The gravitation force cannot be decomposed into components due to a natural reason [7,8]; it is the gravitation of Earth sphere with the force being the measure. It is the ability to attract solids, resulting from the specific nature of Earth kernel/core being the metallic alloy of iron and nickel (named as „nife”). Thus the gravitation force has the only one, radial direction, and directed into the Earth core [7]. Naturally, in a small scale it has been treated as a vertical direction.

The motion of simple mathematical pendulum does not result from a tangent action of gravitation as it used to have been explained under the existent theory. It is the direction of inertia of the pendulum weight. It makes the pendulum oscillates around the position of its stable equilibrium. The real force of gravitation [8] should be in that place where the tangent gravity force was placed.

#### 4. CONCLUSION

Presented in the paper existent theory of a simple mathematical pendulum has many essential shortcomings. In the framework of critics there were several arguments presented against the existent theory. This paper aims at revealing the fact of that theory is still in force and functioning in science for centuries.

A cure and restoration of the existent theory is needed, even necessary. It has to reach the very root; one should master the content first and then find the descriptive words. It is surely a philosophical approach to the subject, leading to understand the reality and then – adequate its description. Thus the *philosophication* of physics is needed with the *mathematization* shifted on the second plan.

It is worth admitting that the Authors of the work have elaborated a new theory of the simple mathematical pendulum. It has an adequate character and corresponds with the studied reality. That theory, due to its vastness, has been the subject of a separate paper.

#### References

- [1] Zdzisław Pluta, Tadeusz Hryniewicz, *International Letters of Chemistry, Physics and Astronomy* 8(2) (2013) 113-122.
- [2] Zdzisław Pluta, Tadeusz Hryniewicz, *International Letters of Chemistry, Physics and Astronomy* 2 (2012) 28-34.
- [3] Zdzisław Pluta, Tadeusz Hryniewicz, *International Letters of Chemistry, Physics and Astronomy* 3 (2012) 1-10.
- [4] Zdzisław Pluta, Tadeusz Hryniewicz, *International Letters of Chemistry, Physics and Astronomy* 3 (2012) 11-23.
- [5] Zdzisław Pluta, Tadeusz Hryniewicz, *International Letters of Chemistry, Physics and Astronomy* 4 (2012) 1-7.

- [6] M. Chown, Quantum theory does not bite. Guide for the universe. Zysk & Co. edition, Poznań 2009 (transl. from English), 1<sup>st</sup> edition.
- [7] Zdzisław Pluta, Tadeusz Hryniewicz, *International Letters of Chemistry, Physics and Astronomy* 4 (2012) 8-16.
- [8] Zdzisław Pluta, Tadeusz Hryniewicz, *International Letters of Chemistry, Physics and Astronomy* 5 (2012) 35-45.