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Fuzzy Bicontinuous Maps in Fuzzy Biclosure Spaces

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Abstract. The purpose of this paper is to introduce the notion of fuzzy bicontinuous maps and fuzzy biclosed (fuzzy biopen) maps in fuzzy biclosure spaces and study some of their properties.

1. Introduction

The concept of fuzzy set was first introduced by Zadeh [1] in his classical paper in 1965. Zadeh's introduction of the notion of a fuzzy set in a universe has inspired many mathematicians to generalize the main concepts and structures of present day mathematics into the framework of fuzzy sets. The theory of general topology is based on the set operation of union, intersection and complementation. Fuzzy sets do have the same kind of operations. Inspired by these observations Chang [2] extended the concepts of point set topology to fuzzy sets and laid the foundation of the fuzzy topology.

In recent years, fuzzy topology has been found to be very useful in solving many practical problems. Du.et. al. [3] fuzzified the very successful 9-intersection Egenhofer model [4] for depicting topological relations in Geographic Information Systems (GIS) query. Today fuzzy topology has firmly established as one of the basic discipline of fuzzy mathematics, and have a fundamental role to play in pure and applied sciences.

Closure spaces which is a generalization of topological spaces were introduced by E. Čech [5]. Today, the theory of closure space is one of the most popular theory of mathematics which finds many interesting applications in the areas of fuzzy sets, combinatorics, genetics or quantum mechanics. In 1985 fuzzy closure spaces were first studied by A.S. Mashhour and M.H. Ghanim [6].

Boonpok [7] studied the notion of biclosure spaces. Such spaces are equipped with two arbitrary closure operators. He extended some of the standard results of separation axioms from closure spaces to biclosure spaces. Thereafter a large number of papers have been written to generalize the concept of closure space to biclosure space.

Recently, Tapi and Navalakhe [8] have introduced the notion of fuzzy biclosure spaces and generalized the concept of topological spaces to fuzzy closure space and fuzzy biclosure space.

In this paper we have fuzzified the paper of Boonpok [9] in which he had studied the concept of Bicontinuous maps in biclosure spaces. Tapi and Navalakhe [10] had already worked on the concept of Pairwise Fuzzy Bicontinuous maps in Fuzzy biclosure spaces.

The focus of this paper is to introduce and study the concept of fuzzy bicontinuous maps and fuzzy biclosed (fuzzy biopen) maps in fuzzy biclosure spaces. We have also investigated some of the important characterizations and properties of fuzzy bicontinuous maps and fuzzy biclosed (fuzzy biopen) maps in fuzzy biclosure spaces

2. Preliminaries

In order to make this paper self contained, we briefly recall certain definitions and results. Let X be an arbitrary set, $I=[0,1]$ and I^X be a family of all fuzzy sets of X. For a fuzzy set μ of X, cl(μ), int(μ) and $1-\mu$ will denote the closure of μ, the interior of μ and the complement of μ respectively whereas the constant fuzzy sets taking on the values 0 and 1 on X are denoted by 0_X and 1_X respectively.

Definition 2.1 [8]. A *fuzzy biclosure space* is a triple (X,u_1,u_2) where X is a non empty set and u_1,u_2 are two fuzzy closure operators on X which satisfy the following properties:

(i) $u_1(0_X)=0_X$ and $u_2(0_X)=0_X$

(ii) $\mu\le u_1\mu$ and $\mu\le u_2\mu$ for all $\mu\le I^X$

(iii) $u_1(\mu\vee\upsilon)=u_1\mu\vee u_1\upsilon$ and $u_2(\mu\vee\upsilon)=u_2\mu\vee u_2\upsilon$ for all $\mu,\upsilon\le I^X$.

Definition 2.2 [8]. A subset μ of a fuzzy biclosure space (X,u_1,u_2) is called fuzzy closed if $u_1u_2\mu=\mu$. The complement of fuzzy closed set is called fuzzy open.

Definition 2.3 [8]. Let (X,u_1,u_2) be a fuzzy biclosure space. A fuzzy biclosure space (Y,v_1,v_2) is called a subspace of (X,u_1,u_2) if $Y\le X$ and $v_i\mu=u_i\mu\wedge 1_Y$ for each $i\in\{1,2\}$ and each subset $\mu\le Y$. If 1_Y is fuzzy closed in (X,u_1) and (X,u_2) then the fuzzy subspace (Y,v_1,v_2) is also said to be fuzzy closed.

Definition 2.4 [9]. Let (X,u_1,u_2) and (Y,v_1,v_2) be fuzzy biclosure spaces. A map $f:(X,u_1,u_2)\to(Y,v_1,v_2)$ is called fuzzy continuous if $f^{-1}(\mu)$ is a fuzzy closed subset of (X,u_1,u_2) for every fuzzy closed subset μ of (Y,v_1,v_2).

Clearly, it is easy to prove that a map $f:(X,u_1,u_2)\to(Y,v_1,v_2)$ is fuzzy continuous if and only if $f^{-1}(\upsilon)$ is a fuzzy open subset of (X,u_1,u_2) for every fuzzy open subset υ of (Y,v_1,v_2).

Definition 2.5 [9]. Let (X,u_1,u_2) and (Y,v_1,v_2) be fuzzy biclosure spaces. A map $f:(X,u_1,u_2)\to(Y,v_1,v_2)$ is said to be fuzzy closed (resp. fuzzy open) if $f(\mu)$ is fuzzy closed (resp. fuzzy open) subset of (Y,v_1,v_2) whenever μ is a fuzzy closed (resp. fuzzy open) subset of (X,u_1,u_2).

Definition 2.6 [9]. The product of a family $\{(X_\alpha,u_\alpha^1,u_\alpha^2):\alpha\in J\}$ of fuzzy biclosure spaces denoted by $\prod_{\alpha\in J}(X_\alpha,u_\alpha^1,u_\alpha^2)$ is the fuzzy biclosure space $\left(\prod_{\alpha\in J}X_\alpha,u^1,u^2\right)$ where $\left(\prod_{\alpha\in J}X_\alpha,u^i\right)$ for $i\in\{1,2\}$ is the product of the family of fuzzy closure spaces $\{X_\alpha,u^i:\alpha\in J\}$.

Remark 2.7. Let $\prod_{\alpha\in J}(X_\alpha,u_\alpha^1,u_\alpha^2)=\left(\prod_{\alpha\in J}X_\alpha,u^1,u^2\right)$. Then for each $\mu\le\prod_{\alpha\in J}X_\alpha$, $u^1u^2\mu=\prod_{\alpha\in J}u_\alpha^1u_\alpha^2\pi_\alpha(\mu)$.

Proposition 2.8. Let $\left\{\left(X_{\alpha}, u_{\alpha}^{1}, u_{\alpha}^{2}\right): \alpha \in J\right\}$ be a family of fuzzy biclosure spaces. Then for each $\beta \in J$, the projection map $\pi_{\beta}: \prod_{\alpha \in J}\left(X_{\alpha}, u_{\alpha}^{1}, u_{\alpha}^{2}\right) \rightarrow\left(X_{\beta}, u_{\beta}^{1}, u_{\beta}^{2}\right)$ is fuzzy continuous.

Proof. Let $\mu \leq \prod_{\alpha \in J} X_{\alpha}$. Then $\pi_{\beta}\left(\prod_{\alpha \in J} u_{\alpha}^{1} \pi_{\alpha}(\mu)\right)=u_{\beta}^{1} \pi_{\alpha}(\mu)$. Hence, $\pi_{\beta}: \prod_{\alpha \in J}\left(X_{\alpha}, u_{\alpha}^{1}\right) \rightarrow \prod_{\alpha \in J}\left(X_{\beta}, u_{\beta}^{1}\right)$ is fuzzy continuous. Similarly, since $\pi_{\beta}\left(\prod_{\alpha \in J} u_{\alpha}^{2} \pi_{\alpha}(\mu)\right)=u_{\beta}^{2} \pi_{\beta}(\mu)$.

Therefore, $\pi_{\beta}: \prod_{\alpha \in J}\left(X_{\alpha}, u_{\alpha}^{2}\right) \rightarrow \prod_{\alpha \in J}\left(X_{\beta}, u_{\beta}^{2}\right)$ is fuzzy continuous. Consequently, $\pi_{\beta}: \prod_{\alpha \in J}\left(X_{\alpha}, u_{\alpha}^{1}, u_{\alpha}^{2}\right) \rightarrow\left(X_{\beta}, u_{\beta}^{1}, u_{\beta}^{2}\right)$ is fuzzy continuous.

Proposition 2.9 [8]. Let $\left\{\left(X_{\alpha}, u_{\alpha}^{1}, u_{\alpha}^{2}\right): \alpha \in J\right\}$ be a family of fuzzy biclosure spaces and let $\beta \in J$. Then $\eta \leq X_{\beta}$ is a fuzzy closed subset of $\left(X_{\beta}, u_{\beta}^{1}, u_{\beta}^{2}\right)$ if and only if $\eta \times \prod_{\substack{\alpha \neq \beta \\ \alpha \in J}} X_{\alpha}$ is a fuzzy closed subset of $\prod_{\alpha \in J}\left(X_{\alpha}, u_{\alpha}^{1}, u_{\alpha}^{2}\right)$.

3. Fuzzy bicontinuous maps

In this section, we introduce the concept of fuzzy bicontinuous maps in fuzzy biclosure spaces and investigate some of the properties of fuzzy bicontinuous maps in fuzzy biclosure spaces.

Definition 3.1. Let $\left(X, u_{1}, u_{2}\right)$ and $\left(Y, v_{1}, v_{2}\right)$ be fuzzy biclosure spaces and let $i \in\{1,2\}$. A map $f:\left(X, u_{1}, u_{2}\right) \rightarrow\left(Y, v_{1}, v_{2}\right)$ is called $i-$fuzzy *continuous* if the map $f:\left(X, u_{i}\right) \rightarrow\left(Y, v_{i}\right)$ is fuzzy continuous. A map f is called fuzzy *continuous* if f is $i-$fuzzy *continuous* for each $i \in\{1,2\}$.

Definition 3.2. Let $\left(X, u_{1}, u_{2}\right)$ and $\left(Y, v_{1}, v_{2}\right)$ be fuzzy biclosure spaces. Then $f:\left(X, u_{1}, u_{2}\right) \rightarrow\left(Y, v_{1}, v_{2}\right)$ is called fuzzy bicontinuous if the map $f:\left(X, u_{1}\right) \rightarrow\left(Y, v_{2}\right)$ is fuzzy continuous.

Proposition 3.3. Let $\left(X, u_{1}, u_{2}\right)$ and $\left(Y, v_{1}, v_{2}\right)$ be fuzzy biclosure spaces. A map $f:\left(X, u_{1}, u_{2}\right) \rightarrow\left(Y, v_{1}, v_{2}\right)$ is fuzzy bicontinuous if and only if $u_{1} f^{-1}(\upsilon) \leq f^{-1}\left(v_{2} \upsilon\right)$ for every $\upsilon \leq Y$.

Proof. Let $\upsilon \leq Y$. Then $f^{-1}(\upsilon) \leq X$. Since f is fuzzy bicontinuous, $f\left(u_{1} f^{-1}(\upsilon)\right) \leq v_{2} f\left(f^{-1}(\upsilon)\right) \leq v_{2} \upsilon$. Therefore, $u_{1} f^{-1}(\upsilon) \leq f^{-1}\left(v_{2} \upsilon\right)$. Conversely, let $\mu \leq X$. Then $f(\mu))(\leq Y$. Thus $u_{1} f^{-1}(f(\mu)) \leq f^{-1}\left(v_{2} f(\mu)\right)$.

Consequently, $f\left(u_{1} \mu\right) \leq f\left(u_{1} f^{-1}(f(\mu))\right) \leq f\left(f^{-1}\left(v_{2} f(\mu)\right)\right) \leq v_{2} f(\mu)$. Hence, the map f is fuzzy bicontinuous .

Proposition 3.4. Let $(X,u_1,u_2),(Y,v_1,v_2)$ and (Z,w_1,w_2) be fuzzy biclosure spaces. If $f:(X,u_1,u_2)\to(Y,v_1,v_2)$ is fuzzy bicontinuous and $h:(Y,v_1,v_2)\to(Z,w_1,w_2)$ is 2-fuzzy continuous, then $h\circ f:X\to Z$ is fuzzy bicontinuous.

Proof. Let $\mu\le X$. Since $h\circ f(u_1\mu)=h(f(u_1\mu))$ and the map f is fuzzy bicontinuous, $h(f(u_1\mu))\le h(v_2f(\mu))$. Since the map h is 2-fuzzy continuous, $h(v_2f(\mu))\le w_2h(f(\mu))$. Thus $h\circ f(u_1\mu)\le w_2h\circ f(\mu)$. Consequently, the map $h\circ f$ is fuzzy bicontinuous.

Proposition 3.5. Let (X,u_1,u_2) be a fuzzy biclosure space, $\{(Y_\alpha,v_\alpha^1,v_\alpha^2):\alpha\in J\}$ be a family of fuzzy biclosure spaces and $f:(X,u_1,u_2)\to\prod_{\alpha\in J}(Y_\alpha,v_\alpha^1,v_\alpha^2)$ be a map. Then f is fuzzy bicontinuous if and only if $\pi_\alpha\circ f$ is fuzzy bicontinuous for each $\alpha\in J$.

Proof. Let f be fuzzy bicontinuous. Since π_α is fuzzy bicontinuous for each $\alpha\in J$, it follows that $\pi_\alpha\circ f$ is fuzzy bicontinuous for each $\alpha\in J$.

Conversely, let the map $\pi_\alpha\circ f$ be fuzzy bicontinuous for each $\alpha\in J$. Suppose that the map f is not fuzzy bicontinuous. Then there exists a fuzzy subset η of X such that $f(u_1\eta)\not\le\prod_{\alpha\in J}v_\alpha^2\pi_\alpha(f(\eta))$. Therefore, there exists $\beta\in J$ such that $\pi_\beta(f(u_1\eta))\not\le v_\beta^2\pi_\beta(f(\eta))$. This contradicts the fuzzy bicontinuity of the map $\pi_\beta\circ f$. Consequently, the map f is fuzzy bicontinuous.

Proposition 3.6. Let $\{(X_\alpha,u_\alpha^1,u_\alpha^2:\alpha\in J)\}$ and $\{(Y_\alpha,v_\alpha^1,v_\alpha^2):\alpha\in J\}$ be families of fuzzy biclosure spaces. For each $\alpha\in J$, let $f_\alpha:(X_\alpha,u_\alpha)\to(Y_\alpha,v_\alpha)$ be a map and let $f:\prod_{\alpha\in J}(X_\alpha,u_\alpha^1,u_\alpha^2)\to\prod_{\alpha\in J}(Y_\alpha,v_\alpha^1,v_\alpha^2)$ be defined by $f((x_\alpha))_{\alpha\in I}=(f_\alpha(x_\alpha))_{\alpha\in I}$. Then the map f is fuzzy bicontinuous if and only if f_α is fuzzy bicontinuous for each $\alpha\in J$.

Proof. Let the map f be fuzzy bicontinuous, let $\beta\in J$ and $\mu\le X_\beta$. Then

$$f_\beta(u_\beta^1\mu)=\pi_\beta\left(f_\beta(u_\beta^1\mu)\times\prod_{\substack{\alpha\ne\beta\\ \alpha\in J}}f_\alpha(u_\alpha^1X_\alpha)\right)=\pi_\beta\left(f\left(u_\beta^1\mu\times\prod_{\substack{\alpha\ne\beta\\ \alpha\in J}}u_\alpha^1X_\alpha)\right)\right)$$

$$=\pi_\beta\left(f\left(\prod_{\alpha\in J}u_\alpha^1\pi_\alpha\left(\mu\times\prod_{\substack{\alpha\ne\beta\\ \alpha\in J}}X_\alpha\right)\right)\right)\le\pi_\beta\left(\prod_{\alpha\in J}v_\alpha^2\pi_\alpha\left(f\left(\mu\times\prod_{\substack{\alpha\ne\beta\\ \alpha\in J}}X_\alpha)\right)\right)\right)$$

$$=\pi_\beta\left(\prod_{\alpha\in J}v_\alpha^2\pi_\alpha\left(f_\beta(\mu)\times\prod_{\substack{\alpha\ne\beta\\ \alpha\in J}}f_\alpha(X_\alpha)\right)\right)=\pi_\beta\left(v_\beta^2f_\beta(\mu)\times\prod_{\substack{\alpha\ne\beta\\ \alpha\in J}}v_\alpha^2f_\alpha(X_\alpha)\right)=v_\beta^2f_\beta(\mu)$$

Hence, the map f_β is fuzzy bicontinuous.

Conversely, let the map f_α be fuzzy bicontinuous for each $\alpha\in J$ and let $\mu\le\prod_{\alpha\in J}X_\alpha$.

Then $f\left(\prod_{\alpha\in J} u_\alpha^1 \pi_\alpha(\mu)\right) = \prod_{\alpha\in J} f_\alpha\left(\prod_{\alpha\in J} u_\alpha^1 \pi_\alpha(\mu)\right)$

$$= \prod_{\alpha\in J} f_\alpha\left(u_\alpha^1 \pi_\alpha(\mu)\right) \le \prod_{\alpha\in J} v_\alpha^2 f_\alpha\left(\pi_\alpha(\mu)\right)$$

$$= \prod_{\alpha\in J} v_\alpha^2 \pi_\alpha\left(f(\mu)\right)$$

Therefore, the map f is fuzzy bicontinuous.

4. Fuzzy biclosed maps

This section of the paper is aimed to introduce the notion of fuzzy biclosed (fuzzy biopen) maps in fuzzy biclosure spaces and to investigate some of the important characterization of fuzzy biclosed (fuzzy biopen) maps in fuzzy biclosure spaces.

Definition 4.1. Let (X,u_1,u_2) and (Y,v_1,v_2) be fuzzy biclosure spaces and let $i\in\{1,2\}$. A map $f:(X,u_1,u_2)\to(Y,v_1,v_2)$ is called $i-$fuzzy *closed* (resp. $i-$fuzzy *open*) if the map $f:(X,u_i)\to(Y,v_i)$ is fuzzy *closed* (resp. fuzzy *open*). A map f is called fuzzy *closed* (resp. fuzzy *open*) if f is $i-$fuzzy *closed* (resp. $i-$fuzzy *open*) for each $i\in\{1,2\}$.

Definition 4.2. Let (X,u_1,u_2) and (Y,v_1,v_2) be fuzzy biclosure spaces. A map $f:(X,u_1,u_2)\to(Y,v_1,v_2)$ is called fuzzy biclosed (resp. fuzzy biopen) if the map $f:(X,u_1)\to(Y,v_2)$ is fuzzy closed (resp. fuzzy open).

Proposition 4.3. Let (X,u_1,u_2), (Y,v_1,v_2) and (Z,w_1,w_2) be fuzzy biclosure spaces. If the map $f:(X,u_1,u_2)\to(Y,v_1,v_2)$ is 1-fuzzy closed and the map $h:(Y,v_1,v_2)\to(Z,w_1,w_2)$ is fuzzy biclosed, then the map $h\circ f:(X,u_1,u_2)\to(Z,w_1,w_2)$ is fuzzy biclosed.

Proof. Let η be a fuzzy closed subset of (X,u_1). Since the map f is 1-fuzzy closed, $f(\eta)$ is a fuzzy closed subset of (Y,v_1). Since the map h is fuzzy biclosed, $h(f(\eta))$ is a fuzzy closed subset of (Z,w_2). Hence, $h\circ f(\eta)$ is a fuzzy closed subset of (Z,w_2). Consequently, the map $h\circ f$ is fuzzy biclosed.

Proposition 4.4. Let (X,u_1,u_2), (Y,v_1,v_2) and (Z,w_1,w_2) be fuzzy biclosure spaces. Let $f:(X,u_1,u_2)\to(Y,v_1,v_2)$ and $h:(Y,v_1,v_2)\to(Z,w_1,w_2)$ be maps. Then

(i) If $h\circ f$ is fuzzy biclosed and f is surjective and 1- fuzzy continuous then h is fuzzy biclosed.

(ii) If $h\circ f$ is fuzzy biclosed and h is injective and 2- fuzzy continuous, then f is fuzzy biclosed.

Proof. (i) Let η be a fuzzy closed subset of (Y,v_1). Since the map f is 1-fuzzy continuous, $f^{-1}(\eta)$ is a fuzzy closed subset of (X,u_1). Since $h\circ f$ is fuzzy biclosed and the map f is surjective, $h\circ f\left(f^{-1}(\eta)\right)=h(\eta)$ is a fuzzy closed subset of (Z,w_2). Hence, the map h is fuzzy biclosed.

(ii) Let η be a fuzzy closed subset of (X,u_1). Since the map $h\circ f$ is fuzzy biclosed, $h\circ f(\eta)$ is a fuzzy closed subset of (Z,w_2). Since h is 2-fuzzy continuous and injective, $h^{-1}(h\circ f(\eta))=f(\eta)$ is a fuzzy closed subset of (Y,v_2). Therefore, the map f is fuzzy biclosed.

The following statement is evident.

Proposition 4.5. Let $\{(X_\alpha,u_\alpha^1,u_\alpha^2:\alpha\in J)\}$ and $\{(Y_\alpha,v_\alpha^1,v_\alpha^2):\alpha\in J\}$ be families of fuzzy biclosure spaces. For each $\alpha\in J$, let $f_\alpha:(X_\alpha,u_\alpha^1,u_\alpha^2)\to(Y_\alpha,v_\alpha^1,v_\alpha^2)$ be a surjection and let $f:\prod_{\alpha\in J}(X_\alpha,u_\alpha^1,u_\alpha^2)\to\prod_{\alpha\in J}(Y_\alpha,v_\alpha^1,v_\alpha^2)$ be defined by $f((x_\alpha))_{\alpha\in I}=(f_\alpha(x_\alpha))_{\alpha\in I}$. Then the map f is fuzzy biclosed if and only if the map f_α is fuzzy biclosed for each $\alpha\in J$.

Proof. Let $\beta\in J$ and let η be a fuzzy closed subset of (X_β,u_β^1). Then $\eta\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in J}}X_\alpha$ is a fuzzy closed subset of $\prod_{\alpha\in J}(X_\alpha,u_\alpha^1)$. Since the map f is fuzzy biclosed, $f\left(\eta\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in J}}X_\alpha\right)$ is a fuzzy closed subset of $\prod_{\alpha\in J}(Y_\alpha,v_\alpha^2)$. But $f\left(\eta\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in J}}X_\alpha\right)=f_\beta(\eta)\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in J}}Y_\alpha$, hence $f_\beta(\eta)\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in J}}Y_\alpha$ is a fuzzy closed subset of $\prod_{\alpha\in J}(Y_\alpha,v_\alpha^2)$. By Proposition 2.9, $f_\beta(\eta)$ is a fuzzy closed subset of (Y_β,v_β^2). Hence, the map f_β is fuzzy biclosed.

Conversely, let the map f_β be fuzzy biclosed for each $\beta\in J$. Suppose that the map f is not fuzzy biclosed. Then there exists a fuzzy closed subset η of $\prod_{\alpha\in J}(X_\alpha,u_\alpha^1)$ such that $\prod_{\beta\in J}v_\beta^2\pi_\beta(f(\eta))\not\leq f(\eta)$. Therefore, there exists $\beta\in J$ such that $v_\beta^2 f_\beta(\pi_\beta(\eta))\not\leq f_\beta(\pi_\beta(\eta))$. But $\pi_\beta(\eta)$ is a fuzzy closed subset of (X_β,u_β^1) and f_β is fuzzy biclosed, $f_\beta(\pi_\beta(\eta))$ is a fuzzy closed subset of (Y_β,v_β^2). This is a contradiction. Therefore, the map f is fuzzy biclosed.

Conclusion

In this paper we have studied the concept of bicontinuous maps in biclosure spaces which was earlier studied by C.Boonpok in the frame work of fuzzy set theory and introduce the new concept of Fuzzy Bicontinuous Maps in Fuzzy Biclosure Spaces. We have also introduced and investigated fuzzy biclosed (fuzzy biopen) maps in fuzzy biclosure spaces. Thus we can say that we have extended the concept of bicontinuous maps in biclosure spaces to fuzzy sets and give rise to the concept of Fuzzy Bicontinuous Maps in Fuzzy Biclosure Spaces.

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