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A Three Species Ecological Model with a Predator and Two Preying Species

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Abstract. The present paper is devoted to an analytical study of a three species ecological model in which a predator is preying on the other two species which are mutually helping each other. In addition to that, all the three species are provided with an alternate food. The model is characterized by a set of first order non-linear differential equations. All the possible equilibrium points of the model have been derived and the local and global stability for the positive equilibrium point is discussed and supported by the numerical simulation using the MATLAB.

1. Introduction

Mathematical Modeling of a three species ecosystem is a study of understanding mechanisms which affects the growth of the species and also their beinghood and stability. The dynamic relationship between predator and prey continuous to be predominant in mathematical ecology which can be traced back to the end of 18th century, a milestone of its inception as a scientific discipline. The extensive studies of prey-predation interaction carried out by Lotka [4] and Volterra [6] is highly remarkable. Many researchers, mathematicians and ecologists namely Freedman [3], Kapur [7], Meyer[5], Cushing[2], have contributed to the growth of this area. The subsequent upswing by well known researchers like Paparao, Lakshmi Narayan [8] and Vidyanath [9] is a benefaction for this field of mathematical ecology. Also authors like Temple H. Fay, Johanna C. Greeff [1] have adapted several methods to study the dynamics among one predator and two mutualistic prey species. Differential equation models for interaction among species are one of the classical applications of mathematics and biology.

The present paper is devoted to an analytical study of a three species ecological model in which a predator (N_3) is preying on other two species (N_1) and (N_2) which are mutually helping each other. In addition to that total all three species are provided with alternative food. The model is characterized by a set of first order non-linear ordinary differential equations. All the eight equilibrium points of the model are identified and local stability of positive equilibrium was discussed by Routh-Hurwitz criteria and global stability was discussed by constructing suitable Lyapunov function. Further exact solutions of perturbed equations have been derived. The stability analysis is supported by Numerical simulation using Mat lab.

We consider three species model by the following set of nonlinear ordinary differential equations.

(i) Equation for the growth rate of first species (N_1):

$$\frac{dN_1}{dt} = a_1 N_1 - \alpha_{11} N_1^2 + \alpha_{12} N_1 N_2 - \alpha_{13} N_1 N_3 \tag{1.1}$$

(ii) Equation for the growth rate of second species (N_2):

$$\frac{dN_2}{dt} = a_2 N_2 - \alpha_{22} {N_2}^2 + \alpha_{21} N_1 N_2 - \alpha_{23} N_2 N_3 \tag{1.2}$$

(iii) Equation for the growth rate of third species (N_3):

$$\frac{dN_3}{dt} = a_3 N_3 - \alpha_{33} N_3^2 + \alpha_{31} N_1 N_3 + \alpha_{32} N_2 N_3 \tag{1.3}$$

with the following notations.
N_i's are the strengths of the i th species.
a_i's are the natural growth rates of N_i's; i=1,2,3.
α_{ii}'s are the natural death rates of the species N_i's due to inter-competitions.
α_{12} is the rate of increase of N_1 due to interaction with N_2.
α_{13} is the rate of decrease of N_1 due to inhibitions by N_3.
α_{21} is the rate of increase of N_2 due to interaction with N_1.
α_{23} is the rate of decrease of N_2 due to inhibitions by N_3.
α_{31} is the rate of increase of N_3 due to inhibition on N_1.
α_{32} is the rate of increase of N_3 due to inhibition on N_2.

The rest of the paper is organized as follows. In section 2 we obtain all the possible equilibrium points and local stability of the positive equilibrium point is discussed in section 3. In section 4, we carry out global stability of the system by constructing a suitable Lyapunov function. The fifth section presents different computer simulations of the system and conclusions are given in section 6.

2. Equilibrium states

The system under investigation has eight equilibrium states. They are

I.E_1: The extinct state

$$\overline{N}_1 = 0; \overline{N}_2 = 0, \overline{N}_3 = 0. \tag{2.1}$$

II.E_2: The state in which only the predator (N_3) survives and the other two are extinct

$$\overline{N}_1 = 0; \overline{N}_2 = 0, \overline{N_3} = \frac{a_3}{\alpha_{33}}. \tag{2.2}$$

III. E_3: The state in which only the second species (N_2) exist and the remaining two are extinct

$$\overline{N_1} = 0, \overline{N_2} = \frac{a_2}{\alpha_{22}}, \overline{N_3} = 0. \tag{2.3}$$

IV. E_4: The state in which only the first species (N_1) exist and the remaining two are extinct

$$\overline{N_1} = \frac{a_1}{\alpha_{11}}, \overline{N_2} = 0, \overline{N_3} = 0. \tag{2.4}$$

V. E_5: The state in which both (N_2) and (N_3) exist

$$\overline{N_1} = 0 ; \overline{N_2} = \left(\frac{a_2\alpha_{33} - a_3\alpha_{23}}{\alpha_{22}\alpha_{33} + \alpha_{23}\alpha_{32}} \right); \ \overline{N_3} = \left(\frac{a_3\alpha_{22} + a_2\alpha_{32}}{\alpha_{22}\alpha_{33} + \alpha_{23}\alpha_{32}} \right). \tag{2.5}$$

This case arise only when $\frac{a_2}{a_3} > \frac{\alpha_{23}}{\alpha_{33}}$.

VI. E_6 :The state in which both (N_1) and (N_3) exist,

$$\overline{N_1} = \frac{(a_1\alpha_{33} - a_3\alpha_{13})}{(\alpha_{11}\alpha_{33} + \alpha_{13}\alpha_{31})} \; ; \overline{N_2} = 0 \; ; \; \overline{N_3} = \frac{(a_3\alpha_{11} + a_1\alpha_{31})}{(\alpha_{11}\alpha_{33} + \alpha_{13}\alpha_{31})} \; . \tag{2.6}$$

The equilibrium state exist only when $\frac{a_1}{a_3} > \frac{\alpha_{13}}{\alpha_{33}}$.

VII. E_7: The state in which both the mutually helping species (N_1) and (N_2) exist and predator is extinct

$$\overline{N_1} = \frac{(a_1\alpha_{22} + a_2\alpha_{12})}{\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21}} \; ; \; \overline{N_2} = \frac{(a_2\alpha_{11} + a_1\alpha_{21})}{\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21}} \; ; \; \overline{N_3} = 0 \; . \tag{2.7}$$

This state exist only when $\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21} > 0$.

VIII. E_8 :The state in which all the three species exist

$$\overline{N_1} = \frac{a_1(\alpha_{22}\alpha_{33} + \alpha_{23}\alpha_{32}) + a_2(\alpha_{12}\alpha_{33} - \alpha_{13}\alpha_{32}) - a_3(\alpha_{12}\alpha_{23} + \alpha_{13}\alpha_{22})}{\alpha_{11}(\alpha_{22}\alpha_{33} + \alpha_{23}\alpha_{32}) + \alpha_{12}(\alpha_{21}\alpha_{33} - \alpha_{31}\alpha_{23}) + \alpha_{13}(\alpha_{21}\alpha_{32} + \alpha_{31}\alpha_{22})}$$

$$\overline{N_2} = \frac{a_1(\alpha_{31}\alpha_{23} - \alpha_{21}\alpha_{33}) + a_2(\alpha_{11}\alpha_{33} + \alpha_{13}\alpha_{31}) - a_3(\alpha_{11}\alpha_{23} + \alpha_{21}\alpha_{13})}{\alpha_{11}(\alpha_{22}\alpha_{33} + \alpha_{23}\alpha_{32}) + \alpha_{12}(\alpha_{21}\alpha_{33} - \alpha_{31}\alpha_{23}) + \alpha_{13}(\alpha_{21}\alpha_{32} + \alpha_{31}\alpha_{22})}$$

$$\overline{N_3} = \frac{a_1(\alpha_{21}\alpha_{32} + \alpha_{31}\alpha_{22}) + a_2(\alpha_{11}\alpha_{32} + \alpha_{31}\alpha_{22}) + a_3(\alpha_{11}\alpha_{22} - \alpha_{21}\alpha_{12})}{\alpha_{11}(\alpha_{22}\alpha_{33} + \alpha_{23}\alpha_{32}) + \alpha_{12}(\alpha_{21}\alpha_{33} - \alpha_{31}\alpha_{23}) + \alpha_{13}(\alpha_{21}\alpha_{32} + \alpha_{31}\alpha_{22})} \; . \tag{2.8}$$

The equilibrium state exist only when,

$$a_1(\alpha_{22}\alpha_{33} + \alpha_{23}\alpha_{32}) + a_2(\alpha_{12}\alpha_{33} - \alpha_{13}\alpha_{32}) > a_3(\alpha_{12}\alpha_{23} + \alpha_{13}\alpha_{22}) \, ,$$

$$a_1(\alpha_{31}\alpha_{23} - \alpha_{21}\alpha_{33}) + a_2(\alpha_{11}\alpha_{33} + \alpha_{13}\alpha_{31}) > a_3(\alpha_{11}\alpha_{23} + \alpha_{21}\alpha_{13}) \; . \tag{2.8a}$$

3. Local Stability of the positive equilibrium state

Let $N = (N_1, N_2, N_3)^T \; = \overline{N} + U$

where U = $(u_1, u_2, u_3)^T$ is the perturbation over the equilibrium state. $\overline{N} = (\overline{N_1}, \overline{N_2}, \overline{N_3})^T$.
The x-basic equations are linearized to obtain the equations for the perturbed state.

$$\begin{aligned} \frac{du_1}{dt} &= -\alpha_{11}\overline{N_1}u_1 + \alpha_{12}\overline{N_1}u_2 - \alpha_{13}\overline{N_1}u_3 \\ \frac{du_2}{dt} &= \alpha_{21}\overline{N_2}u_1 - \alpha_{22}\overline{N_2}u_2 - \alpha_{23}\overline{N_2}u_3 \\ \frac{du_3}{dt} &= \alpha_{31}\overline{N_3}u_1 + \alpha_{32}\overline{N_3}u_2 - \alpha_{33}\overline{N_3}u_3 \end{aligned} \tag{3.1}$$

With the characteristic equation

$$\begin{aligned} &\lambda^3 + \left(\alpha_{11}\overline{N_1} + \alpha_{22}\overline{N_2} + \alpha_{33}\overline{N_3}\right)\lambda^2 + \left[\begin{array}{r} \left(\alpha_{11}\alpha_{33} + \alpha_{13}\alpha_{31}\right)\overline{N_1}\,\overline{N_3} + \left(\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21}\right)\overline{N_1}\,\overline{N_2} \\ + \left(\alpha_{33}\alpha_{22} + \alpha_{23}\alpha_{32}\right)\overline{N_2}\,\overline{N_3} \end{array}\right]\lambda \\ &+ \left(\alpha_{11}\alpha_{33}\alpha_{22} - \alpha_{21}\alpha_{12}\alpha_{33} + \alpha_{23}\alpha_{12}\alpha_{31} + \alpha_{13}\alpha_{31}\alpha_{22} + \alpha_{11}\alpha_{23}\alpha_{32} + \alpha_{21}\alpha_{32}\alpha_{13}\right)\overline{N_1}\,\overline{N_2}\,\overline{N_3} = 0 \end{aligned} \tag{3.2}$$

Let

$$a_1 = \alpha_{11}\overline{N_1} + \alpha_{22}\overline{N_2} + \alpha_{33}\overline{N_3} > 0$$

$$a_2 = \left(\alpha_{11}\alpha_{33} + \alpha_{13}\alpha_{31}\right)\overline{N_1}\,\overline{N_3} + \left(\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21}\right)\overline{N_1}\,\overline{N_2} + \left(\alpha_{33}\alpha_{22} + \alpha_{23}\alpha_{32}\right)\overline{N_2}\,\overline{N_3} \tag{3.3}$$

$$a_3 = [\left(\alpha_{11}\alpha_{33}\alpha_{22} - \alpha_{21}\alpha_{12}\alpha_{33} + \alpha_{23}\alpha_{12}\alpha_{31} + \alpha_{13}\alpha_{31}\alpha_{22} + \alpha_{11}\alpha_{23}\alpha_{32} + \alpha_{21}\alpha_{32}\alpha_{13}\right)]\overline{N_1}\,\overline{N_2}\,\overline{N_3}\,.$$

By Routh-Hurwitz criteria, when all Eigen values of the above characteristic equation have negative real parts if only if $D_1 = a_1 > 0$, $D_2 = \left(a_1 a_2 - a_3\right) > 0$ and $D_3 = a_3 > 0$. Clearly $a_1 > 0$ and $a_3 > 0$ if $(\alpha_{22}\alpha_{33} - \alpha_{23}\alpha_{32}) > 0$.

And based on certain algebraic deductions applicable in this case, it can be verified that $D_2 > 0$
Hence the positive equilibrium point E_8 is locally asymptotically stable.
The trajectories for the equilibrium are

$$u_1 = A_1 e^{s_1 t} + B_1 e^{s_2 t} + C_1 e^{s_3 t}$$

$$u_2 = A_2 e^{s_1 t} + B_2 e^{s_2 t} + C_2 e^{s_3 t}$$

$$u_3 = A_3 e^{s_1 t} + B_3 e^{s_2 t} + C_3 e^{s_3 t} \tag{3.4}$$

where $s_{1,}s_2$ & s_3 are roots of the equation (3.2) and

$$A_1 = \left[\frac{u_{10}(s_1 + \alpha_{22}\overline{N_2})(s_1 + \alpha_{33}\overline{N_3}) + u_{20}\alpha_{12}\overline{N_1}(s_1 + \alpha_{33}\overline{N_3}) - \left[u_{30}\alpha_{13}\overline{N_1}(s_1 + \alpha_{22}\overline{N_2})\right]}{\left(s_1 - s_2\right)\left(s_1 - s_3\right)}\right]$$

$$+\left[\frac{u_{10}\alpha_{23}\alpha_{32}\overline{N_2}\,\overline{N_3} - u_{30}\alpha_{12}\alpha_{23}\overline{N_1}\,\overline{N_2} - u_{20}\alpha_{13}\alpha_{32}\overline{N_1}\,\overline{N_3}}{\left(s_1 - s_2\right)\left(s_1 - s_3\right)}\right]$$

$$B_1 = \left[\frac{u_{10}(s_2 + \alpha_{22}\overline{N_2})(s_2 + \alpha_{33}\overline{N_3}) + u_{20}\alpha_{12}\overline{N_1}(s_2 + \alpha_{33}\overline{N_3}) - \left[u_{30}\alpha_{13}\overline{N_1}(s_2 + \alpha_{22}\overline{N_2})\right]}{\left(s_2 - s_1\right)\left(s_2 - s_3\right)}\right]$$

$$+\left[\frac{u_{10}\alpha_{23}\alpha_{32}\overline{N_2}\,\overline{N_3} - u_{30}\alpha_{12}\alpha_{23}\overline{N_1}\,\overline{N_2} - u_{20}\alpha_{13}\alpha_{32}\overline{N_1}\,\overline{N_3}}{\left(s_2 - s_1\right)\left(s_2 - s_3\right)}\right]$$

$$C_1 = \left[\frac{u_{10}(s_3 + \alpha_{22}\overline{N_2})(s_3 + \alpha_{33}\overline{N_3}) + u_{20}\alpha_{12}\overline{N_1}(s_3 + \alpha_{33}\overline{N_3}) + \left[u_{30}\alpha_{13}\overline{N_1}(s_3 + \alpha_{22}\overline{N_2})\right]}{\left(s_3 - s_1\right)\left(s_3 - s_2\right)}\right]$$

$$+\left[\frac{u_{10}\alpha_{23}\alpha_{32}\overline{N_2}\,\overline{N_3} - u_{30}\alpha_{12}\alpha_{23}\overline{N_1}\,\overline{N_2} - u_{20}\alpha_{13}\alpha_{32}\overline{N_1}\,\overline{N_3}}{\left(s_3 - s_1\right)\left(s_3 - s_2\right)}\right]$$

$$A_2 = \left[\frac{u_{20}(s_1 + \alpha_{11}\overline{N_1})(s_1 + \alpha_{33}\overline{N_3}) - u_{30}\alpha_{23}\overline{N_2}(s_1 + \alpha_{11}\overline{N_1}) + u_{10}\alpha_{21}\overline{N_2}(s_1 + \alpha_{33}\overline{N_3})}{\left(s_1 - s_2\right)\left(s_1 - s_3\right)}\right]$$

$$-\left[\frac{u_{10}\alpha_{23}\alpha_{31}\overline{N_2}\,\overline{N_3} + u_{30}\alpha_{13}\alpha_{21}\overline{N_1}\,\overline{N_2} - u_{20}\alpha_{13}\alpha_{31}\overline{N_1}\,\overline{N_3}}{\left(s_1 - s_2\right)\left(s_1 - s_3\right)}\right]$$

$$B_2=\left[\frac{u_{20}(s_2+\alpha_{11}\overline{N_1})(s_2+\alpha_{33}\overline{N_3})-u_{30}\alpha_{23}\overline{N_2}(s_2+\alpha_{11}\overline{N_1})+u_{10}\alpha_{21}\overline{N_2}(s_2+\alpha_{33}\overline{N_3})}{(s_2-s_1)(s_2-s_3)}\right]$$
$$-\left[\frac{u_{10}\alpha_{23}\alpha_{31}\overline{N_2}\,\overline{N_3}+u_{30}\alpha_{13}\alpha_{21}\overline{N_1}\,\overline{N_2}-u_{20}\alpha_{13}\alpha_{31}\overline{N_1}\,\overline{N_3}}{(s_2-s_1)(s_2-s_3)}\right]$$

$$C_2=\left[\frac{u_{20}(s_3+\alpha_{11}\overline{N_1})(s_3+\alpha_{33}\overline{N_3})-u_{30}\alpha_{23}\overline{N_2}(s_3+\alpha_{11}\overline{N_1})+u_{10}\alpha_{21}\overline{N_2}(s_3+\alpha_{33}\overline{N_3})}{(s_3-s_1)(s_3-s_2)}\right]$$
$$-\left[\frac{u_{10}\alpha_{23}\alpha_{31}\overline{N_2}\,\overline{N_3}+u_{30}\alpha_{13}\alpha_{21}\overline{N_1}\,\overline{N_2}-u_{20}\alpha_{13}\alpha_{31}\overline{N_1}\,\overline{N_3}}{(s_3-s_1)(s_3-s_2)}\right]$$

$$A_3=\left[\frac{u_{30}(s_1+\alpha_{11}\overline{N_1})(s_1+\alpha_{22}\overline{N_2})+u_{20}\alpha_{32}\overline{N_3}(s_1+\alpha_{11}\overline{N_1})+u_{10}\alpha_{31}\overline{N_3}(s_1+\alpha_{22}\overline{N_2})}{(s_1-s_2)(s_1-s_3)}\right]$$
$$-\left[\frac{u_{30}\alpha_{12}\alpha_{21}\overline{N_1}\,\overline{N_2}-u_{20}\alpha_{12}\alpha_{31}\overline{N_1}\,\overline{N_3}-u_{10}\alpha_{21}\alpha_{32}\overline{N_2}\,\overline{N_3}}{(s_1-s_2)(s_1-s_3)}\right]$$

$$B_3=\left[\frac{u_{30}(s_2+\alpha_{11}\overline{N_1})(s_2+\alpha_{22}\overline{N_2})+u_{20}\alpha_{32}\overline{N_3}(s_2+\alpha_{11}\overline{N_1})+u_{10}\alpha_{31}\overline{N_3}(s_2+\alpha_{22}\overline{N_2})}{(s_2-s_1)(s_2-s_3)}\right]$$
$$-\left[\frac{u_{30}\alpha_{12}\alpha_{21}\overline{N_1}\,\overline{N_2}-u_{20}\alpha_{12}\alpha_{31}\overline{N_1}\,\overline{N_3}-u_{10}\alpha_{21}\alpha_{32}\overline{N_2}\,\overline{N_3}}{(s_2-s_1)(s_2-s_3)}\right]$$

$$C_3=\left[\frac{u_{30}(s_3+\alpha_{11}\overline{N_1})(s_3+\alpha_{22}\overline{N_2})+u_{20}\alpha_{32}\overline{N_3}(s_3+\alpha_{11}\overline{N_1})+u_{10}\alpha_{31}\overline{N_3}(s_3+\alpha_{22}\overline{N_2})}{(s_3-s_1)(s_3-s_2)}\right]$$
$$-\left[\frac{u_{30}\alpha_{12}\alpha_{21}\overline{N_1}\,\overline{N_2}-u_{20}\alpha_{12}\alpha_{31}\overline{N_1}\,\overline{N_3}-u_{10}\alpha_{21}\alpha_{32}\overline{N_2}\,\overline{N_3}}{(s_3-s_1)(s_3-s_2)}\right] \tag{3.5}$$

And u_{10}, u_{20} and u_{30} are the initial strengths of u_1, u_2 and u_3 respectively.

4. Global stability

Let the Lyapunov function for the interior equilibrium points E_8 is:

$$V(\overline{N_1},\overline{N_2},\overline{N_3})=(N_1-\overline{N_1})-\overline{N_1}\ln\left[\frac{N_1}{\overline{\overline{N_1}}}\right]+(N_2-\overline{N_2})-\overline{N_2}\ln\left[\frac{N_2}{\overline{\overline{N_2}}}\right]+(N_3-\overline{N_3})-\overline{N_3}\ln\left[\frac{N_3}{\overline{\overline{N_3}}}\right] \tag{4.1}$$

Then calculate $\frac{dV}{dt}$ which is as follows

$$\frac{dV}{dt}=\frac{dN_1}{dt}\left[1-\frac{\overline{N_1}}{N_1}\right]+\frac{dN_2}{dt}\left[1-\frac{\overline{N_2}}{N_2}\right]+\frac{dN_3}{dt}\left[1-\frac{\overline{N_3}}{N_3}\right] \tag{4.2}$$

$$=[N_1-\overline{N_1}]\left[a_1-\alpha_{11}N_1+\alpha_{12}N_2-\alpha_{13}N_3\right]+[N_2-\overline{N_2}]\left[a_2-\alpha_{22}N_2+\alpha_{21}N_1-\alpha_{23}N_3\right]$$
$$+[N_3-\overline{N_3}]\left[a_3-\alpha_{33}N_3+\alpha_{31}N_1+\alpha_{32}N_2\right].$$

Substitute $a_1 = \alpha_{11}\overline{N_1} - \alpha_{12}\overline{N_2} + \alpha_{13}\overline{N_3}$, $a_2 = -\alpha_{21}\overline{N_1} + \alpha_{22}\overline{N_2} + \alpha_{23}N_3$ &

$$a_3 = \alpha_{33}\overline{N_3} - \alpha_{31}\overline{N_1} - \alpha_{32}N_2 , \tag{4.3}$$

we get

$$\frac{dV}{dt} = -\alpha_{11}\left[N_1 - \overline{N_1}\right]^2 - \alpha_{22}\left[N_2 - \overline{N_2}\right]^2 - \alpha_{33}\left[N_3 - \overline{N_3}\right]^2 - (\alpha_{21} - \alpha_{12})\left[N_1 - \overline{N_1}\right]\left[N_2 - \overline{N_2}\right]$$
$$-(\alpha_{13} + \alpha_{31})\left[N_3 - \overline{N_3}\right]\left[N_1 - \overline{N_1}\right]$$
$$\le -\left(\alpha_{11} + \frac{\alpha_{12} + \alpha_{21} - \alpha_{13} + \alpha_{31}}{2}\right)\left[N_1 - \overline{N_1}\right]^2 - \left(\alpha_{22} + \frac{\alpha_{12} + \alpha_{21} - \alpha_{23} + \alpha_{32}}{2}\right)\left[N_2 - \overline{N_2}\right]^2$$
$$-\left(\alpha_{33} + \frac{\alpha_{32} - \alpha_{23} - \alpha_{13} + \alpha_{31}}{2}\right)\left[N_3 - \overline{N_3}\right]^2 \tag{4.4}$$
$$< 0$$

Hence the system is globally asymptotically stable.

5. Numerical Examples

For the simulation purpose we use the following parameter values:

a_1=0.156; a_{11}=0.85; a_{12}=0.71; a_{13}=0.746; a_2=0.24; a_{21}=0.456; a_{22}=0.145; a_{23}=0.565; a_3=0.163; a_{31}=0.819; a_{32}=0.159; a_{33}=0.641;

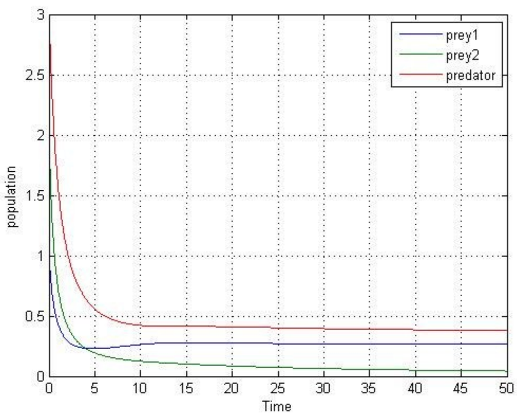

Fig.1

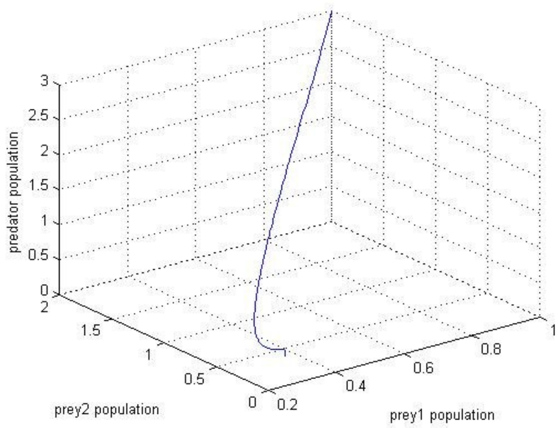

Fig. 2

Fig. 1 represents variations in the growth rate of the population with time.
Fig. 2 represents phase-space trajectories of the population.

6. Conclusion

A three species ecological model with mutualism and prey-predation is considered for investigation. The interior equilibrium point of the model was identified and its local stability is discussed using Routh-Hurwitz criteria. The global stability of the system was established by constructing a suitable Lyapunov's function. Solutions of the perturbed equations shows that the system under consideration was stable. The numerical simulation also shows that the system is globally asymptotically stable.

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