

# ICOSAHEDRAL SUPERSYMMETRY AND DARK MATTER

J.A.de Wet

Box 514, Plettenberg Bay, 6600,  
 South Africa

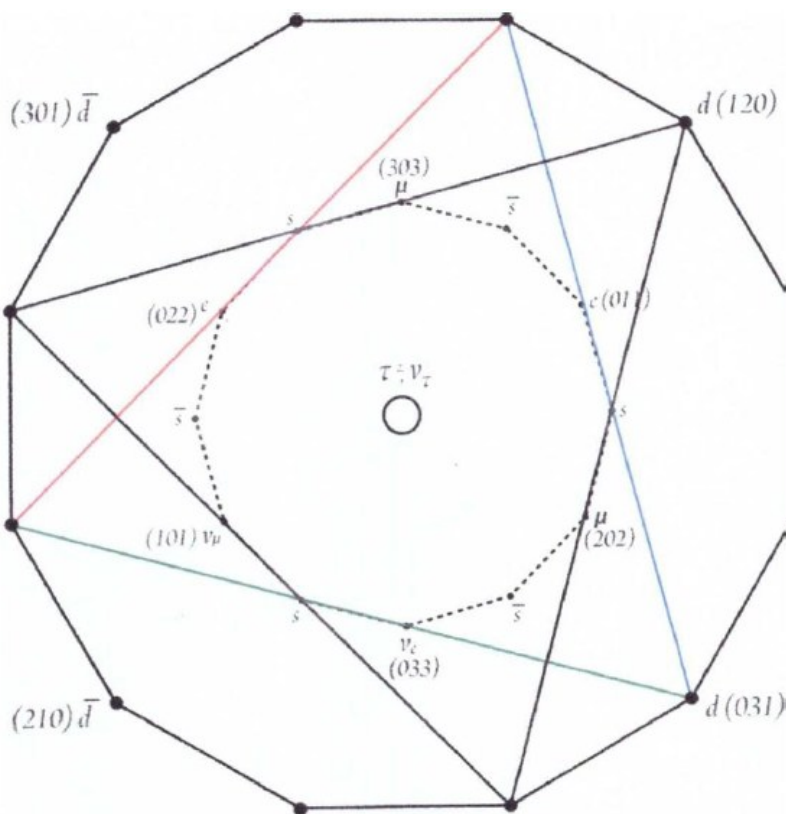
**Keywords:** Coxeter Graph, Torus Knots, EVIII decomposition, WeakInteraction, Dark Matter.

**Abstract.** The Icosahedral group has the Lie algebra  $E_8$  with a graph of 240 vertices and one real and 2 complex forms as well as a non-compact Split Form EVIII that is infinite-dimensional and shown to have 42 vertices that account for the squarks, sleptons and sneutrinos of Dark Matter.

## 1 Introduction

This letter is an appendix of the paper ‘Icosahedral Supersymmetry’[5]. Essentially the Icosahedral group has the Lie algebra  $E_8$  by the McKay correspondence. Here we will analyse the 240 vertices of the Toroidal graph of  $E_8$  illustrated on the Frontispiece of [4] and relate them to the real and complex forms of  $E_8$ , namely: (a) the compact form of the subalgebra  $E_6$  with 72 vertices, (b) The compact subgroup  $E_7 \times su(2)$  with a double cover of 126 vertices in  $E_7$  [2]. The symplectic form could carry 72 supersymmetric particles on 70 vertices leaving  $126-70 = 56$  for the  $W^\pm$  bosons in the double Weyl permutation group  $2W(E_7)$ , (c) the Split Form with  $240-(72+126) = 15+27$  vertices for the squarks, sleptons and sneutrinos of Dark Matter.

These possibilities will be discussed in more detail below and will be related to 4 of the 5 sets of 3 synthemes of  $E_8$ , where a syntheme is a set of 3 commuting Dirac or E-numbers. These are mapped by 15 planes passing through the center of the Icosahedron with 4 vertices according to Baez [1], 5 of which are true and 10 are skew.



**Fig.1**  
 $u(130) \cdot u(012)$

## 1 Representations of E8

The compact form of  $E_8$  has been analysed in several publications (cf.eg.[6]). It utilizes the subalgebra  $su_3 \times su_3 \times su_3$  of  $E_6$  which has the lattice of Fig.1 found in [3] with vertices labeled by the elementary particles of the Standard Model according to a map by Slansky [12]. For example uud are at the 3 vertices of a tritangent plane to the cubic or Del Pezzo surface that is Fig.1. However there are another 45 triads of mutually non-adjacent vertices on tritangent planes on the Del Pezzo surface which appear in Fig.12.B of [4]. For example, apart from the tritangent labeled by (012,023,031) for (uud)there are 2 more tritangents (012,230,103) and (012,203,330) associated with the vertex (012) that account for 2 of the 45 planes and so on. Here Coxeters labeling  $(0\mu\nu, \nu0\mu, \mu\nu0)$  has been employed for the 3 vertices on a tritangent that may be rotated into itself by  $\omega = 120$  degrees. But these labels are only those found on Fig.1, so the remaining vertices are not associated with the elementary particles that are represented by the subalgebra  $(su_3)^3$ .

Turning now to the 15 synthemes of  $E_8$ , the first set of

$$(12, 34, 05), (31, 24, 05), (23, 14, 05) \quad (1)$$

was employed sometime ago by de Wet [7] to represent the many nucleon problem. Here the pair ab of 2 numbers is shorthand for a 4x4 E- matrix employed by Eddington that is isomorphic to a Dirac matrix  $\gamma\nu$ . Specifically  $E_{12}, E_{23}, E_{31}$  are spins about  $x_1, x_3, x_2$  accompanied by parities  $E_{34}, E_{24}, E_{14}$  that are rotations about  $x_4$  which reflect apin, while  $E_{05}$  is a coordinate in isospace carrying charge. In this way equation (1) covers the 12 vertices representing the nucleons and anti-nucleons on the outer shell of Fig.1 that are supposed to survive when the 6d projective space  $CP^3$  of  $E_6$  compactifies to a 3d projective space [5].

The second and third sets of synthemes are

$$(12, 35, 04), (31, 25, 04), (23, 15, 04); (12, 03, 45), (31, 02, 45), (23, 01, 45) \quad (2)$$

which again represent 3 spin axes and coordinates of isospace  $E_{04}, E_{05}$  but have no parity in a 5- or 6-space and therefore represent bosons in a symplectic or quartic space introduced by Manivel([11]pp 10,16). The first set of 3 synthemes in (2) could carry 72 supersymmetric particles on 70 vertices in  $E_7$  where 2 + 2 planes coincide (cf.Brouwer [2]).

This leaves  $126-70 = 56$  vertices and 56 lines lying on a Del Pezzo or cubic surface of degree 2 for the second set,which is defined by Manivel as the double cover of the projective space branched over a quartic (or 4-th order) curve on the surface with a line configuration of index 2. In fact de Wet[8] has associated the  $E_7 \times su(2)$  subalgebra of  $E_8$  with the  $W^\pm$  boson pair and confirmed this by calculating its mass based on the order

$$|W(E_7 + A_1)| = 5806080 = 2W(E_7) \quad (3)$$

of the Weyl group W yielding the number of possible permutations of the 56 lines which is associated with entropy related to eV by the constant 27.7. Finally turning to the non-compact split form of E VIII possibly represented by the synthemes

$$(14, 25, 03), (24, 35, 01), (34, 15, 02) \quad (4)$$

with spin rotations in 4-,5- and 6-space but no charge (cf.[10]).This form could carry  $240-(72+126) = 15+27$  vertices for the squarks, sleptons and sneutrinos of Dark Matter decaying from neutralinos or WIMPs that could be anywhere as it is infinite dimensional. Then 12 vertices are identified with the outer shell of Fig.1 and 3 representing  $e^\pm, e_\nu$  are located at the origin. So E VIII can cover these vertices twice as well as the remaining 12 on the inner shell.

## 2 Conclusion

The paths on the graph of  $E_8$  in the frontispiece of [4] that meet the inner ring of the torus could be gluons as discussed in [9]. Also knots or loops in space may underlie Loop Quantum Gravity according to Sundance Bilson Thompson and Smolin et.al. [13]. Thus knot space is fundamental as is the Exceptional Lie group  $E_8$  because there is no  $E_9$  or Platonic solid larger than the Icosahedron.

## References

- [1] John Baez, Some Thoughts on the Number 6, On line Oct.(2007).
- [2] A.E.Brouwer, Lattices, On line (2002)
- [3] H.S.M. Coxeter, The Polytope 221 whose 27 Vertices Correspond to the Lines on the General Cubic Surface, Am. Mathematical Soc.62(1940)467- 486.
- [4] H.S.M. Coxeter, Regular Complex Polytopes Camb. Univ.Press, Second Ed.(1991).
- [5] J.A.de Wet, Icosahedral Supersymmetry, Online, Published by Int.Soc.of Frontier Science, Oct(2014)
- [6] J.A.de Wet, A Standard Model Algebra, Int. Mathematical Forum Vol.8 no,23(2013)1111-1117 HIKARI Ltd, <http://dx.doi.org/10.12988/imf.2013.3477>
- [7] J.A.de Wet, A group theoretical approach to the many nucleon problem, Proc.Camb.Phil.Soc. vol. 70(1971)485-496
- [8] J.A.de Wet, Strong Force without QCD and the Origin of Mass without the Higgs. International Mathematical Forum, vol.7(2012)1419-1425, HIKARI Ltd
- [9] J.A.deWet, Particle Knots in Toric Modular Space, Online, Published by Bulletin of Soc. for Mathematical Services and Standards vol 3 No 2 (2014)54-59.
- [10] Paulo Faccin and Willem A de Graaf, Constructing Semi- Simple Subalgebras of Real Semi-Simple Algebras, arXiv 1310.0172v1 [math.RA] Oct(2013)
- [11] L.Manivel, Configuration of Lines and Models of Lie Algebras, arXiv:math. AG/0507188v1(2005)
- [12] R.Slansky, Group Theory for Unified Model Building, Reprinted in "Unity of Forces in the Universe" Ed A.Lee, World Scientific(1997)
- [13] Sundance O. Bilson-Thompson, Fotini Markopoulo and Lee Smolin, Quantum Gravity and the Standard Model, Online, June(2014)