International Frontier Science Letters *Online: 2015-01-01*
ISSN: 2349-4484, Vol. 3, pp 19-31
doi:10.18052/www.scipress.com/IFSL.3.19

Simultaneity, Relativistic Time and Galileo Transformations

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ABSTRACT. It is shown that the theory of Restrict Relativity it's not free of contradictions, being one of them related to the relativity of simultaneity. Another contradiction occurs when we calculate the light speed in relation to a moving reference using the contraction of space and dilation of time, because it is verified that the speed of light depends on the speed of the referential. It is also shown that for slow speeds, but great distances, that Lorentz's transformation for the time does not reduces itself to their Galileo transformation, subject not explore further on most books of scientific disclosure and even academic.

I - Introduction

When Einstein initiated the Special Relativity (S.R.), in 1905, he intended to eliminate the asymmetries contained in Maxwell's electrodynamics to moving bodies and create, from two postulates, an electrodynamics for moving bodies, simple and free of contradictions, unlinked from the sense of "luminiferous Ether" and based on Maxwell's theory to bodies in rest. This can be deduced from the first two paragraphs from the article that originated the S.R., EINSTEIN (1905).

Although it is undeniable the success achieved by the S.R. and it's abidance with many experimental results, it is known, according to ASSIS (1999, pp. 77-79), that "the asymmetry of electromagnetic induction quoted in the first paragraph by Einstein does not appear in Maxwell's electromagnetism, contrary to his statement. It only appears as a specific interpretation of Lorentz's formulation to the electrodynamics. This asymmetry would not exist to Faraday, that discovered the phenomenon." (...) "Maxwell had the same points of view in relation to this subject and did not see any "clear distinction" to the explanation of Faraday's experiences, does not concerning if it was the circuit or the magnet that moved in relation to the laboratory." (...) "This asymmetry pointed by Einstein also does not appears on Weber's electrodynamics".

Furthermore, unlike Einstein said, the S.R. is not free of contradictions, being one of the contradictions related to the relativity of simultaneity, what will try to be proved on the Section II. Another contradiction occurs when we calculated the light speed in relation to a moving referential, using the space contraction and time dilation, because it is verified that the speed of light depends on the speed of the referential. It is what is shown on Section III.

In Section IV will be shown that the statement that says that Lorentz's transformations are reduced to Galileo's transformations in the limit of slow speeds is not true, deserving such statement a certain correction. The Section V will be dedicated to the critique of Professor Piza to this article and the Section VI will conclude the present work.

II - Synchronism of clocks and Simultaneity

According to the S.R., two clocks A and B, motionless in relation to another and in uniform rectilinear motion (U.R.M.) will be synchronous if

$$\tau_B - \tau_A = \tau'_A - \tau_B, \tag{1}$$

where τ_A is the instant marked by A in which a light ray leaves A going to B, τ_B the instant marked by B in which the light ray reaches B and go back to A and τ'_A the instant marked by A in which the ray gets to A (EINSTEIN, 1905, p. 894). The relation (1) must be true to any constant speed inferior, in module, to the speed of light and to any distance between A and B. Although we know that it is not so simple in the Theory of General Relativity, let us limit ourselves to the S.R.

Suppose that A and B do execute a movement of constant speed v from the instant $t = 0$ along the axis of abscissa of a triorthogonal system of rectangular coordinates considered fix, S(x, y, z, t), where (x, y, z) and t the coordinated position and the instant, respectively, from an event E measured in S, and that S'(ξ, η, ζ, τ) is another triorthogonal system of rectangular coordinates, which A and B are considered fixes where (ξ, η, ζ) and τ are the coordinate of position and the instant, respectively, of this same event E when measured in S'.

If we admit that in the instant $t = 0$ the origins of both systems were coincident and $\tau(x=0, t=0) = 0$, that the axis of x and ξ are coincident and the axis of y and z are respectively parallel to the axis of η and ζ and further, that the clocks which measure the instants t and τ are synchronous according to rule (1), then between S e S', according to the S.R., are valid the Lorentz's transformations:

$$\tau = \beta(t - vx/c^2); \qquad (2)$$
$$\xi = \beta(x - vt); \qquad (3)$$
$$\eta = y; \qquad (4)$$
$$\zeta = z; \qquad (5)$$

where $\beta = 1/(1-v^2/c^2)^{1/2}$ and c is the speed of light in the vacuum, considered independent of the speed of the observers and the luminous source.

Let us, by hypothesis, suppose that A and B are clocks that mark the time in system S' according to transformation (2), in function of t and x. If the clock A leaves in $t = 0$ from the point xA, measured with the system in rest, then it's time equation in S will be $x = x_A + vt$. Using this value of x in the transformation (2) we obtain

$$\tau_A = t/\beta - \beta v x_A/c^2, \qquad (6)$$

that will be the indication of the clock A in S', in function of t, during its whole movement of velocity v.

Similarly, B leaving from point x_B in $t = 0$ we have

$$\tau_B = t/\beta - \beta v x_B/c^2. \qquad (7)$$

From the deduction of Lorentz's transformations made by EINSTEIN (1905, pp. 898-902) it is ensured that the clocks A and B obey to the condition of synchronism gave in (1) when mark, respectively, the values given in (6) and (7), what can also be verified through simple cinematic calculations, according the shown in GODOI (1997, pp. 323-324). For briefness, we will omit here such demonstration.

Assuming $x_B > x_A$ and $v > 0$ we have $\tau_A > \tau_B$.

Effecting the difference between the times marked by A and B we obtain

$$\Delta\tau_{AB} = \tau_A - \tau_B = \beta v(x_B - x_A)/c^2 > 0. \qquad (8)$$

From (8) it would be possible to conclude, through the gumption and our intuition, that, indeed, our clocks A and B cannot be synchronous, nor in relation to S, nor in relation to S', because the difference pointed in (8) is never equal to zero and what is expected, *a priori*, from synchronous clocks is that they mark the same time, without any differences. It is what EINSTEIN (1905, p. 894) called of "common time" to A and B (*A und B gemeinsame "Zeit"*).

How can be refuted such argumentation starting from the principle that, according to S.R., two simultaneous events in a referential are not in another animated referential of non-null constant speed in relation to the first (relativity of simultaneity), let us prove the existence of contradiction in the S.R., analyzing the relativity of simultaneity. Before, however, let us analyze the difference (8) to a specific case of low speeds, because in the domain of slow speeds the S.R. should be reduced to the classic theory of Newton, which the time is absolute, or, said in a more rigorous way, the transformations of Lorentz should be reduced to the transformations of Galileo:

$$\tau = t;$$
$$\xi = x - vt;$$
$$\eta = y;$$
$$\zeta = z.$$

Thus, let us admit the inequality $0 < v << c$, but now making that the difference $x_B - x_A$ be of the order of c^2, or even superior order (respecting the respective measuring units). To these cases the difference (8) is reduced to

$$\Delta\tau_{AB} \approx v(x_B - x_A)/c^2, \qquad (9)$$

that is never equal to zero for $v > 0$, coming to reinforce even more our initial idea that the clocks A and B are no synchronous, nor in relation to S, nor in relation to S', which may even mark a great difference on its times.

In other words, who at low speed, from equal distance to A and B, and to which both clocks are fix, could observe simultaneously the times marked by A and B, would verify that it's markings are not the same, *i.e.*, $\tau_A \neq \tau_B$. We can imagine that this observer possesses 2 great telescopes to aid him in the comparison, one reflecting the image of clock A and the other reflecting the image of clock B, being equal the distances between each scope and the respective clock.

Exemplifying, in the instant $\tau = \tau_0$ and position $\xi = 0$ from the referential S' in movement our observer wish to know if A and B, symmetrically located and also motionless in relation to him, are synchronous to each other in relation to S'. The time equation of the observer, fix in the origin of the referential S' and possessing a clock obeying (2), will be $x = vt$, the time equation of A will be $x = -x_B + vt$ and of B will be $x = x_B + vt$, $x_B > 0$, all these measured in relation to the referential S, considered fix.

If the clock of the observer mark the instant $\tau = \tau_0$, satisfied the conditions needed to the validity of (2), then in relation to S the corresponding instant t is $t = \beta\,\tau_0 \approx \tau_0$. Thus, from (6) we'll have $\tau_A \approx \tau_0 + vx_B/c^2$, and from (7) we'll have $\tau_B \approx \tau_0 - vx_B/c^2$, so, $\tau_A \neq \tau_B \neq \tau_0$ to $0 < v << c$, instead of the expected result $\tau_A = \tau_B = \tau_0$.

This means: when the clock of the observer in the origin of S' marks the instant τ_0 the clock A will not be marking τ_0, nor B, and beyond that, A and B will not be marking the same time, though they are synchronous "by definition", according to (1).

After an interval of time of order x_B/c, our observer (if could live so much time and did not interfere in the light trajectory of the clocks to the telescopes) would verify, at long last, the difference between the times of the 2 clocks occurred in the long past, such that $\Delta\tau_{AB} = \tau_A - \tau_B \approx 2vx_B/c^2$. It is worth mentioning that will not arrive simultaneously to the telescopes and the observer these times marked by A and B, arriving first the time of B and then of A.

See how it is strange the proposition of the S.R.: if on the instant $t = 0$ of S A and B start to move will not be able to mark $\tau_A = \tau_B = 0$, as the clock of the observer on the origin of S' will. Instead, they should mark $\tau_A = -\tau_B \approx vx_B/c^2$, yet that a gigantic observer moving along the two clocks would be capable to guarantee that both he and the clocks A and B started to move simultaneously, for example, for not to have in S' variation of the distance or speed related to him, A and B.

To the justification that we are only against one more simple case of relativity of simultaneity, we need to make understand that is more important than that: in case A be next to the origin of S in $t = 0$ we'll have $\tau_A \approx t$, at least as long A is working correctly (do not consider more the observer lately mentioned), on the other hand, B, supposedly very distant from the origin of S (for example, distant $(3.10^8)^3$ m) and about $1,0$ m/s, will mark all the instant $\tau_B << t$, also working properly. If any experience in these conditions, of gigantic dimensions, took just a brief interval of time to be performed (for example, a single $1,0$ s), in any moment of the experience we would have $\tau_B \approx \tau_A$ (for example, τ_B at the end of the experience, $\approx \tau_A$ at the beginning of the experience), much less the expected result $\tau_B = \tau_A \approx t$. In other words: during the execution of the experience there would not exist the "common time" to A and B mentioned by Einstein.

To the justification that this happens because we treat of great distances, and for great distances the information (of light) takes more time to get to the observers (to its clocks), it will be needed to justify the following: why in a referential considered fix all the synchronous clocks show times in common (hours, minutes, seconds, etc.), whatever the distance between them, even from the order of c^2 or more, but for slow speeds, for example, 0,5 or 1,0 m/s, speeds of a pedestrian in a walk, there can be so much difference in the times marked by a clock considered fix and other in movement with these speeds, based on the Lorentz's transformations? And why this difference on the time rises how farther we are from the origin of the fix referential, even for a value of t in the Lorentz's transformations? This does not seem natural, nor real, as long as we know that "Physics is an attempt to conceptually grasp reality as something that is considered to be independent of its being observed", in the words of the EINSTEIN (1982, p. 78) himself.

Let us now give a general proof, valid to any distances and velocities ($0 < v < c$). As mentioned, according to S.R., two simultaneous events in a referential are not related to another referential moving in U.R.M. with relation with the first (relativity of simultaneity). If our two clocks A and B start their movement with constant velocity v in relation to S in the instant $t = 0$ of S, in relation to this referential A and B start the movement simultaneously.

Thus, in relation to S' the beginning of the movement of the clock it is not simultaneous, according to S.R.: the clock A starts the movement in $\tau_A = 0$, assuming $x_A = 0$, and B in $\tau_B = -\beta vx_B/c^2$, $\tau_B < \tau_A$ to $x_B > x_A$ and $v > 0$, therefore, before A. Here we are using for τ_A, x_A, τ_B e x_B and x_B the same meanings used in (6) e (7).

Now, if the relation to S' the clock B starts the movement before A, in this referential the distance between A and B would increase in function of time, until remains constant, when A started to move, what does not happen, because the distance between then is the same during the whole movement (measuring βx_B in relation to S').

Furthermore, the speed of B in relation to A equals zero, both in relation to S' and to S, therefore, there is no initial distance between B in relation to A, so we have got to a contradiction: events non-simultaneous by definition become simultaneous by logic deduction.

Similarly, we can offer a proof to the instant of stopping. If in relation to S the clocks A and B stop simultaneously the movement of constant velocity v, suppose that, in the instant $t = tf$, in relation to S' do not stop the move simultaneously in relation to the S.R.

If in relation to S' they do not stop simultaneously the movement of one has stopped the move in S' first. As $\tau_B < \tau_A$ assuming $x_B > x_A$ e $v > 0$, B has stopped the movement before A, in relation to S' (if in S they start and stop the movement simultaneously, in S' the clock B starts and stops to move before A). Then, in relation to S', after the stop of B the clock A has approached B, until the moment that A also stopped its movement. But the position of A in relation to S' is equal to $\xi A = 0$ during the whole movement $x_A = 0$, while B's is $\xi_B = \beta x_B$, so, keep between each other, since $t = 0$ until $t = tf$ of S, the same distance $\Delta\xi = \xi_B - \xi_A = \beta x_B$ in S', not having the expected final approach from A in direction to B. Beyond that, the speed of B in relation to A, in S' and S, is equal to zero during the whole movement, and not another speed, how it should be so that A could approach B.

Ergo, we have got to a contradiction.

It is easy to extend the two previous proofs to the case of negative velocities. If $-c < v < 0$ e $x_B > x_A$ we have $\tau_A < \tau_B$ so it will be the clock A that starts or stops the movement before B. The contradiction occurs once again because there is no variation of the distance or of the relative speed between A and B, nor in relation to S, nor in relation to S', even using the S.R.

III – Constancy of the speed of light

Also to any speeds and distances, let us analyze an idealized experience, prepared to verify if the speed of light depends or not on the speed of the observers (clocks or chronometers). We will use both space contraction and time dilation, according to the S.R.

In order to avoid the contradictions shown in the last section, let us admit that two events, simultaneous in relation to S, considered fix, are also simultaneous in relation to S', the referential that moves with constant velocity v in relation to S. Our "new" Lorentz's transformations will become

$$\tau = t/\beta;$$
$$\xi = \beta(x - vt);$$
$$\eta = y;$$
$$\zeta = z.$$

Assume that, so, two chronometers A and B, fix over an object that moves with constant speed in relation to the ground, the ground being considered fix. Being A and B fixed over the object, for example a treadmill, will also move with speed v in relation to the ground.

With the treadmill stopped we will synchronize the two chronometers zeroing its markings, and will remain zeroed until the treadmill moves. An electronic device, located in same distance from the two chronometers, will be the responsible to allow the running of both chronometers and treadmill, as soon as the chronometers start to work. From this moment our chronometers should mark the time of an entirely natural way, assuming that both work normally when fixed.

A ray of light will depart from A in direction to B in the moment the treadmill starts to move, and based on the register of chronometer B when the arrival of the light in B we will calculate the speed of light in relation to the treadmill, knowing that in relation to the ground its speed is c.

If the distance between A and B measures L when the treadmill is in movement, measured in relation to the fix referential (ground), it will measure βL in relation to the moving referential (treadmill). Thereby we are admitting the contraction of space, or Lorentz's.

In relation to the fix referential, the light will take an interval f time $tf = L/(c - v)$ to complete its course, therefore, in relation to the treadmill it will take $\tau_f = t_f/\beta = L/[\beta(c - v)]$, being this the expected indication on chronometer B when the light reaches it. This way we are admitting the time dilation.

The speed of light γ in relation to the treadmill will so be $\gamma = \beta L/\tau_f = c^2/(c+v)$, $\gamma < c$ to $v > 0$, in other words, the speed of light did depend on the speed of the observers (clocks or chronometers), contradicting the second postulate of the S.R., although we have used both space contraction and time dilation. Thus, one more time, the S.R. brought us to a contradiction.

See that if it was $\tau_f = t_f/\beta - \beta\, vL/c^2$, obeying (2) and according to (7), we would obtain $\gamma = c$, the expected independence of the speed of light in relation to the speed of the observers, however, this would imply that our chronometers could register negative times (just notice in (2), (6) or (7) that is possible $\tau < 0$), what is an absurd from the experimental point of view, even that accepted or explained in theory.

Every chronometer in proper functioning, moving or not, show only positive and crescent times , without discontinuity in its markings that could skip from 0 to $-\beta\, vL/c^2$ and go back after some time to 0 again, continuing its crescent marking. Another argument against the using of this last expression to τ_f is that our chronometers need to previously know the distance between them, or in which point of the fix referential are in $t = 0$ (according to (6) and (7)), something very artificial to be accepted. Thus, both A and B shall register in function of t the instant $\tau = t/\beta$, because this is the more reasonable indication from the experimental point of view, admitting valid the time dilation.

IV – Relativistic Time and Galileo Transformations

Independently of the acceptation of the previous conclusions, sections II and III, what can't be forgot to be understood is that the assertion that Lorentz's transformations are reduced to Galileo's transformations at low speeds is not true. To low speeds, but great distances, the transformation (2) is reduced to

$$\tau \approx t - vx/c^2, \qquad (10)$$

different from the respective Galileo transformation, deserving correction what already told us the very creator of the S.R. and one of his most close contributors (EINSTEIN & INFELD, 1980, pp. 157-158): "We can only expect discordance between the experience and the classic transformation with speeds that are close to the speed of light. Only in cases of very high speeds the Lorentz's transformations can be put to the test." (...) "This more general theory does not contradict the classic transformation and the classic mechanics. On the contrary, we go back to the old concepts as a limitative case when the speeds are low."

Without Einstein's partnership, INFELD (1950, pp. 56-57) wrote:

"To slow speeds there is no difference between the Galileo's transformation and Lorentz's".

And without Infeld's partnership, in a more advanced text, EINSTEIN (1999, p. 34) wrote:

"Galileo's transformation is obtained from Lorentz's equating on this last one the speed of light c to an infinitely great value."

This time it is not brought to consideration that if it is possible to make the speed of light infinitely great (we would be in an hypothetic universe, with the allowing of infinite energy) so it is also possible to make velocity v to tend to the infinite, keeping the condition $|v|/c < 1$ (the overall energy $E = mc^2$ will diverge due to the value of c, but not due to the value of the relativistic mass m), and yet the Lorentz's transformations are not reduced to Galileo's transformations (in the case of time specifically).

As an example, let us see the particular case $v = c/n$, $n > 1$ and finite. From (2) we have

$$\tau = n/(n^2-1)^{1/2}\ (t - x/(nc)).$$

If $c \to \infty$, to every finite x, we have

$$\tau \to n/(n^2-1)^{1/2}\ t,$$

instead of the expected result $\tau \to t$.

To the position ξ, effecting the reason between the respective transformations of Lorentz and Galileo, assuming $v = c/n$, $n > 1$, finite n, and $c\ n$, $n > 1$, we find

$$\xi Lorentz/\xi Galileo = \beta = n/(n^2-1)^{1/2},$$

different from the expected result $\xi_{Lorentz}/\xi_{Galileu} \to 1$.

Many college books also need correction, for example:

1) KITTEL *et al* (1973, p. 332): "This is Lorentz's transformation. It is linear in x and t; reduces to Galileo transformation to $V/c \to 0$";

2) EISBERG (1979, p. 23): "It is seen that Lorentz's transformation is reduced to Galileo's transformation when $v/c << 1$";

3) TIPLER (1981, p. 17): "The equations of the classic transformation must so be modified to become consistent with Einstein's postulates; but must be reduced to the classic equations when v is much lower than c.";

4) HALLIDAY & RESNICK (1984, p. 318): "The Newtonian mechanics reveals itself as a particular case of the S.R. in the limit of slow speeds. In fact, a S.R. test consists in allow $c \to \infty$ (in which case, $v << c$ is always valid) and find that derive the corresponding formula of the Newton Mechanics."

Even acclaimed authors, winners of the Nobel prize, committed similar mistakes, for example:

1) BORN (1965, p. 237): "Particular interest attaches to the limiting case in which the velocity v of the two systems becomes very small in comparison with the velocity of light. We then

arrive directly at the Galileo transformation (formula (29), p.74). For if v/c can be neglected in comparison with 1, we get from (70) $x'=x-vt, y'=y, z'=z, t'=t.$";

2) LANDAU & LIFSHITZ (1979, p. 133): "En (36,3) se ve claramente que cuando en el limite $c \rightarrow \infty$ se pasa a la mecânica clásica, las fórmulas de la transformación de Lorentz se convierten en la transformación de Galileo.";

3) SEGRÈ (1980, p. 87): "Lorentz tinha descoberto uma transformação de coordenadas, a famosa transformação de Lorentz, que deixa as equações de Maxwell invariantes e, quando $v << c$, reduz-se à transformação de Galileu."

Fortunately, not all committed the mistakes previously described: FRENCH (1974, p. 88) says that the Galileo transformation $\tau = t$ is obtained from the respective Lorentz's transformation, making $x << ct$ e $v/c << 1$. NUSSENZVEIG (1998, p.190) follow the same inequalities of French (more exactly, we should have $|v| << c$ and $|vx| << c^2$ (or $vx/c^2 \rightarrow 0$), because French's inequalities fail to $x << 0$ or $v << 0$).

Although they admit $c \rightarrow \infty$ without telling about $v \rightarrow \infty$, LANDAU & LIFCHITZ (1974, p.21), in § 4 from volume 2 of their complete course of Theoretical Physics, get to transformation (10) to small velocities v in comparison with the speed of light, instead using directly Galileo's transformation to the time. As it can be realized, this cases are isolated.

V – Critique of Professor Piza

Professor Piza, very nice person and titular of the Departamento de Física Matemática of the Instituto de Física da USP (Department of Mathematical Physics of the Physics Institute of the University of São Paulo), researcher in Theoretical Nuclear Physics, gently accepted to read a previous version of this article, but does not believe to have any contradiction in the S.R.

Said that this article contains a subtle Relativity problem, but probably there is solution within the theory. He could not, however, formalize an explanation or proof of what is wrong (that Prof. Piza forgive me if I am not being true in the reproduction of his words or misunderstood his explanations).

Regarding the comments and examples I gave before the general proof (section II), when I treated about the low speeds and great distances, mentioned four problems, criticism or suggestions that could be useful:

1.1) cone of light and causality;
1.2) great distances;
1.3) Lorentz contraction;
1.4) intuition.

In relation to the general proof (thus far, general proofs because I analyzed the simultaneous beginning of the movement, simultaneous ending of the movement and negative speeds), mentioned three problems:

2.1) transient;
2.2) the twins paradox;
2.3) variation of the distance between the clocks.

Let us answer to his critique and comments, thus far:

1.1) Cone of light and causality

In LANDAU & LIFCHITZ (1974, pp. 10-16), on § 2 from volume 2 of their complete course of Theoretical Physics, it is exposed the concept of interval between events and explains what is called by line of universe and cone of light, terms already used by Minkowski in 1908.

The examples of low speeds and great distances belong, to small values of the time t, to the region of absolute distance of the light cone, which intervals are of the space type, and necessarily there is no fluke relation between an event O in ($x=0$, $t=0$) and an event E in (x, t), $|t| < |x|/c$, because the light would take more time than $|t|$ to go to the origin O to the position x (in other words, the transmission of the interaction should possess speed higher than the light speed). However, it is possible to find in this region a referential S' such the event E be either previous, simultaneous or posterior to the event O, and on this referential S', O and E will always happen in distinct points, if $(x, t) \neq (0, 0)$.

After this brief exposition, to a two-dimensional version of the light cone, I answer that in any moment said to have any fluke relation between the clocks A and B, although the movement of both have the same cause: the movement in relation to S from the referential S', support of the clocks, from the instant $t = 0$.

1.2) Great distances

I used, at first, great distances, because only for great distances is evident that in the limit of low speeds Lorentz's transformations to the time do not reduces to the previous Galileo transformations, thus exposing a few explored aspect of the S.R. Beyond that, in the S.R. the Universe can be considered plan and infinite, therefore not having any limit to the distances involved.

1.3) Lorentz contraction

As for slow speeds ($0 < v << c$) the contraction of Lorentz can be despicable when proportionally compared to the same distance non-contracted, even for great values for x (for example, even for $x = x_B = (3.10^8)^3\ m$), it is clear the in an interval of time exactly equal to $\beta\, x_B/c = (x_B/c)/\beta$ is on the order of x_B/c.

A numerical example: to $\alpha = 3.10^8$, $c = \alpha\ m/s$, $x_B = \alpha^3\ m$ and $v = 1\ m/s$ we have $\beta\, x_B/c \approx (1 + ½\ v^2/c^2)\ x_B/c = (\alpha^2 + 1/2)\ s$, that is given on the order of $x_B/c = \alpha^2\ s$.

1.4) Intuition

Prof. Piza said I was wrong all the times I used the intuition. And I didn't use the intuition. I have just mentioned that "From (8) it would be possible to conclude, through the gumption and our intuition, that, indeed, our clocks A and B cannot be synchronous, nor in relation to S, nor in relation to S', because the difference pointed in (8) is never equal to zero and what is expected, *a priori*, from synchronous clocks is that they mark the same time, without any differences." In the case, the common sense would come to confirm the intuition.

Once I opted for the discursive method, obviously, deductive, to expose this work, made through "a series of acts, from a series of successive efforts", in the words of Manuel Garcia Morente (Fundamentos de Filosofia, pg. 48, ed. Mestre Jou, 1980), it stays very clear the non-using of the intuition.

2.1) Transient

The S.R. would not be the current Einstein's S.R. if there was any transient to be considered in its transformations, but the theory analyzed here is Einstein's S.R. and not another.

Yet that there was a transient in the supposedly "true" transformations of Lorentz (an example is given in the answer 2.3), originated from a molecular effect or due to forces of friction and inertia, and affecting on the contraction of space or dilation of time, its values could not be applicable or relevant, otherwise the S.R. would not receive experimental confirmations (time dilation, space contraction, Doppler effect, momentum and relativistic energy, etc.).

Furthermore, when EINSTEIN (1905, § 4) applies the Lorentz's transformations to rigid bodies in move and to moving clocks do not use any transient, the same occurring with other authors, nor when deduced the transformations, thus I do not commit any mistake in despise supposed transients of insignificant values, experimentally unknown and that do not make part of the theory.

2.2) The twins paradox

The study of the twins paradox make us believe that it is not the movement of the axis of an hypothetic referential arbitrarily located the responsible to alter the march of a clock, but the movement of the clock. If a clock (or chronometer) has a certain march in relation to a inertial referential S, where it is found fix, and starts to move with constant speed in relation to this referential, so its new march is determined by the speed that it has in relation to S, related to the march it had before. This can also be deducted from EINSTEIN (1905, § 4).

The explanation usually given to the twins paradox refers to accelerations from the change of referential, when the direction of velocity v is altered to $-v$, however, it is not calculated for how long these accelerations act nor what effect it produces in the time dilation or any variation on it, while EINSTEIN (1905, § 4) uses for the dilation of time of a clock in curvilinear motion of constant linear speed the same formula used for a clock in U.R.M., without mentioning any effect produced on the time due to centripetal or centrifugal accelerations.

Although the analysis of the twins' paradox deserves an article dedicated exclusively to it, here it is only needed to clarify that I did not introduced changes of referential, inversions on the direction of velocity, nor long-lasting acceleration, so the analogy with the paradox is not applicable.

2.3) Variation of the distances between the clocks

Finally, Professor Piza believes that the distance between the clocks A and B varies in relation to S', at the simultaneous beginning and simultaneous ending of the movement of both in relation to S.

Now, during the whole movement of any clocks in relation to S, while $v \neq 0$, v is constant, and following Lorentz transformations, what can be the position of A in S' if not $\xi_A = 0$ (to $x_A = 0$), and what can be the position of B in S' if not $\xi_B = \beta\, x_B$? In other words, what can be the distance between A and B if not the constant distance $\Delta\xi = \beta\, x_B$?

It is true that we assumed implicitly a discontinuity in the position of B, caused by the instant variation of its position xB in S to $\xi_B = \beta\, x_B$ in S', on the instant $t = 0$, occurring another discontinuity in the instant of stop $t = tf$, but this does not make part of the theory: the constant speed v comes to exist on the instant $t = 0$, when the origins of the referential S and S' are common, and ceases to exist on the instant $t = tf$, instant here considered as the one of referential stop S'. In the S.R. Lorentz's contraction occurs immediately at the beginning of the movement, *i.e.*, as long as it is needed to say of the referential S' ≠ S, $v \neq 0$.

If it were not so, how could be calculated the contraction of Lorentz in function of time t using only Lorentz's transformation, that are, after all, the first result of the S.R.? Maybe we would need to go back to the theory of the electron from Lorentz that Einstein did not use?

If we change, for example, in (2), (3) e β the constant speed v for the variable speed $v = at$, the constant, $a > 0$, valid transformations to the interval of time $0 \leq t \leq t_0$, $0 < t_0 \approx 0$, would not have discontinuity in the value of position of B in $t = 0$, when there happens to exist the referential S' (to $x = x_B + at^2$ we would have $\xi_B = (1-a^2t^2/c^2)^{-1/2}\, x_B$, an example of transient), but the equation of the waves (amongst others) would not be covariant and would be applied the principle of the constancy of the speed of light, so the S.R. would lose its validity during this brief interval of time. This way it would be needed to find a way to prove the validity of new transformation obtained and that were able to provide which are the correct values to a and t_0. The S.R., indeed, does not solve this problem, due to the substitutions of an inertial referential to an accelerated referential.

After these comments, maybe useful, we need to go back to the essence of the demonstration: one of the clocks would need to stay motionless in relation to S (or S') and only the other to move in S (or S'), what does not happen, because nor ξ_A, nor ξ_B, vary on the time while $v \neq 0$ (nor, by hypothesis, if $v = 0$); this is the base of the contradiction. Do not interest the discontinuities of the position of B on the instants $t = 0$ e $t = t_f$, nay, the "appearance" and "disappearance" of the position of B in the referential S'.

If we draw a graph $\tau X\, \xi$ from the beginning of the movement in $t = 0$ up to the end of the movement in $t = tf$, the movement of A in S' will be represented in the graph by a line segment in the horizontal position going from point ($\tau = 0$, $\xi = 0$) to point ($\tau = t_f/\beta$, $\xi = 0$), *i.e.*, we will represent the immobility of A in S', and the movement of B in S' will be represented in this graph by another horizontal segment going from point ($\tau = -\beta v x_B/c^2$, $\xi = \beta x_B$) to point ($\tau = t_f/\beta - \beta v x_B/c^2$, $\xi = \beta x_B$), *i.e.*, we will represent the immobility of B in S'. Of course, to be immobile in relation to S' means to be moving with constant velocity v in relation to S.

In this graph, supposing $v > 0$ e $x_B > 0$, where would be represented A during the interval f time $I_1 = \{-\beta v x_B/c^2 \leq \tau < 0\}$ and where would be represented B during the interval of time $I_2 = \{ t_f/\beta - \beta v x_B/c^2 < \tau \leq t_f/\beta \}$ so that we can verify the movement of any of them in relation to the other to the same instant of time τ since according to S.R., the beginning and the ending of the movement of the clocks is not simultaneous in relation to S' if in relation to S they are simultaneous?

It is easy to verify that A and B remains motionless in S' at the interval of time $0 \leq \tau \leq t_f/\beta - \beta v x_B/c^2$, *i.e.*, move with the same speed in relation to S, but, any that were the position of A in I_1 and of B in I_2 (we could define them as the position in S' without the contraction of Lorentz), there is no how to assume that, in relation to S, A remains immobile in I_1 and only B moves or that B remains immobile in I_2 and only A moves.

Remember: A remain immobile in I_1 and B moves in this interval means that there is some value of t in $0 \leq t \leq t_f$, related to some value of τ in I_1, such that A has speed 0 in relation to S and only B has a velocity v, what does not happen, because, by hypothesis, A and B started the movement simultaneously in relation to S. Similar reasoning can be made to the interval I_2.

In other words: there is no how A and B to move in relation to S ($v > 0$, $0 \leq t \leq t_f$), but an observer in S' consider A (or B) in rest in relation to S and only B (or A) in movement in relation to S. Then, I believe that the proofs shown here thus far are correct.

VI - Conclusion

It stands to reason draw the attention to so much false statements about the Lorentz transformations (section IV), notoriously with relation to the relativistic time. The list of quotes could be increased, without adding anything of interest.

We saw that it is not so immediate the reduction of Lorentz transformations to Galileo transformations, and French and Nussenzveig, from the mentioned authors, are the ones which better teach us to do that, if we limit to positive speeds and positions.

Unfortunately, the textbooks often used in our universities do not treat this subject on the right way. For example, allow $c \rightarrow \infty$, but deny $v \rightarrow \infty$, it gets to be a counter-sense, as we all know that the speed of light is finite, as well as all the energy we meddle with. There are no reasons n Physics to allow infinite luminous energy (generated by photons of energy $E = hc/\lambda$, assuming the constant of Planck h and the wavelength λ finite and non-null), infinite electric forces (with module $F = 10^{-7}\, c^2\, q_1\, q_2 / r^2$, assuming non-null electrical charges q_1 and q_2 on the vacuum), overall infinite energies (with value $E = mc^2$, supposing the non-null relativistic mass m), but to prohibit infinite kinetic energy ($K = (\beta - 1)\, m_0 c^2$ is infinite if the speeds v e c, $|v|/c < 1$, are infinite and $m_0 > 0$. Evidently, the condition $|v|/c < 1$ was used on this work to do not make imaginary, nor infinite, the value of β).

This article also intended to show something more important: The S.R., on itself, contains contradictions from the most fundamentals, contrary, once again, to the stated by Einstein.

If two clocks starts or stop simultaneously a movement of constant velocity $v \neq 0$ in relation to a fix referential (S), in relation to the referential in which the clocks are fix (S') one of them has started or stopped the movement before the other, according to the S.R. If so, during certain interval of time there would be a non-null speed of one in relation to the other, with variation of the distance between them, what does not happen, even using the S.R.

But if we admit that simultaneous events in an inertial referential are also simultaneous in relation to another inertial referential moving in relation to the first, even admitting the time dilation and Lorentz contraction we obtain the dependence of the speed of light on the speed of the referential, contrary the second postulate of the S.R. So, one way or the other we find contradictions on the S.R.

Acknowledgements

To teachers of the Institute of Physics of USP A.F.R. de Toledo Piza, for the criticisms of this work, forcing me to make it more rigorous and explanatory, and Henrique Fleming, for the reading the section IV.

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