

ICOSAHEDRAL SUPERSYMMETRY AND DARK MATTER

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Abstract. The Icosahedral group has the Lie algebra E_8 with a graph of 240 vertices and one real and 2 complex forms as well as a non-compact Split Form EVIII that is infinite-dimensional and shown to have 42 vertices that account for the squarks, sleptons and sneutrinos of Dark Matter.

1 Introduction

This letter is an appendix of the paper ‘Icosahedral Supersymmetry’[5]. Essentially the Icosahedral group has the Lie algebra E_8 by the McKay correspondence. Here we will analyse the 240 vertices of the Toroidal graph of E_8 illustrated on the Frontispiece of [4] and relate them to the real and complex forms of E_8 , namely: (a) the compact form of the subalgebra E_6 with 72 vertices, (b) The compact subgroup $E_7 \times su(2)$ with a double cover of 126 vertices in E_7 [2]. The symplectic form could carry 72 supersymmetric particles on 70 vertices leaving $126-70 = 56$ for the W^\pm bosons in the double Weyl permutation group $2W(E_7)$, (c) the Split Form with $240-(72+126) = 42$ vertices for the squarks, sleptons and sneutrinos of Dark Matter.

These possibilities will be discussed in more detail below and will be related to 4 of the 5 sets of 3 synthemes of E_8 , where a syntheme is a set of 3 commuting Dirac or E-numbers. These are mapped by 15 planes passing through the center of the Icosahedron with 4 vertices according to Baez [1], 5 of which are true and 10 are skew.

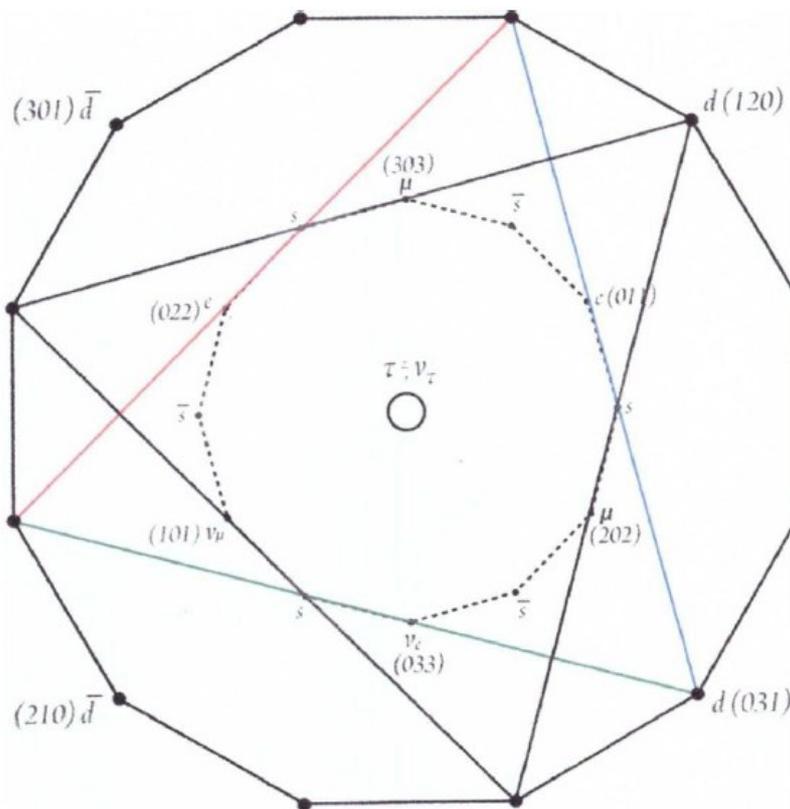


Fig.1
 $u(130) \cdot u(012)$

1 Representations of E8

The compact form of E_8 has been analysed in several publications (cf.eg.[6]). It utilizes the subalgebra $su_3 \times su_3 \times su_3$ of E_6 which has the lattice of Fig.1 found in [3] with vertices labeled by the elementary particles of the Standard Model according to a map by Slansky [12]. For example uud are at the 3 vertices of a tritangent plane to the cubic or Del Pezzo surface that is Fig.1. However there are another 45 triads of mutually non-adjacent vertices on tritangent planes on the Del Pezzo surface which appear in Fig.12.B of [4]. For example, apart from the tritangent labeled by (012,023,031) for (uud)there are 2 more tritangents (012,230,103) and (012,203,330) associated with the vertex (012) that account for 2 of the 45 planes and so on. Here Coxeters labeling $(0\mu\nu, \nu0\mu, \mu\nu0)$ has been employed for the 3 vertices on a tritangent that may be rotated into itself by $\omega = 120$ degrees. But these labels are only those found on Fig.1, so the remaining vertices are not associated with the elementary particles that are represented by the subalgebra $(su_3)^3$.

Turning now to the 15 synthemes of E_8 , the first set of

$$(12, 34, 05), (31, 24, 05), (23, 14, 05) \quad (1)$$

was employed sometime ago by de Wet [7] to represent the many nucleon problem. Here the pair ab of 2 numbers is shorthand for a 4x4 E- matrix employed by Eddington that is isomorphic to a Dirac matrix $\gamma\nu$. Specifically E_{12}, E_{23}, E_{31} are spins about x_1, x_3, x_2 accompanied by parities E_{34}, E_{24}, E_{14} that are rotations about x_4 which reflect apin, while E_{05} is a coordinate in isospace carrying charge. In this way equation (1) covers the 12 vertices representing the nucleons and anti-nucleons on the outer shell of Fig.1 that are supposed to survive when the 6d projective space CP^3 of E_6 compactifies to a 3d projective space [5].

The second and third sets of synthemes are

$$(12, 35, 04), (31, 25, 04), (23, 15, 04); (12, 03, 45), (31, 02, 45), (23, 01, 45) \quad (2)$$

which again represent 3 spin axes and coordinates of isospace E_{04}, E_{05} but have no parity in a 5- or 6-space and therefore represent bosons in a symplectic or quartic space introduced by Manivel([11]pp 10,16). The first set of 3 synthemes in (2) could carry 72 supersymmetric particles on 70 vertices in E_7 where 2 + 2 planes coincide (cf.Brouwer [2]).

This leaves $126-70 = 56$ vertices and 56 lines lying on a Del Pezzo or cubic surface of degree 2 for the second set,which is defined by Manivel as the double cover of the projective space branched over a quartic (or 4-th order) curve on the surface with a line configuration of index 2. In fact de Wet[8] has associated the $E_7 \times su(2)$ subalgebra of E_8 with the W^\pm boson pair and confirmed this by calculating its mass based on the order

$$|W(E_7 + A_1)| = 5806080 = 2W(E_7) \quad (3)$$

of the Weyl group W yielding the number of possible permutations of the 56 lines which is associated with entropy related to eV by the constant 27.7. Finally turning to the non-compact split form of E VIII possibly represented by the synthemes

$$(14, 25, 03), (24, 35, 01), (34, 15, 02) \quad (4)$$

with spin rotations in 4-,5- and 6-space but no charge (cf.[10]).This form could carry $240-(72+126) = 15+27$ vertices for the squarks, sleptons and sneutrinos of Dark Matter decaying from neutralinos or WIMPs that could be anywhere as it is infinite dimensional. Then 12 vertices are identified with the outer shell of Fig.1 and 3 representing e^\pm, e_ν are located at the origin. So E VIII can cover these vertices twice as well as the remaining 12 on the inner shell.

2 Conclusion

The paths on the graph of E_8 in the frontispiece of [4] that meet the inner ring of the torus could be gluons as discussed in [9]. Also knots or loops in space may underlie Loop Quantum Gravity according to Sundance Bilson Thompson and Smolin et.al. [13]. Thus knot space is fundamental as is the Exceptional Lie group E_8 because there is no E_9 or Platonic solid larger than the Icosahedron.

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