International Journal of Pure Mathematical Sciences
ISSN: 2297-6205, Vol. 19, pp 1-9
doi:10.18052/www.scipress.com/IJPMS.19.1
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Submitted: 2016-05-31
Accepted: 2017-06-09
Online: 2017-12-07

Locally $g\omega\alpha$ -Closed Sets in Topological Spaces

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Keywords: $g\omega\alpha$ -closed, $g\omega\alpha$ -continuous, $g\omega\alpha$ -irresolute, locally closed, $g\omega\alpha$ -locally closed.

Abstract. In the year 2014, the present authors introduced and studied the concept of $g\omega\alpha$ -closed sets in topological spaces. The purpose of this paper to introduce a new class of locally closed sets called $g\omega\alpha$ -locally closed sets (briefly $g\omega\alpha lc$-sets) and study some of their properties. Also $g\omega\alpha$ -locally closed continuous (briefly $g\omega\alpha lc$-continuous) functions and its irresolute functions are introduced and studied their properties in topological spaces.

1. Introduction

The notion of locally closed sets was introduced by Bourbaki [6]. According to him, a subset of a topological space X is locally closed in X if it is the intersection of an open set and closed set in X. Kuratowski and Sierpinski [10] considered the difference of two closed subsets of an n-dimensional euclidean space. Implicit in their work is the notion of a locally closed subset of a topological space X. Stone [17] has used the term FG for a locally closed subset as the spaces that in every embedding are locally closed. The results of Borges [5] show that locally closed sets play an important role in the context of simple extension. Ganster and Reilly [8] has introduced locally closed sets, which are weaker forms of both open and closed sets and they used locally closed sets to define LC-continuity and LC-irresoluteness. Sundaram [18] introduced the concept of generalized locally closed sets. After that Balachandran et al. [3], Gnanambal [9], Arokhiarani et al. [1], Pushpalatha [14], Shaik John [15] and P.G. Patil [13] have introduced α -locally closed, generalized locally semi closed, semi generalized locally closed, regular generalized locally closed, strongly locally closed, ω -locally closed and $\omega\alpha$ -locally closed sets and their continuous maps in topological spaces respectively. Also various authors have contributed to the development of generalizations of locally closed sets and locally continuous maps in topological spaces.

In this paper, we introduced the notion of $g\omega\alpha$ -locally closed sets which are denoted by $g\omega\alpha - LC$ sets and study some of the fundamental properties of $g\omega\alpha - LC$ sets in generalized topological spaces.

2. Preliminary

Throughout this paper (X,τ) or simply X represents topological space on which no separation axioms are assumed unless and otherwise mentioned. For a subset A of X. cl(A), int(A) and A^c denote the closure of A, interior of A and complement of A respectively. If A is a subset of a space τ, then $C_\tau(A)$ is the smallest τ -closed set containing A and $I_\tau(A)$ is the largest τ -open set contained in A.

For our analysis, we require the following basic definitions.

Definition 2.1. *A subset A of a topological space (X, τ) is called*
(i) α -open set [12] if A $\subseteq$ int(cl(int(A))).
(ii) Semi-open set [11] if A $\subseteq$ cl(int(A)).
(iii) Regular open set [16] if A = int(cl(A)).

The complements of the above mentioned sets are called their respective closed sets.

Definition 2.2. [4] *A subset A of X is $g\omega\alpha$-closed if $\alpha\, cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\omega\alpha$-open in X.*

The family of all $g\omega\alpha$-closed subsets of the space X is denoted by $G\omega\alpha$C(X).

Definition 2.3. [4] *The intersection of all $g\omega\alpha$-closed sets containing a set A is called $g\omega\alpha$-closure of A and is denoted by $g\omega\alpha$-cl(A).*

A set A is $g\omega\alpha$-closed set if and only if $g\omega\alpha$-cl(A) = A.

Definition 2.4. [4] *The union of all $g\omega\alpha$-open sets contained in A is called $g\omega\alpha$-interior of A and is denoted by $g\omega\alpha$-int(A).*

A set A is $g\omega\alpha$-open if and only if $g\omega\alpha$-int(A) = A.

Definition 2.5. *A topological space X is said to be a*

1. locally closed [8] if $A = U \cap V$ where U is open set and V is closed set in X.
2. generalized locally closed (briefly glc-closed) [2] if $A = U \cap V$ where U is g-open set and V is g-closed set in X.
3. generalized locally semi-closed (briefly $gslc$-closed) [9] if $A = U \cap V$ where U is g-open set and V is semi-closed set in X.
4. strongly generalized locally closed (briefly $g^{*}lc$-closed) [14] if $A = U \cap V$ where U is strongly g-open set and V is strongly g-closed set in X.
5. α-locally closed (briefly αlc-closed) [9] if $A = U \cap V$ where U is α-open set and V is α-closed set in X.
6. ω-locally closed (briefly ωlc-closed) [15] if $A = U \cap V$ where U is ω-open set and V is ω-closed set in X.
7. $\omega\alpha$-locally closed (briefly $\omega\alpha lc$- closed[13] If $A = U \cap V$ where U is $\omega\alpha$-open set and V is $\omega\alpha$-closed set in X.

Definition 2.6. *A topological space X is said to be a*

1. sub maximal space [7] if every dence subset of X is open in X.
2. door space [2] if every subset of X is either open or closed in X.

Definition 2.7. *A function f: $X \to Y$ is called*

1. LC-continuous [8] if $f^{-1}(G)$ is locally closed set in X for each open set G of Y.
2. LC-irresolute [8] if $f^{-1}(G)$ is locally closed set in X for locally closed set G of Y.

3. Locally $g\omega\alpha$-Closed Set

In this section, we introduce $g\omega\alpha lc$-sets and study some of their properties.

Definition 3.1. *Let A be a subset of X. Then A is called locally $g\omega\alpha$-closed if there exists an open set U and $g\omega\alpha$-closed set F of X such that $A = U \cap F$.*

The collection of all locally $g\omega\alpha$-closed sets is denoted by $LG\omega\alpha C(X)$.

Definition 3.2. *A space is said to have the $g\omega\alpha$-closure preserving property if $g\omega\alpha - cl(A)$ is always $g\omega\alpha$-closed.*

Theorem 3.1. *Suppose X has the $g\omega\alpha$-closure preserving property and let A be a subset of X. Then $A \in LG\omega\alpha C(X)$ if and only if $A = U \cap g\omega\alpha - cl(A)$ for some open set U.*

Proof. Let $A = LG\omega\alpha C(X)$. Then $A = U \cap F$ where U is an open and F is $g\omega\alpha$-closed. By definition 3.2, $g\omega\alpha - cl(A)$ is $g\omega\alpha$-closed in X, $A \subseteq F$ implies $g\omega\alpha - cl(A) \subseteq F$. Now,

$A = A \cap g\omega\alpha - cl(A) = U \cap F \cap g\omega\alpha - cl(A) = U \cap g\omega\alpha - cl(A)$. Therefore, $A = U \cap g\omega\alpha - cl(A)$ for some open set U.
Conversely. Assume that $A = U \cap g\omega\alpha - cl(A)$ for some open set U. By definition 3.2, $g\omega\alpha - cl(A)$ is $g\omega\alpha$-closed in X. Therefore, $A \in LG\omega\alpha C(X)$. This proves the theorem.

Definition 3.3. *Let A be a subset of X. Then A is called $g\omega\alpha$-locally closed if there exists $g\omega\alpha$-open set U and a $g\omega\alpha$-closed set F of X such that $A = U \cap F$.*

The collection of all $g\omega\alpha$-locally closed sets of X will be denoted by $G\omega\alpha LC(X)$.

Theorem 3.2. *For a submaximal space X, $LG\omega\alpha C(X) \subseteq GSLC(X)$.*

Proof. Let $A = LG\omega\alpha C(X)$. Then there exist an open set U and a $g\omega\alpha$-closed set F of X such that $A = U \cap F$. In a submaximal space X, every $g\omega\alpha$-closed set is gs-closed. Therefore F is gs-closed. Since every open set is gs-open, it follows that A is an intersection of gs-open set U and a gs-closed set F of X. Therefore, $A \in GSLC(X)$. This proves the theorem.

Theorem 3.3. *Every locally closed set is $g\omega\alpha$-locally closed set but not conversely.*

Proof. From [4] every closed set is $g\omega\alpha$-closed. Hence the proof follows.

Example 3.1. *Let $X = \{a,b,c\}$ and $\tau = \{X, \phi, \{a,b\}\}$. Then $g\omega\alpha LC(X) = P(X)$ and $LC(X) = \{X, \phi, \{c\}, \{a,b\}\}$. Then the set $A = \{a,c\}$ is $g\omega\alpha$-locally closed set but not locally closed set in X.*

Remark 3.1. *If A is α-locally closed set in X, then A is $g\omega\alpha$-locally closed in X but not conversely.*

Example 3.2. *In example 3.1 $\alpha LC(X) = \{X, \phi, \{c\}, \{a,b\}\}$. Then the set $A = \{a,c\}$ is $g\omega\alpha lc$-closed but not αlc-closed in X.*

Theorem 3.4. *For a subset A of X the followings are equivalent.*
1. A is $g\omega\alpha$-locally closed.
2. $A = U \cup c_\tau(A)$, for some $g\omega\alpha$-open set U.

Proof. $(i) \Rightarrow (ii)$ Let $A \in g\omega\alpha lc(X)$ then there exists a $g\omega\alpha$-open set U and a $g\omega\alpha$-closed set V such that $A = U \cap V$. Since $A \subset U$ and $A \subset c_\tau(A)$, we have $A \subset U \cap c_\tau(A)$.
Conversely. Since $c_\tau(A) \subset V, U \cap c_\tau(A) \subset U \cap V = A$, which implies that $A = U \cap c_\tau(A)$.
$(ii) \Rightarrow (ii)$ Since U is $g\omega\alpha$-open and $c_\tau(A)$ is $g\omega\alpha$-closed, $U \cap c_\tau(A) \in g\omega\alpha lc(X)$

Theorem 3.5. *Suppose a space X has the $g\omega\alpha$-closure preserving property. Then the followings are equivalent.*
1. $A \in G\omega\alpha LC(X)$.
2. $A = U \cap g\omega\alpha - cl(A)$ for some $g\omega\alpha$-open set U.

Proof. $(i) \Rightarrow (ii)$ Let $A \in G\omega\alpha LC(X)$. Then there exist a $g\omega\alpha$-open subset U and a $g\omega\alpha$-closed subset F such that $A = U \cap F$. Since $A \subseteq U$ and since $A \subseteq g\omega\alpha - cl(A)$. By definition 3.2, $g\omega\alpha - cl(A)$ is $g\omega\alpha$-closed, $g\omega\alpha - cl(A) \subseteq F$ and hence $U \cap g\omega\alpha - cl(A) \subseteq U \cap F = A$. Therefore, $A = U \cap g\omega\alpha - cl(A)$. Thus proves (ii).
$(ii) \Rightarrow (i)$ By definition 3.2, $g\omega\alpha - cl(A)$ is $g\omega\alpha$-closed. Therefore. $A = U \cap g\omega\alpha - cl(A) \in G\omega\alpha LC(X)$.

Definition 3.4. *Let A be a subset of X. Then A is called $g\omega\alpha^*$-locally closed if there exist $g\omega\alpha$-open set U and an α-closed set F of X such that $A = U \cap F$.*

The collection of all $g\omega\alpha^*$-locally closed sets of X will be denoted by $G\omega\alpha LC^*(X)$

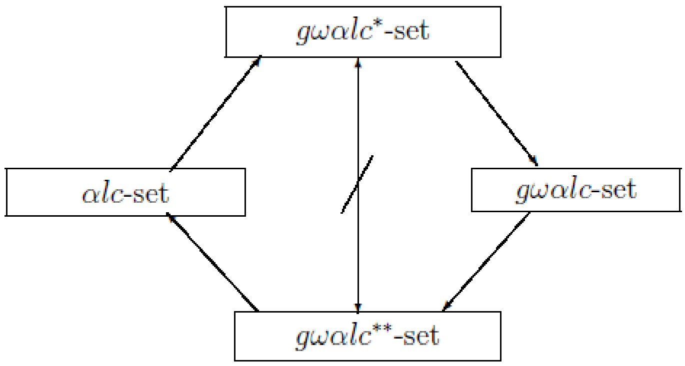
Definition 3.5. *Let A be a subset of X. Then A is called $g\omega\alpha lc^{**}$-set if there exist an α-open set U and a $g\omega\alpha$-closed set F of X such that $A = U \cap F$*

The collection of all $g\omega\alpha lc^{**}(X)$ will be denoted by $G\omega\alpha LC^{**}(X)$.

Proposition 3.1. *For a topological space X the following inclusions hold:*
1. $\alpha LC(X) \subseteq G\omega\alpha LC(X)$.
2. $\alpha LC(X) \subseteq G\omega\alpha LC^{**}(X) \subseteq G\omega\alpha LC(X)$.
3. $\alpha LC(X) \subseteq G\omega\alpha LC^*(X) \subseteq G\omega\alpha LC(X)$.

Proof. It follows from the fact that every α-closed set is $g\omega\alpha$-closed in X.

Remark 3.2. *We have the following diagram.*



The reverse implications are not true shown in the following example.

Example 3.3. *Let $X = \{a,b,c\}$ and $\tau = \{X, \phi, \{a,b\}\}$. Then we have:*

$\alpha LC(X) = \{X, \phi, \{c\}, \{a,b\}\}$

$G\omega\alpha LC(X) = P(X)$

$G\omega\alpha LC^*(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a,b\}\}$.

$G\omega\alpha LC^{**}(X) = \{X, \phi, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}\}$

(1) The reverse implication need not be true as seen from the above sets.

(2) It can be also seen that $G\omega\alpha LC^*(X)$ and $G\omega\alpha LC^{**}(X)$ are independent.

Definition 3.6. *A topological space X is said to be a $g\omega\alpha$-door space if each subset of X is either $g\omega\alpha$-open or $g\omega\alpha$-closed in X.*

Example 3.4. *Let $X = \{a,b,c\}$ and $\tau = \{X, \phi, \{a\}, \{b,c\}\}$. Then X is $g\omega\alpha$-door space*

Remark 3.3. *If X is $g\omega\alpha$-door space, then $G\omega\alpha LC(X) = P(X)$.*

Proposition 3.2. *Let X be a $T_{g\omega\alpha}$-space. Then $\alpha LC(X) = G\omega\alpha LC(X)$.*

Proof. Since X is $T_{g\omega\alpha}$-space, every $g\omega\alpha$-open set is α-open and every $g\omega\alpha$-closed set is α-closed. Hence we have $G\omega\alpha LC(X) \subseteq \alpha LC(X)$. By proposition 3.1(i), $\alpha LC(X) \subseteq G\omega\alpha LC(X)$. Hence $G\omega\alpha LC(X) = \alpha LC(X)$.

Proposition 3.3. *For a subset A of a space X the following statements are equivalent:*
1. $A \in G\omega\alpha LC(X)$.
2. $A = U \cap g\omega\alpha - cl(A)$ for some $g\omega\alpha$-open set U in X.

Proof. $(i) \Rightarrow (ii)$ Let $A \in G\omega\alpha LC(X)$. Then there exist a $g\omega\alpha$-open set U and a $g\omega\alpha$-closed set F of X such that $A = U \cap F$. Since $A \subseteq U$ and $A \subseteq g\omega\alpha - cl(A)$, $A \subseteq U \cap g\omega\alpha - cl(A)$. Conversely. By definition of $g\omega\alpha$-closure, $g\omega\alpha - cl(A) \subseteq F$ and hence $U \cap g\omega\alpha - cl(A) \subseteq U \cap F = A$. Therefore $A = U \cap g\omega\alpha - cl(A)$.

$(ii) \Rightarrow (i)$ Assume $A = U \cap g\omega\alpha - cl(A)$ for some $g\omega\alpha$-open set U. Since $g\omega\alpha - cl(A)$ is $g\omega\alpha$-closed, $A = U \cap g\omega\alpha - cl(A) \in G\omega\alpha LC(X)$.

Theorem 3.6. *Let X be a $T_{g\omega\alpha}$-space. For a subset A of X the following statements are equivalent.*

1. $A \in G\omega\alpha LC(X)$.
2. $A = U \cap g\omega\alpha - cl(A)$ for some $g\omega\alpha$-open set U in X.
3. $(g\omega\alpha - cl(A)) - A$ is $g\omega\alpha$-closed.
4. $A \cup (X - g\omega\alpha - cl(A))$ is $g\omega\alpha$-open.

Proof. $(i) \Leftrightarrow (ii)$ Follows from the proposition 3.2.

$(ii) \Rightarrow (iii)$ Let $A = U \cap g\omega\alpha - cl(A)$ for some $g\omega\alpha$-open set U in X.

Now, $g\omega\alpha - cl(A) - A = g\omega\alpha - cl(A) \cap A^c$

$= g\omega\alpha - cl(A) \cap (U \cap g\omega\alpha - cl(A))^c$

$= g\omega\alpha - cl(A) \cap [U^c \cup (g\omega\alpha - cl(A))^c]$

$= [g\omega\alpha - cl(A) \cap U^c] \cup [g\omega\alpha - cl(A) \cap (g\omega\alpha - cl(A))^c]$

$= (g\omega\alpha - cl(A) \cap U^c)$.

Here U^c is $g\omega\alpha$-closed. Since X is $T_{g\omega\alpha}$-space, U^c is α-closed. From [4], the intersection of a $g\omega\alpha$-closed set and an α-closed set is $g\omega\alpha$-closed, $g\omega\alpha - cl(A) - A$ is $g\omega\alpha$-closed.

$(iii) \Rightarrow (ii)$ Let $U = X - (g\omega\alpha - cl(A) - A)$. By (iii), U is a $g\omega\alpha$-open set in X and $A = U \cap g\omega\alpha - cl(A)$ holds.

$(iii) \Rightarrow (iv)$ Let $F = g\omega\alpha - cl(A) - A$. Then $X - F = A \cup (X - g\omega\alpha - cl(A))$ holds. Also, $X - F$ is $g\omega\alpha$-open, since F is $g\omega\alpha$-closed by (iii). Hence $A \cup (X - g\omega\alpha - cl(A))$ is $g\omega\alpha$-open.

$(iv) \Rightarrow (iii)$ Let $U = A \cup (X - g\omega\alpha - cl(A))$. Then $X - U = g\omega\alpha - cl(A)$, which is $g\omega\alpha$-closed, Since U is $g\omega\alpha$-open by (iv). Hence $g\omega\alpha - cl(A) - A$ is $g\omega\alpha$-closed.

Theorem 3.7. *For a subset A of X the following statements are equivalent:*

1. $A \in G\omega\alpha LC^*(X)$.
2. $A = U \cap \alpha cl(A)$ for some $g\omega\alpha$-open set U in X.
3. $\alpha cl(A) - A$ is $g\omega\alpha$-closed.
4. $A \cup (X - \alpha cl(A))$ is $g\omega\alpha$-open.

Proof. Using the fact every α-closed set is $g\omega\alpha$-closed and from theorem 3.6, the proof follows.

Theorem 3.8. *For a subset A of X, $A \in G\omega\alpha LC^{**}(X)$, if and only if $A = U \cap g\omega\alpha - cl(A)$ for some α-open set U in X.*

Proof. Necessity. Let $A \in G\omega\alpha LC^{**}(X)$. Then by definition $A = U \cap F$ where U is an α-open set and F is a $g\omega\alpha$-closed set containing A. Since F is a $g\omega\alpha$-closed set, we have $g\omega\alpha - cl(A) \subseteq F$, which implies that $U \cap g\omega\alpha - cl(A) \subseteq U \cap F = A$. Since $A \subseteq U$ and $A \subseteq g\omega\alpha - cl(A)$ we have $A \subseteq U \cap g\omega\alpha - cl(A)$. Therefore $A = U \cap g\omega\alpha - cl(A)$, where U is an α-open.

Sufficient. Assume that $A = U \cap g\omega\alpha - cl(A)$ for some α-open set U in X. Since $g\omega\alpha - cl(A)$ is $g\omega\alpha$-closed, we have $A \in G\omega\alpha LC^{**}(X)$.

Definition 3.7. *A subset A of X is called $g\omega\alpha$ -dense if $g\omega\alpha - cl(A) = X$.*

Example 3.5. *Let $X = \{a,b,c\}$ and $\tau = \{X, \phi, \{a\}\}$. The only $g\omega\alpha$ -closed set containing $\{a\}$ is X and hence $g\omega\alpha - cl\{a\} = X$. Then the subset $\{a\}$ is $g\omega\alpha$ -dense in X.*

Proposition 3.4. *In a topological space X, every $g\omega\alpha$ -dense set is α -dense but not conversely.*

Proof. Let A be $g\omega\alpha$ -dense set in X. Then $g\omega\alpha - cl(A) = X$. Since $g\omega\alpha - cl(A) \subseteq \alpha cl(A)$, we have $\alpha cl(A) = X$. Hence A is α -dense.

Example 3.6. *Let $X = \{a,b,c\}$ and $\tau = \{X, \phi, \{a\}, \{b,c\}\}$. Then the subset $\{a,b\}$ is α -dense in X but not $g\omega\alpha$ -dense in X, as $g\omega\alpha - cl\{a,b\} = \{a,b\} \neq X$*

Definition 3.8. *A topological space X is called $g\omega\alpha$ -submaximal (resp. $g\omega\alpha^*$ -submaximal) if every $g\omega\alpha$ -dense (resp. α -dense) subset is $g\omega\alpha$ -open in X.*

Proposition 3.5. *Every $g\omega\alpha^*$ -submaximal space is $g\omega\alpha$ -submaximal.*

Proof. Let X be a $g\omega\alpha^*$ -submaximal space and A be a $g\omega\alpha$ -dense subset of X. By proposition 3.4, A is α -dense in X. By assumption, A is $g\omega\alpha$ -open and hence X is $g\omega\alpha$ -submaximal.

Theorem 3.9. *Let X be a $T_{g\omega\alpha}$-space. Then X is $g\omega\alpha$ -submaximal if and only if $P(X) = G\omega\alpha LC(X)$.*

Proof. Let X be $g\omega\alpha$ -submaximal and $A \in P(X)$. Consider $V = A \cup (X - g\omega\alpha - cl(A) = X - (g\omega\alpha - cl(A) - A)$. Then $g\omega\alpha - cl(V) = X$. That is V is $g\omega\alpha$ -dense in X. By assumption, V is $g\omega\alpha$ -open. Then by theorem 3.6, $A \in G\omega\alpha LC(X)$ and hence $P(X) = G\omega\alpha LC(X)$.

Conversely. Let A be a $g\omega\alpha$ -dense set in X and $P(X) = G\omega\alpha LC(X)$. Since A is $g\omega\alpha$ -dense, $g\omega\alpha - cl(A) = X$. Then $A = A \cup \phi = A \cup (X - g\omega\alpha - cl(A))$. Since $A \in G\omega\alpha LC(X)$, $A = A \cup (X - g\omega\alpha - cl(A))$ is $g\omega\alpha$ -open, by theorem 3.6. Hence X is $g\omega\alpha$ -submaximal.

Theorem 3.10. *Let A and B be any two subsets of X and let $A \subseteq B$. Suppose that the collection of all $g\omega\alpha$ -open subsets of X is closed under finite intersection. If B is $g\omega\alpha$ -open and $A \in G\omega\alpha LC^*(B, \tau \setminus B)$, then $A \in G\omega\alpha LC^*(X)$.*

Proof. If $A \in G\omega\alpha LC^*(B, \tau/B)$, then there exist $g\omega\alpha$ -open set G in $(B, \tau/B)$ such that $A = G \cap cl_B(A)$ where $cl_B(A) = B \cap cl(A)$. Since G and B are $g\omega\alpha$ -open then $G \cap B$ is $g\omega\alpha$ -open. This implies that $A = G \cap (B \cap cl(A)) = (G \cap B) \cap cl(A)$ is $g\omega\alpha lc^*$ set in X. So $A \in G\omega\alpha LC^*(X)$.

Theorem 3.11. *If the collection of all $g\omega\alpha$ -closed subset of X is closed under finite intersection if B is $g\omega\alpha$ -closed, open in X and $A \in G\omega\alpha LC^*(B, \tau/B)$ then $A \in G\omega\alpha LC(X)$.*

Proof. Let $A \in G\omega\alpha LC^*(B, \tau/B)$. Then there exist $g\omega\alpha$ -open set G in $(B, \tau/B)$ and closed set F in $(B, \tau/B)$ such that $A = G \cap F$. Since F is closed in $(B, \tau/B)$, $F = V \cap B$ for some closed set V of X. So V and B are $g\omega\alpha$ -closed sets in X. Therefore F is the intersection of $g\omega\alpha$ -closed sets V and B. So F is also $g\omega\alpha$ -closed set in X. Therefore $A = G \cap (V \cap B) \in G\omega\alpha LC(X)$.

4. $g\omega\alpha$ -Locally Continuous Functions

In this section, $g\omega\alpha LC$-continuous functions, $g\omega\alpha LC^{*}$-continuous functions and $g\omega\alpha LC^{**}$-continuous functions are defined and their properties are obtained. Also $g\omega\alpha LC$-irresolute functions, $g\omega\alpha^{*}$-irresolute functions and $g\omega\alpha LC^{**}$-irresolute functions are defined and their properties are discussed.

Definition 4.1. *Let $f: X \to Y$ be a function. Then f is called,*

1. $G\omega\alpha LC$-continuous if $f^{-1}(V) \in G\omega\alpha LC(X)$ for each open set V of Y.
2. $G\omega\alpha LC^{*}$-continuous if $f^{-1}(V) \in G\omega\alpha LC^{*}(X)$ for each open set V of Y.
3. $G\omega\alpha LC^{**}$-continuous if $f^{-1}(V) \in G\omega\alpha LC^{**}(X)$ for each open set V of Y.

Theorem 4.1. *Let $f: X \to Y$ be a function. Then the following statements are true:*

1. If f is αLC-continuous then it is $G\omega\alpha LC$-continuous, $G\omega\alpha LC^{*}$-continuous and $G\omega\alpha LC^{**}$-continuous.
2. If f is $G\omega\alpha LC^{*}$-continuous or $G\omega\alpha LC^{**}$-continuous then it is $G\omega\alpha LC$-continuous.

Proof. 1. Follows from the fact that every αlc-set is $g\omega\alpha lc$-set, $g\omega\alpha lc^{*}$-set and $g\omega\alpha lc^{**}$-set by proposition 3.1.
2. Since every $g\omega\alpha lc^{*}$-set is $g\omega\alpha lc$ set and every $g\omega\alpha lc^{**}$-set is $g\omega\alpha lc$-set, the proof follows.
The converse of the above theorem need not be true as seen from the following example.

Example 4.1. *Let $X = Y = \{a,b,c\}$ and $\tau = \{X,\phi,\{a\},\{b,c\}\}$ and $\mu = \{X,\phi,\{a\},\{a,c\}\}$. Let $f: X \to Y$ be an identity function then f is $G\omega\alpha LC$-continuous, $G\omega\alpha LC^{*}$-continuous and $G\omega\alpha LC^{**}$-continuous but not αLC-continuous. Since for the open set $\{a,c\}$ in Y, $f^{-1}\{a,c\} = \{a,c\} \notin \alpha LC(X)$.*

Definition 4.2. *Let $f: X \to Y$ be a function. Then f is called,*

1. $G\omega\alpha LC$-irresolute if $f^{-1}(V) \in G\omega\alpha LC(X)$ for each $V \in G\omega\alpha LC(Y)$.
2. $G\omega\alpha LC^{*}$-irresolute if $f^{-1}(V) \in G\omega\alpha LC^{*}(X)$ for each $V \in G\omega\alpha LC^{*}(Y)$.
3. $G\omega\alpha LC^{**}$-irresolute if $f^{-1}(V) \in G\omega\alpha LC^{**}(X)$ for each $V \in G\omega\alpha LC^{**}(Y)$.

Proposition 4.1. *Let $f: X \to Y$ be a $g\omega\alpha$-irresolute map. If $B \in G\omega\alpha LC(Y)$, then $f^{-1}(B) \in G\omega\alpha LC(X)$.*

Proof. Let $f: X \to Y$ be a $g\omega\alpha$-irresolute function. Let $B \in G\omega\alpha LC(Y)$. Then there exist a $g\omega\alpha$-open set G and a $g\omega\alpha$-closed set H such that $B = G \cap H$ which implies that $f^{-1}(B) = f^{-1}(G) \cap f^{-1}(H)$. Since f is a $g\omega\alpha$-irresolute, $f^{-1}(G)$ and $f^{-1}(H)$ are $g\omega\alpha$-open and $g\omega\alpha$-closed in X respectively. Hence $f^{-1}(B) \in G\omega\alpha LC(X)$.

Theorem 4.2. *Let $f: X \to Y$ be a $g\omega\alpha$-irresolute. Then f is $G\omega\alpha LC$-irresolute.*

Proof. Follows from the above proposition 4.1.
The converse of the above proposition need not be true as seen from the following example.

Example 4.2. *Let $X = Y = \{a,b,c\}$ and $\tau = \{X,\phi,\{b\},\{a,b\},\{b,c\}\}$ and $\mu = \{X,\phi,\{a\},\{a,c\}\}$. Let $f: X \to Y$ be an identity function then f is $G\omega\alpha LC$-irresolute, but not $g\omega\alpha$-irresolute. Since for the $g\omega\alpha$-closed set $\{b,c\}$ in Y, $f^{-1}\{b,c\} = \{b,c\}$ is not $g\omega\alpha$-closed in X.*

Theorem 4.3. *If* $f: X \to Y$ *is* $G\omega\alpha LC$*-continuous (*$G\omega\alpha LC^*$*-continuous or* $G\omega\alpha LC^{**}$*-continuous) and* X *is a* $T_{g\omega\alpha}$ *then* f *is* αLC*-continuous.*

Proof. Let $f: X \to Y$ be $G\omega\alpha LC$-continuous and V be an open set in Y. Since f is $G\omega\alpha LC$ ($G\omega\alpha LC^*$, $G\omega\alpha LC^{**}$) continuous, $f^{-1}(V)$ is $g\omega\alpha lc$-set ($g\omega\alpha lc^*$-set, $g\omega\alpha lc^{**}$-set) in X. Since X is $T_{g\omega\alpha}$-space, $f^{-1}(V)$ is αLC set in X. By proposition 3.2. Hence f is αLC-continuous.

Theorem 4.4. *Any function defined on an* α*-door space is* $G\omega\alpha LC$*-irresolute.*

Proof. Let $f: X \to Y$ be a function where X is an α-door space and Y is any space. Let $A \in G\omega\alpha LC(Y)$. Then by assumption on X, $f^{-1}(A)$ is either α-open or α-closed. Since every α-closed set is $g\omega\alpha$-closed, $f^{-1}(V) \in G\omega\alpha LC(X)$. Therefore f is $G\omega\alpha LC$-irresolute.

Theorem 4.5. *Let* X *be a* $T_{g\omega\alpha}$*-space. If* X *is* $g\omega\alpha$*-submaximal then every function having* X *as its domain is* $G\omega\alpha LC$*-irresolute.*

Proof. Let X be $T_{g\omega\alpha}$-space and $g\omega\alpha$-submaximal. Let $f: X \to Y$ be any map. Then by theorem 3.9 $P(X) = G\omega\alpha LC(X)$. If U is $g\omega\alpha lc$-set of Y, then $f^{-1}(V) \in P(X) = G\omega\alpha LC(X)$ and hence f is $G\omega\alpha LC$-irresolute.

Theorem 4.6. *Let* $f: X \to Y$ *and* $g: Y \to Z$ *be any two maps. Then,*

1. $g \circ f: X \to Z$ is $G\omega\alpha LC$-irresolute (resp. $G\omega\alpha LC^*$-irresolute, $G\omega\alpha LC^{**}$-irresolute) if f is $G\omega\alpha LC$-irresolute (resp $G\omega\alpha LC^*$-irresolute, $G\omega\alpha LC^{**}$-irresolute) and g is $G\omega\alpha LC$-irresolute (resp $G\omega\alpha LC^*$-irresolute, $G\omega\alpha LC^{**}$-irresolute).
2. $g \circ f: X \to Z$ is $G\omega\alpha LC$-continuous if f is $G\omega\alpha LC$-irresolute and g is $G\omega\alpha LC$-continuous.

Proof. (i) Let $V \in G\omega\alpha LC(Z)$ (resp. $V \in G\omega\alpha LC^*(Z)$, $V \in G\omega\alpha LC^{**}(Z)$). Since g is $G\omega\alpha LC$-irresolute (resp. $G\omega\alpha LC^*$-irresolute, $G\omega\alpha LC^{**} - irresolute$), $g^{-1}(V) \in G\omega\alpha LC(Y)$ (resp. $g^{-1}(V) \in G\omega\alpha LC^*(Y)$, $g^{-1}(V) \in G\omega\alpha LC^{**}(Y)$. Since f is $G\omega\alpha LC$-irresolute (resp. $G\omega\alpha LC^*$-irresolute, $G\omega\alpha LC^{**}$-irresolute), $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1} \in G\omega\alpha LC(X)$ (resp $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1} \in G\omega\alpha LC^*(X)$, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1} \in G\omega\alpha LC^{**}(X)$). Therefore $g \circ f$ is $G\omega\alpha LC$-irresolute (resp. $G\omega\alpha LC^*$-irresolute, $G\omega\alpha LC^{**}$-irresolute).
(ii) Let V be any open set in Z. Since g is $G\omega\alpha LC$-continuous, $g^{-1}(V) \in G\omega\alpha LC(Y)$. Since f is $G\omega\alpha LC$-irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1} \in G\omega\alpha LC(X)$. Therefore $g \circ f$ is $G\omega\alpha LC$-continuous.

Acknowledgement

The authors are grateful to the University Grants Commission, New Delhi, India for its financial support under UGC-SAP-III DRS to the Department of Mathematics, Karnatak University, Dharwad, India. Also this research was supported by the University Grants Commission, New Delhi, India. under No.F.4-1/2006(BSR)/7-101/2007(BSR) dated: 20th June, 2012.

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