International Journal of Pure Mathematical Sciences
ISSN: 2297-6205, Vol. 19, pp 20-29
doi:10.18052/www.scipress.com/IJPMS.19.20
CC BY 4.0. Published by SciPress Ltd, Switzerland, 2017

Submitted: 2017-06-02
Revised: 2017-08-11
Accepted: 2017-10-06
Online: 2017-12-07

Certain Types of Continuity via $I_g^{**\alpha}$- Closed Sets

C. Santhini[1,a*] , M. Suganya[1,b]

[1]Department of Mathematics, V.V.Vanniaperumal College for Women, Virudhunagar-626 001, India

[a]santhinimdu@gmail.com, [b]suganyaksa97@gmail.com

Keywords: $I_g^{**\alpha}$ - continuous function, $*^{\alpha}$- continuous function, almost $I_g^{**\alpha}$ - continuous function, $A_{I^{\alpha}}$ - continuous function.

Abstract. In this paper we introduce and investigate the notion of $I_g^{**\alpha}$- continuous functions, almost $I_g^{**\alpha}$- continuous functions and discussed the relationship with other continuous functions and obtained their characteristics. Finally we obtain the decomposition of $*^{\alpha}$ - continuity.

Introduction

The subject of ideals in topological spaces has been studied by Kuratowski [8] and Vaidyanathaswamy [15], Jankovic and Hamlett [5] investigated further properties of ideal space. The importance of continuity and generalized continuity is significant in various areas of Mathematics and related Sciences. The decomposition of continuity has been studied by many authors. Husain [3] introduced the notion of almost continuous functions. Recently Santhini et al [14] introduced $I_g^{**\alpha}$ - closed sets by using α - local functions. In this paper we introduce $I_g^{**\alpha}$- continuous functions and Almost $I_g^{**\alpha}$ - continuous functions via $I_g^{**\alpha}$ -closed sets . Some characterizations of these continuous functions along with their relationships with certain other types of continuous functions are given. Finally we obtain a decomposition of $*^{\alpha}$-continuity.

Preliminaries

First we recall some basic definitions and some properties.

A collection $I \subseteq P(X)$ is called an ideal on X if it satisfies the following two conditions: (a): $A \in I$ and $B \subseteq A \Rightarrow$ B∈ I. (b): $A \in I$ and $B \in I \Rightarrow A \cup B \in I$. An ideal topological space is a topological space (X, τ) with an ideal I on X and it is denoted by (X, τ, I). For $A \subseteq X$, $A^*(I, \tau)$={$x \in X : U \cap A \notin I$ for every $U \in \tau(x)$} where $\tau(x)$={$U \in \tau : x \in U$} is called the local function [8] of A with respect to τ and I. We simply write A^* instead of $A^*(I, \tau)$. A Kuratowski closure operator $cl^*(.)$ for a topology $\tau^*(I, \tau)$ called the $*$-topology finer than τ is defined by $cl^*(A)$=$A \cup A^*$. A subset A of an ideal space (X, τ, I) is $*$-closed (τ^* - closed) [5] if $A^* \subseteq A$. By a space, we always mean a topological space (X, τ, I) with no separation properties assumed. If $A \subseteq X$, $cl(A)$ and $int(A)$ will respectively, denoted the closure and interior of A in (X, τ) and $cl^*(A)$, $int^*(A)$ will denote the interior of A in (X, τ^*). A subset A of a topological space (X, τ) is α - open if $A \subseteq int(cl(int(A)))$. Let A be a subset of an ideal topological space (X, τ, I). Then $A^{*\alpha}$ ={$x \in X : U \cap A \notin I$ for every $U \in \tau^{\alpha}(x)$} is called α - local function of A [6] with respect to I and τ^{α}. we simply write $A^{*\alpha}$ instead of $A^{*\alpha}(I, \tau)$ in this case there is no ambiguity. A Kuratowski α - closure operator $cl^{*\alpha}(.)$ for a topology $\tau^*(I, \tau)$ called the $\tau^{*\alpha}$ - topology, finer than τ^*, $\tau^{*\alpha}$ and τ is defined by $cl^{*\alpha}(A) = A \cup A^{*\alpha}$. A subset A of an ideal topological space (X, τ, I) is $*^{\alpha}$-closed [10] if $A^{*\alpha} \subseteq A$.

Definition 1. [7] For $A \subseteq X$, $A_*(I, \tau)$={$x \in X : U \cap A \notin I$ for every $U \in \tau^s(x)$} is called the semi local function of A with respect to I and τ where $\tau^s(x) = \{U \in \tau^s(x) : x \in U\}$. We simply write A_* instead of $A_*(I, \tau)$ in this case there is no ambiguity.

Definition 2. A subset A of an ideal topological space (X, τ, I) is said to be semi I-closed [2] if $int(cl^*(A)) \subseteq A$.

Theorem 3. *[6] Let (X, τ, I) be an ideal topological space and A be a subset of X. Then $A_* \subseteq A^{*\alpha} \subseteq A^*$.*

Definition 4. A subset A of an ideal topological space (X, τ, I) is said to be

(i) I_g - closed [1] if $A^* \subseteq$ U whenever $A \subseteq U$ and U is open in X.

(ii) $I_g^{*\alpha}$ - closed [11] if $A^{*\alpha} \subseteq$ U whenever $A \subseteq U$ and U is open in X.

(iii) gI - closed [7] if $A_* \subseteq U$ whenever $A \subseteq U$ and U is open in X.

(iv) rgI - closed [9] if $A_* \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.

Definition 5. [11] A subset A of an ideal topological space (X, τ, I) is said to be an $I^{*\alpha}$- locally closed (briefly $I^{*\alpha} - LC$) -set if $A = U \cap V$ where U is open and V is $*^{\alpha}$-closed.

Definition 6. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be almost continuous [3] if $f^{-1}(V)$ is closed in X for every regular closed set V in Y.

Definition 7. A function $f : (X, \tau, I) \to (Y, \sigma)$ is said to be

(i) $*$- continuous [4] if $f^{-1}(V)$ is $*$- closed in X for every closed set V in Y.

(ii) $*^{\alpha}$- continuous [11] if $f^{-1}(V)$ is $*^{\alpha}$- closed in X for every closed set V in Y.

(iii) $I_g^{*\alpha}$- continuous [11] if $f^{-1}(V)$ is $I_g^{*\alpha}$- closed in X for every closed set V in Y.

(iv) rgI- continuous [9] if $f^{-1}(V)$ is rgI- closed in X for every closed set V in Y.

(v) gI - continuous [13] if $f^{-1}(V)$is gI- closed in X for every closed set V in Y.

(vi) I_g - continuous [4] if $f^{-1}(V)$ is I_g- closed in X for every closed set V in Y.

(vii) sI - continuous [2] if $f^{-1}(V)$ is sI- closed in X for every closed set V in Y.

(viii) $I^{*\alpha} - LC$ continuous [11] if $f^{-1}(V)$ is $I^{*\alpha} - LC$ set in X for every closed set V in Y.

Definition 8. A function $f : (X, \tau, I) \to (Y, \sigma)$ is said to be

(i) almost $*$-continuous [12] if $f^{-1}(V)$ is $*$-closed in X for every regular closed set V in Y.

(ii) almost gI- continuous [13] if $f^{-1}(V)$ is gI- closed in X for every regular closed set V in Y.

Definition 9. [14] A subset A of an ideal topological space (X, τ, I) is said to be an $I_g^{**\alpha}$- closed set if $A^{*\alpha} \subseteq U$ whenever $A \subseteq U$ and U is $*$-open in X.

The complement of $I_g^{**\alpha}$- closed set are called $I_g^{**\alpha}$- open set. The family of all $I_g^{**\alpha}$-open and $I_g^{**\alpha}$-closed subsets of a space (X, τ, I) are denoted by $I_g^{**\alpha}O(X)$ and $I_g^{**\alpha}C(X)$.

Remark 10. [14]

(i) Every closed set is $I_g^{**\alpha}$-closed set.

(ii) Every $*$-closed set is $I_g^{**\alpha}$-closed set.

(iii) Every $*^{\alpha}$-closed set is an $I_g^{**\alpha}$-closed set.

(iv) Every $I_g^{**\alpha}$-closed set is an $I_g^{*\alpha}$-closed set.

(v) Every $I_g^{**\alpha}$-closed set is a gI-closed set.

(vi) Every $I_g^{**\alpha}$-closed set is a rgI-closed set.

Definition 11. [14] A subset A of an ideal topological space (X, τ, I) is said to be a $*^{\alpha}$- perfect if $A = A^{*\alpha}$

Definition 12. [14] A subset A of an ideal topological space (X, τ, I) is said to be A_{I^α}- set if $A = U \cap V$ where U is open and V is a $*^{\alpha}$- perfect.

Remark 13. $I_g^{**\alpha}$-open set is not closed under any union from the following example.

Example 1. Let $X = \{a, b, c, d\}$, $\tau = \{\phi, X, \{a\}, \{a, b\}\}$, $I = \{\phi\}$. Then A={c}, B={d} are $I_g^{**\alpha}$-open sets but $A \cup B$={c,d} is not $I_g^{**\alpha}$-open set.

$I_g^{**\alpha}$- continuous functions

In this section, we introduce the notion of $I_g^{**\alpha}$-continuity and obtain several properties of $I_g^{**\alpha}$-continuity.

Definition 14. A function $f : (X, \tau, I) \to (Y, \sigma)$ is said to be $I_g^{**\alpha}$- continuous if $f^{-1}(V)$ is $I_g^{**\alpha}$-closed in X for every closed set V in Y.

Remark 15.

(i) Every continuous is $*^{\alpha}$-continuous.

(ii) Every $*$-continuous is $*^{\alpha}$-continuous.

Proof. (i): Let $f : (X, \tau, I) \to (Y, \sigma)$ be a continuous function and V be closed in Y. Then $f^{-1}(V)$ is closed in X. Since every closed set is $*^{\alpha}$-closed set, $f^{-1}(V)$ is $*^{\alpha}$-closed in X. Hence f is $*^{\alpha}$-continuous.
(ii): Let $f : (X, \tau, I) \to (Y, \sigma)$ be a $*$-continuous function and V be closed in Y. Then $f^{-1}(V)$ is $*$-closed in X. Since every $*$-closed set is $*^{\alpha}$-closed set, $f^{-1}(V)$ is $*^{\alpha}$-closed in X. Hence f is $*^{\alpha}$-continuous.

Theorem 16.

(i) Every continuous function is an $I_g^{**\alpha}$ - continuous function.

(ii) Every $*$-continuous function is an $I_g^{**\alpha}$ - continuous function.

(iii) Every $*^{\alpha}$- continuous function is an $I_g^{**\alpha}$ - continuous function.

(iv) Every $I_g^{**\alpha}$-continuous function is an $I_g^{*\alpha}$ - continuous function.

(v) Every $I_g^{**\alpha}$-continuous function is a rgI - continuous function.

(vi) Every $I_g^{**\alpha}$-continuous function is a gI - continuous function.

Proof. (i): Let $f : (X, \tau, I) \to (Y, \sigma)$ be a continuous function and V be closed in Y. Then $f^{-1}(V)$ is closed in X. By remark 10, $f^{-1}(V)$ is $I_g^{**\alpha}$ closed in X. Hence f is $I_g^{**\alpha}$- continuous.
(ii)-(vi): Similar to the proof of (i).

Remark 17. The converse of the above theorem is not true as seen from the following examples.

Example 2.

(i) Let $X = Y = \{a, b, c, d\}$, $\tau = \{\phi, X, \{a\}, \{a, b\}\}$, $I = \{\phi\}$, $\sigma = \{\phi, Y, \{b\}, \{b, c, d\}\}$. Define $f : (X, \tau, I) \to (Y, \sigma)$ by $f(a) = c, f(b) = a, f(c) = b, f(d) = d$ is $I_g^{**\alpha}$- continuous but not continuous.

(ii) Let $X = Y = \{a, b, c, d\}$, $\tau = \{\phi, X, \{a\}, \{a, b\}\}$, $I = \{\phi\}$, $\sigma = \{\phi, Y, \{b\}, \{b, c, d\}\}$. Define $f : (X, \tau, I) \to (Y, \sigma)$ by $f(a) = c, f(b) = a, f(c) = b, f(d) = d$. Then f is $I_g^{**\alpha}$- continuous but not $*$- continuous.

(iii) Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b, c\}\}$, $I = \{\phi, \{a\}\}$ $\sigma = \{\phi, Y, \{b, c\}\}$. Define $f : (X, \tau, I) \to (Y, \sigma)$ by $f(a) = c, f(b) = a, f(c) = b$. Then f is $I_g^{**\alpha}$- continuous but not $*^{\alpha}$- continuous.

(iv) Let $X = Y = \{a, b, c, d\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$, $I = \{\phi, \{b\}, \{c\}, \{b, c\}\}$, $\sigma = \{\phi, Y, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. Define $f : (X, \tau, I) \to (Y, \sigma)$ by $f(a) = a, f(b) = d, f(c) = b, f(d) = c$. Then f is $I_g^{*\alpha}$- continuous but not $I_g^{**\alpha}$- continuous.

(v) Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{a, c\}\}$, $I = \{\phi\}$, $Y = \{a, b\}$, $\sigma = \{\phi, Y, \{b\}\}$. Define $f : (X, \tau, I) \to (Y, \sigma)$ by $f(a) = a, f(b) = f(c) = b$. Then f is rgI- continuous but not $I_g^{**\alpha}$- continuous.

(vi) Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$, $I = \{\phi\}$, $\sigma = \{\phi, Y, \{b\}, \{a, b\}, \{b, c\}\}$. Define $f : (X, \tau, I) \to (Y, \sigma)$ by $f(a) = b, f(b) = c, f(c) = a$. Then f is gI- continuous but not $I_g^{**\alpha}$- continuous.

Remark 18. We have the following diagram among several continuities defined above.

$$\begin{tikzcd}
 & 1 \arrow[dl] \arrow[d] \arrow[dr] & \\
2 \arrow[r] \arrow[d] & 3 \arrow[d] & 4 \arrow[l] \arrow[d] \\
5 & 6 \arrow[l] \arrow[r] & 7
\end{tikzcd}$$

1: Continuous 2: $*$-continuous 3: $I_g^{**\alpha}$-continuous 4: $*^{\alpha}$-continuous 5: gI-continuous 6: $I_g^{*\alpha}$-continuous 7: rgI- continuous

Remark 19. $I_g^{**\alpha}$ -continuous function and I_g- continuous function are independent of each other as seen from the following examples.

Example 3. Let $X = \{a, b, c, d\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$, $I = \{\phi, \{b\}, \{c\}, \{b, c\}\}$, $Y = \{a, b, c\}$, $\sigma = \{\phi, Y, \{a\}, \{a, b\}$. Define $f : (X, \tau, I) \to (Y, \sigma)$ by $f(a) = d, f(b) = a, f(c) = b, f(d) = c$. Then f is I_g- continuous but not $I_g^{**\alpha}$- continuous.

Example 4. Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{a, b\}\}$, $I = \{\phi\}$, $\sigma = \{\phi, Y, \{c\}, \{a, c\}, \{b, c\}\}$. Define $f : (X, \tau, I) \to (Y, \sigma)$ by $f(a) = c, f(b) = b, f(c) = a$. Then f is $I_g^{**\alpha}$- continuous but not I_g- continuous.

Remark 20. $I_g^{**\alpha}$- continuous function and sI- continuous function are independent of each other as seen from the following examples.

Example 5. Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{a, c\}\}$, $\sigma = \{\phi, Y, \{a\}, \{b\}, \{a, b\}\}$, $I = \{\phi\}$. Define $f : (X, \tau, I) \to (Y, \sigma)$ by $f(a) = b, f(b) = c, f(c) = a$. Then f is $I_g^{**\alpha}$- continuous but not sI - continuous.

Example 6. Let $X = Y = \{a, b, c, d\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$, $I = \{\phi, \{c\}\}$, $\sigma = \{\phi, Y, \{a\}, \{a, b\}, \{a, b, c\}\}$. Define $f : (X, \tau, I) \to (Y, \sigma)$ by $f(a) = b, f(b) = d, f(c) = c, f(d) = a$. Then f is sI - continuous but not $I_g^{**\alpha}$- continuous.

Remark 21. $I_g^{**\alpha}$ -continuous function and $I^{*\alpha} - LC$-continuous function are independent of each other as seen from the following examples.

Example 7. Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, X, \{c\}\}$, $I = \{\phi\}$, $\sigma = \{\phi, Y, \{a\}, \{b, c\}\}$. Define $f : (X, \tau, I) \to (Y, \sigma)$ by $f(a) = b, f(b) = c, f(c) = a$. Then f is $I^{*\alpha} - LC$-continuous but not $I_g^{**\alpha}$-continuous.

Example 8. Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b, c\}\}$, $I = \{\phi, \{a\}\}$, $\sigma = \{\phi, Y, \{c\}, \{b, c\}, \{a, c\}\}$. Define $f : (X, \tau, I) \to (Y, \sigma)$ by $f(a) = c, f(b) = a, f(c) = b$. Then f is $I_g^{**\alpha}$-continuous but not $I^{*\alpha} - LC$-continuous.

Characterizations of $I_g^{**\alpha}$- continuous functions

Theorem 22. *For a function $f : (X, \tau, I) \to (Y, \sigma)$ the following are equivalent. Assume that $I_g^{**\alpha}O(X)$ is closed under any union.*

(i) f is $I_g^{**\alpha}$- continuous function .

(ii) For each $x \in X$ and $V \in \sigma$ containing $f(x)$ there exists a $W \in I_g^{**\alpha}O(X)$ containing x such that $f(W) \subseteq V$.

Proof. (i) $\Rightarrow$ (ii): Let f be $I_g^{**\alpha}$- continuous and V be any open set in Y containing $f(x)$ so that $x \in f^{-1}(V)$. Since f is $I_g^{**\alpha}$- continuous, $f^{-1}(V)$ is $I_g^{**\alpha}$- open in X. Let $W = f^{-1}$(V) then W is $I_g^{**\alpha}$- open in X and $x \in W$. Also $f(W) = f(f^{-1}(V)) \subseteq V$.
(ii)$\Rightarrow$ (i): Let V be open in Y and $x \in f^{-1}(V)$. Then $f(x) \in V$. By (ii), there exist an $I_g^{**\alpha}$- open set W_x containing x such that $f(W) \subseteq V$ which implies $W \subseteq f^{-1}(V)$. Therefore $f^{-1}(V) = \cup\{W_x : x \in f^{-1}(V)\}$. Since W_x is $I_g^{**\alpha}$ -open and $I_g^{**\alpha}O(X)$ is closed under any union, $f^{-1}(V)$ is $I_g^{**\alpha}$- open in X.

Theorem 23. *A function $f : (X, \tau, I) \to (Y, \sigma)$ is an $I_g^{**\alpha}$- continuous function if $f^{-1}(V)$ is $I_g^{**\alpha}$ - open in X for every open set V in Y.*

Proof. Since $f^{-1}(V^c) = (f^{-1}(V))^c$, proof follows.

Remark 24. Composition of two $I_g^{**\alpha}$- continuous functions is not an $I_g^{**\alpha}$- continuous function.

Example 9. Let $X = Y = Z = \{a, b, c, d\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$, $I = \{\phi, \{b\}, \{c\}, \{b, c\}\}$, $\sigma = \{\phi, Y, \{a\}, \{a, b\}\}$, $J = \{\phi\}$, $\eta = \{\phi, Z, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. Define $f : (X, \tau, I) \to (Y, \sigma, J)$ by $f(a) = c, f(b) = b, f(c) = d, f(d) = a$ and $g : (Y, \sigma, J) \to (Z, \eta)$ is an identity map. Then $g \circ f : (X, \tau, I) \to (Z, \eta)$ is not an $I_g^{**\alpha}$- continuous functions.

Theorem 25. *Let $f : (X, \tau, I) \to (Y, \sigma, J)$ and $g : (Y, \sigma, J) \to (Z, \eta)$ be a function. Then the following are hold*

(i) If f is $I_g^{**\alpha}$- continuous and g is continuous then $g \circ f$ is $I_g^{**\alpha}$- continuous.

(ii) If f is $I_g^{**\alpha}$- continuous and g is continuous then $g \circ f$ is $I_g^{*\alpha}$- continuous.

(iii) If f is $I_g^{**\alpha}$- continuous and g is continuous then $g \circ f$ is rgI-continuous.

(iv) If f is $I_g^{**\alpha}$- continuous and g is continuous then $g \circ f$ is gI- continuous.

Proof. (i): Let V be closed in Z. Since g is continuous, $g^{-1}(V)$ is closed in Y. Also f is $I_g^{**\alpha}$- continuous, then $f^{-1}(g^{-1}(V))$= (g∘f)$^{-1}$(V) is $I_g^{**\alpha}$- closed in X. Hence $g \circ f$ is $I_g^{**\alpha}$- continuous.
(ii)-(iv): Similar to the proof of (i).

Definition 26. A function $f : (X, \tau, I) \to (Y, \sigma, J)$ is said to be $I_g^{**\alpha}$- irresolute if $f^{-1}(V)$ is $I_g^{**\alpha}$- closed in X for every $I_g^{**\alpha}$ -closed set V in Y.

Theorem 27. *Every $I_g^{**\alpha}$- irresolute function is an $I_g^{**\alpha}$- continuous function.*

Proof. Let $f : (X, \tau, I) \to (Y, \sigma, J)$ be $I_g^{**\alpha}$- irresolute and V be closed in Y. By remark 10, V is $I_g^{**\alpha}$- closed in Y. Since f is $I_g^{**\alpha}$- irresolute, $f^{-1}(V)$ is $I_g^{**\alpha}$- closed in X. Hence f is an $I_g^{**\alpha}$- continuous function.

Remark 28. The converse of the above theorem is not true as seen from the following example.

Example 10. Let $X = \{a, b, c, d\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$, $I = \{\phi, \{c\}\}$, $Y = \{a, b, c\}$, $\sigma = \{\phi, Y, \{a\}, \{a, c\}\}$, $J = \{\phi\}$. Define $f : (X, \tau, I) \to (Y, \sigma, J)$ by $f(a) = d, f(b) = c, f(c) = b, f(d) = a$. Then f is $I_g^{**\alpha}$- continuous but not $I_g^{**\alpha}$- irresolute.

Theorem 29. *Let $f : (X, \tau, I) \to (Y, \sigma, J)$ and $g : (Y, \sigma, J) \to (Z, \eta)$ be a functions Then the following are hold*

(i) If f is $I_g^{**\alpha}$- irresolute and g is $*^{\alpha}$- continuous then $g \circ f$ is $I_g^{**\alpha}$- continuous.

(ii) If f is $I_g^{**\alpha}$- irresolute and g is $*^{\alpha}$- continuous then $g \circ f$ is $I_g^{*\alpha}$- continuous.

(iii) If f is $I_g^{**\alpha}$- irresolute and g is $*^{\alpha}$- continuous then $g \circ f$ is gI- continuous.

(iv) If f is $I_g^{**\alpha}$- irresolute and g is $*^{\alpha}$- continuous then $g \circ f$ is rgI- continuous.

Proof. (i): Let V be closed in Z. Since g is $*^{\alpha}$-continuous, $g^{-1}(V)$ is $*^{\alpha}$ -closed in Y. By remark 10, $g^{-1}(V)$ is $I_g^{**\alpha}$- closed in Y. Now f is $I_g^{**\alpha}$- irresolute, implies $f^{-1}(g^{-1}(V))$= $(g \circ f)^{-1}(V)$ is $I_g^{**\alpha}$- closed in X. Hence $g \circ f$ is $I_g^{**\alpha}$- continuous.
(ii) - (iv): Proof is similar to (i)

Remark 30. The above theorem is also true whenever g is $*$-continuous and g is continuous, by remark 15.

Theorem 31. *Composition of two $I_g^{**\alpha}$- irresolute function is an $I_g^{**\alpha}$- irresolute function.*

Proof. Let $f : (X, \tau, I) \to (Y, \sigma, J)$ and $g : (Y, \sigma, J) \to (Z, \eta)$ be an $I_g^{**\alpha}$- irresolute functions and V be $I_g^{**\alpha}$ -closed in Z. Then $g^{-1}(V)$ is $I_g^{**\alpha}$- closed in Y and so $f^{-1}(g^{-1}(V))$= $(g \circ f)^{-1}$(V) is $I_g^{**\alpha}$- closed in X. Hence $g \circ f$ is $I_g^{**\alpha}$- irresolute.

Theorem 32. *A function $f : (X, \tau, I) \to (Y, \sigma, J)$ is $I_g^{**\alpha}$- irresolute iff $f^{-1}(V)$ is $I_g^{**\alpha}$ - open in X for every $I_g^{**\alpha}$- open set V in Y.*

Proof. Since f$^{-1}(V^c)$ = (f^{-1}(V))^c , proof follows.

Almost $I_g^{**\alpha}$- continuous function

In this section, we introduce and obtain some characterizations of almost $I_g^{**\alpha}$-continuous functions.

Definition 33. A function $f:(X,\tau,I)\rightarrow(Y,\sigma)$ is said to be almost $I_g^{**\alpha}$- continuous if $f^{-1}(V)$ is $I_g^{**\alpha}$ -closed in X for every regular closed set V in Y

Theorem 34.

(i) Every continuous function is almost $I_g^{**\alpha}$- continuous function.

(ii) Every $*$-continuous function is almost $I_g^{**\alpha}$- continuous function.

(iii) Every $*^\alpha$-continuous function is almost $I_g^{**\alpha}$- continuous function.

(iv) Every $I_g^{**\alpha}$-continuous function is almost $I_g^{**\alpha}$- continuous function.

(v) Every almost continuous function is almost $I_g^{**\alpha}$- continuous function.

(vi) Every almost $*$-continuous function is almost $I_g^{**\alpha}$- continuous function.

(vii) Every almost $I_g^{**\alpha}$-continuous function is almost gI- continuous function.

Proof. (i): Let $f:(X,\tau,I)\rightarrow(Y,\sigma)$ be a continuous function and V be regular closed in Y. Since every regular closed set is closed set, V is closed in Y. Then $f^{-1}(V)$ is closed in X. By remark 10, $f^{-1}(V)$ is $I_g^{**\alpha}$- closed in X. Hence f is almost $I_g^{**\alpha}$- continuous.
(ii): Let $f:(X,\tau,I)\rightarrow(Y,\sigma)$ be a $*$-continuous function and V be regular closed in Y. Since every regular closed set is closed set, V is closed in Y. Then $f^{-1}(V)$ is $*$-closed in X. By remark 10, $f^{-1}(V)$ is $I_g^{**\alpha}$-closed in X. Hence f is almost $I_g^{**\alpha}$-continuous.
(iii): Let $f:(X,\tau,I)\rightarrow(Y,\sigma)$ be a $*^\alpha$-continuous function and V be regular closed in Y. Since every regular closed set is closed set, V is closed in Y. Then $f^{-1}(V)$ is $*^\alpha$-closed in X. By remark 10, $f^{-1}(V)$ is $I_g^{**\alpha}$-closed in X. Hence f is almost $I_g^{**\alpha}$-continuous.
(iv): Let $f:(X,\tau,I)\rightarrow(Y,\sigma)$ be an $I_g^{**\alpha}$-continuous function and V be regular closed in Y. Since every regular closed set is closed set, V is closed in Y. Then $f^{-1}(V)$ is $I_g^{**\alpha}$-closed in X. Hence f is almost $I_g^{**\alpha}$-continuous.
(v): Let $f:(X,\tau,I)\rightarrow(Y,\sigma)$ be a almost continuous function and V be regular closed in Y. Then $f^{-1}(V)$ is closed in X. By remark 10, $f^{-1}(V)$ is $I_g^{**\alpha}$-closed. Hence f is almost $I_g^{**\alpha}$-continuous.
(vi): Let $f:(X,\tau,I)\rightarrow(Y,\sigma)$ be almost $*$-continuous function and V be regular closed in Y. Then $f^{-1}(V)$ is $*$-closed in X. By remark 10, $f^{-1}(V)$ is $I_g^{**\alpha}$-closed. Hence f is almost $I_g^{**\alpha}$-continuous.
(vii): Let $f:(X,\tau,I)\rightarrow(Y,\sigma)$ be almost $I_g^{**\alpha}$-continuous function and V be regular closed in Y. Then $f^{-1}(V)$ is $I_g^{**\alpha}$-closed in X. By remark 10, $f^{-1}(V)$ is gI-closed. Hence f is almost gI-continuous.

Remark 35. The converse of the above theorem is not true as seen from the following examples.

Example 11.

(i) Let $X=Y=\{a,b,c\}$, $\tau=\{\phi,X,\{a,b\}\}$, $I=\{\phi,\{a\},\{b\},\{a,b\}\}$, $\sigma=\{\phi,Y,\{a\},\{b\},\{a,b\}\}$. Define $f:(X,\tau,I)\rightarrow(Y,\sigma)$ by $f(a)=c, f(b)=b, f(c)=a$. Then f is almost $I_g^{**\alpha}$-continuous but not continuous.

(ii) Let $X=\{a,b,c,d\}$, $\tau=\{\phi,X,\{a\},\{b,c,d\}\}$, $I=\{\phi\}$,$Y=\{a,b,c\}$,$\sigma=\{\phi,Y,\{a\},\{b\},\{a,b\}\}$. Define $f:(X,\tau,I)\rightarrow(Y,\sigma)$ by $f(a)=b, f(b)=a, f(c)=f(d)=c$. Then f is almost $I_g^{**\alpha}$- continuous but not $*$-continuous.

(iii) Let $X = \{a, b, c, d\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$, $I = \{\phi, \{b\}, \{c\}, \{b, c\}\}$, $Y = \{a, b, c\}$, $\sigma = \{\phi, Y, \{a\}, \{b\}, \{a, b\}\}$. Define $f : (X, \tau, I) \to (Y, \sigma)$ by $f(a) = c, f(b) = f(d) = a, f(c) = b$. Then f is almost $I_g^{**\alpha}$- continuous but not $*^\alpha$-continuous.

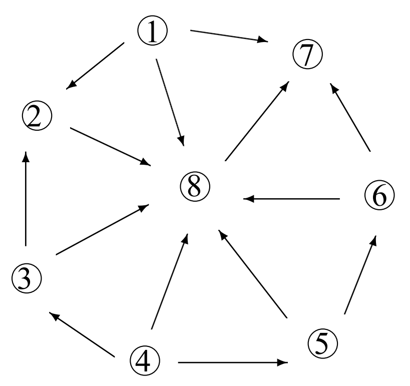
(iv) Let $X = \{a, b, c, d\}$, $\tau = \{\phi, X, \{a\}, \{a, b\}\}$, $I = \{\phi\}$, $Y = \{a, b, c\}$, $\sigma = \{\phi, Y, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$. Define $f : (X, \tau, I) \to (Y, \sigma)$ by $f(a) = c, f(b) = f(c) = b, f(d) = a$. Then f is almost $I_g^{**\alpha}$- continuous but not $I_g^{**\alpha}$- continuous.

(v) Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b, c\}\}$, $I = \{\phi, \{a\}\}$, $\sigma = \{\phi, Y, \{a\}, \{b\}, \{a, b\}\}$. Define $f : (X, \tau, I) \to (Y, \sigma)$ by $f(a) = b, f(b) = c, f(c) = a$. Then f is almost $I_g^{**\alpha}$-continuous but not almost continuous.

(vii) Let $X = Y = \{a, b, c\}$,$\tau = \{\phi, X, \{a\}, \{a, c\}\}$,$I = \{\phi\}$,$\sigma = \{\phi, Y, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$. Define $f : (X, \tau, I) \to (Y, \sigma)$ by $f(a) = c, f(b) = a, f(c) = b$. Then f is almost $I_g^{**\alpha}$- continuous but not almost $*$-continuous.

(vii) Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$, $I = \{\phi\}$, $\sigma = \{\phi, Y, \{a\}, \{b, c\}\}$. Define $f : X \to Y$ by $f(a) = b, f(b) = a, f(c) = c$. Then f is almost gI - continuous but not an almost $I_g^{**\alpha}$-continuous.

Remark 36. The relationships defined above, are shown in the following diagram:



1: Almost continuous 2: almost $*$-continuous 3: $*$-continuous 4: continuous
5: $*^\alpha$- continuous 6: $I_g^{**\alpha}$- continuous 7: almost gI-continuous 8: almost $I_g^{**\alpha}$-continuous.

Theorem 37. *For a function $f : (X, \tau, I) \to (Y, \sigma)$ the following are equivalent.*

(i) f is almost $I_g^{**\alpha}$- continuous.

(ii) $f^{-1}(A) \in I_g^{**\alpha}O(X)$ for every $A \in RO(Y)$.

(iii) $f^{-1}(int(cl(A))) \in I_g^{**\alpha}O(X)$ for every $A \in \sigma$.

(iv) $f^{-1}(cl(int(A))) \in I_g^{**\alpha}C(X)$ for every closed set A of Y.

Proof. (i) ⇒ (ii): Let f be almost $I_g^{**\alpha}$ - continuous and V be regular open set in Y. By (i), $f^{-1}(V)$ is $I_g^{**\alpha}$- open in X.
(ii) ⇒ (i): Let V be regular open in Y. By (ii), $f^{-1}(V) \in I_g^{**\alpha}O(X)$ Then f is almost $I_g^{**\alpha}$ -continuous in X.
(ii) ⇒ (iii): Suppose $A \in \sigma$. We have $int(cl(A)) \in RO(Y)$. By (ii) $f^{-1}(int(cl(A))) \in I_g^{**\alpha}O(X)$.
(iii) ⇒ (ii): Suppose $A \in RO(Y)$. We have $A = int(cl(A))$. Since every regular open set is open set, A is open in Y. By (iii), $f^{-1}(int(cl(A))) \in I_g^{**\alpha}O(X)$ and so $f^{-1}(A) \in I_g^{**\alpha}O(X)$.
(iii) ⇒ (iv): Let A be closed in Y. Then $Y - A$ is open in Y. By (iii), $f^{-1}(int(cl(y - A)))$ is $I_g^{**\alpha}$-open in X. But $f^{-1}(int(cl(Y - A)))$ =$f^{-1}(Y - (cl(int(A))))$ =$X - f^{-1}(cl(int(A)))$ which implies $f^{-1}(cl(int(A)))$ is $I_g^{**\alpha}$- closed in X.
(iv) ⇒ (iii): Let A be open in Y. Then $Y - A$ is closed in Y. By (iv), $f^{-1}(cl(int(y - A)))$ is $I_g^{**\alpha}$-closed in X. But $f^{-1}(cl(int(Y - A))) = f^{-1}(Y - int(cl(A))) = X - f^{-1}(int(cl(A)))$ which implies $f^{-1}(int(cl(A)))$ is $I_g^{**\alpha}$- open in X.

Decomposition of $*^{\alpha}$- continuity

In this section, we obtain a decomposition of $*^{\alpha}$-continuity in ideal topological spaces.

Definition 38. A function $f : (X, \tau, I) \to (Y, \sigma)$ is said to be $A_{I^{\alpha}}$-continuous function if $f^{-1}(V)$ is $A_{I^{\alpha}}$-set in X for every closed set V in Y.

Theorem 39. *Every $A_{I^{\alpha}}$-continuous function is $I^{*\alpha}$-LC-continuous function.*

Proof. Let f be $A_{I^{\alpha}}$- continuous and V be closed in Y. Then $f^{-1}(V)$ is $A_{I^{\alpha}}$- set. By remark 5.3 [14], $f^{-1}(V)$ is $I^{*\alpha} - LC$- set. Therefore f is $I^{*\alpha} - LC$- continuous.

Theorem 40. *If $f : (X, \tau, I) \to (Y, \sigma)$ is $A_{I^{\alpha}}$-continuous and $I_g^{**\alpha}$-continuous then f is $*^{\alpha}$-continuous.*

Proof. Let f be $A_{I^{\alpha}}$-continuous and $I_g^{**\alpha}$-continuous and V be closed in Y. Then $f^{-1}(V)$ is $A_{I^{\alpha}}$-set and $I_g^{**\alpha}$-closed in X. By theorem 5.6 [14], $f^{-1}(V)$ is $*^{\alpha}$-closed in X. Hence f is $*^{\alpha}$- continuous.

Theorem 41. *Let $f : (X, \tau, I) \to (Y, \sigma)$ be a function. Then the following are equivalent.*

(i) $*^{\alpha}$-continuous.

(ii) $I^{*\alpha} - LC$ continuous and $I_g^{**\alpha}$- continuous.

Proof. (i) ⇒ (ii): Let f be $*^{\alpha}$- continuous and V be closed in Y. Then $f^{-1}(V)$ is $*^{\alpha}$- closed in X. By theorem 5.9 [14] , $f^{-1}(V)$ is $I^{*\alpha} - LC$ set and $I_g^{**\alpha}$- closed in X. Hence f is $I^{*\alpha} - LC$ continuous and $I_g^{**\alpha}$- continuous.
(ii) ⇒ (i): Let f be $I^{*\alpha} - LC$ continuous and $I_g^{**\alpha}$- continuous. Then $f^{-1}(V)$ is $I^{*\alpha} - LC$ closed and $I_g^{**\alpha}$- closed. By theorem 5.9 [14], $f^{-1}(V)$ is a $*^{\alpha}$-continuous.

References

[1] J. Dontchev, M. Ganster, T. Noiri, Unified operation approach of generalized closed sets via topological ideal, Math. Japanica. 49 (1999) 395–401.

[2] E. Hatir, T. Noiri, On decomposition of continuity via idealization, Acta Math. Hungar. 96(4) (2002) 341–349.

[3] T. Husain, Almost continuous mappings, Prace Mat. 10(1) (1966) 1–7. .

[4] V. Inthumathi, S. Krishnaprakash, M. Rajamani, Strongly-I-locally closed sets and decompositions of $*$-continuity, Acta Math. Hungar. 130(4) (2011) 358-362.

[5] D. Jankovic, T.R. Hamlett, New topologies from old via ideals, Amer. Math. Monthly. 97(4) (1990) 295–310.

[6] J.G. Kang, C.S. Kim, On P-I open sets, Honam Mathematical J. 31(3) (2009) 293–314.

[7] M. Khan, T. Noiri, On gI-closed sets in ideal topological spaces, Journal of Advanced Studies in Topology. 11 (2010) 29–33.

[8] K. Kuratowski, Topology, Vol. I, Academic Press, New York, 1966.

[9] N.R. Paul, RgI-closed set in ideal topological spaces, International Journal of Computer Applications. 69(4) (2013) 23–27.

[10] M. Rajamani, V. Inthumathi, S. Krishnaprakash, Some stronger local function via ideals, J. Adv. Res. Pure Math. 2 (2010) 48–52.

[11] M. Rajamani, V. Inthumathi, S. Krishnaprakash, On $I_g^{*\alpha}$-closed sets and $I_g^{*\alpha}$-continuity, Jordan Journal of Mathematics and Statistics. 5(3) (2012) 201–208.

[12] M. Rajamani, V. Inthumathi, S. Krishnaprakash, $I_{\pi g}$ -closed sets and $\mathrm{I}_{\pi g}$-continuity, J. Adv. Res. Pure Math. 2(4) (2010) 63–72.

[13] R. Santhini, M. Rameshkumar, On πgI-continuous functions on ideal topological spaces, Scientific Studies and Research Series Mathematics and Informatics. 25(1) (2015) 225–236.

[14] C. Santhini, M. Suganya, New form of generalized closed sets via ideal topology, Proceedings of International Conference on Recent Trends in Applied Mathematics, S.R.N.M. College, Sattur, India, 2017.

[15] R. Vaidynathaswamy, The localization theory in set topology, Proc. Indian Acad. Sci. Math . Sci. 20 (1945) 51–61.