International Frontier Science Letters
ISSN: 2349-4484, Vol. 3, pp 16-18
doi:10.18052/www.scipress.com/IFSL.3.16

Online: 2015-01-01

# The concept of prime number and the Legendre conjecture

Jamel Ghanouchi

Professor of Mathematics
RIME Department of Mathematics
6 Rue Khansa 2070 Marsa Tunisia

**Keywords :** Primes ; Legendre ; Conjecture ; Proof.

**Abstract**
In this paper, we generalize the concept of prime number and define new primes. It allows to apply the new concept to the Legendre conjecture and to prove it.

## Introduction

The prime numbers are called primes because they are the bricks of the numbers : Each number n can be written as $\prod_j p_j^{n_j}$ where $p_j$ are primes and $n_j$ are integers.
This writing is called the decomposition in prime factors of the number n.
In fact, this definition is a very particular case of a much more general case.
Indeed, if $n_j$ are rationals, everything changes.
Considering that the decomposition in prime factors of an integer n when $n_j$ are rationals $\prod_j p_j^{n_j}$. In this writhing, then, the $p_j$ have no reason to be the same than before and they become a convention. For example, if we decide that 16 is conventionally prime, we have $2 = 16^{\frac{1}{4}}$ and each number can be written according to 16 and its rational exponent instead of 2.
If we decide conventionally that each Fermat number is prime, and it is possible by the fact that they are coprime two by two, then each new prime (new primes=bricks with rational exponents in the writing) replaces anther one in the list of the old primes (old primes=bricks with integral exponents in the writing).
Example: If by convention, the 5th Fermat number $F_5 = 2^{2^5} + 1 = 4294967297 = 641.6700417$ is prime, we can decide that it replaces 641 which becomes compound when 6700417 remains prime or 641 remains prime and it replaces 67004147 which becomes compound.
In all cases, the advantage is that we have a formula which gives for each n a prime. And we can see the primes are infinite.
There is another interesting result: Let Ulam spiral. The Fermat numbers are all situated in the same line.

## The Legendre conjecture

The Legendre conjecture states that there is always a prime number between the squares of two consecutive integers. So $\exists p | n^2 \leq p \leq (n+1)^2 ; \forall n \in N$ where p is prime. What does it become with our new definition ? It remains true ! Effectively :

**Proof :**

We have $(2m)^2 \leq 4m^2 + 1 \leq (2m+1)^2 \leq 4m^2 + 8m + 1 \leq (2m+2)^2$
But we will prove now that (CD : common divisor)

$CD(4m^2+1,4k^2+1)=5; m \neq k$

$CD(4m^2+8m+1,4k^2+8k+1)=3; m \neq k$

$GCD(4m^2+1,4p^2+8p+1)=1$

It is true for the two first assertions and for the third, let us suppose d dividing both the two equations, we have

$d \mid 4m^2+1; d \mid 4p^2+8p+1$ hence

$d \mid 4p^2+8p+1-4m^2-1 \Rightarrow d \mid p^2-m^2+2p$

$d \mid 4p^2+8p+1+4m^2+1 \Rightarrow d \mid 2p^2+2m^2+4p+1$

$=2(p+m)^2-2pm+4p+1$

$d \mid (p-m)(p+m)^2+2p(p+m)$

$d \mid 2(p-m)(p+m)^2-2pm(p-m)+4p(p-m)+p-m$

$\Rightarrow d \mid -2pm(p-m)+p-m=-2p^2m+2pm^2+p-m$

$\Rightarrow d \mid -4p^2m-8pm-m+8pm+m+4pm^2+p-p+p-$ [illegible]

$\Rightarrow d \mid pm \Rightarrow d \mid p-m \Rightarrow d \mid m \Rightarrow d=1$

because d is odd. And

$m=5(k+k')\pm 1$

$p=5(k-k')\pm 1 \neq m$

$\Rightarrow 4m^2+1=5(20(k+k')^2 \pm 8(k-k')+1)$

$4p^2+1=5(20(k-k')^2 \pm 8(k-k')+1)$

And

$m=3(k+k')+2$

$p=3(k-k')+2 \neq m$

$4m^2+8m+1=(2m+$ [illegible] $=(6(k+$ [illegible] $)+6)^2-3=3w$

$4p^2+8p+1=(2$ [illegible] $2)^2-3$ [illegible] $6(k-k')+6)^2-3=3w'$

And $4m^2+1$ [illegible] $4m$ [illegible] $+8m+1$ and can be taken primes simultaneously with our definition of the primes [illegible] for [illegible] mple, the first $4m^2+1$ divisible by 5 is for m=4, and then $4m^2+$ [illegible] $=6$ [illegible] $13.5=$ [illegible] $=65.13^{-1}$ is no more prime and 65 is prime, the second is for m=6 and then $4$ [illegible] $=29.5 \Rightarrow 29=145.5^{-1}=145.13.65^{-1}$ is no more prime and 145 is prime, etc… until in[illegible]ity. By the same way, the first $4m^2+8m+1$ ivisible by 3 is for m=2 and then $4m^2+8m+1=33=11.3 \Rightarrow 3=33.11^{-1}$ is no more prime and 33 is prime, etc… until infinity ;

but $(2m)^2 \leq 4m^2+1 \leq (2m+1)^2 \leq 4m^2+8m+1 \leq (2m+2)^2$

The Legendre conjecture is true for the news definition of the primes, we have proved it.

**Back to the traditional definition of primes**

Let now the Legendre conjecture, we have found that for the new definition

$(2n)^2 < p' < (2n+1)^2 < q' < (2n+2)^2$

Where p', q' are new primes. If Legendre conjecture is false for old and true for new primes. There exists x, for witch (p' : new prime, p : old prime, q' : new prime)

$$p' - ux^2 - (1-u)(1+x)^2 = 0; 0 < u < 1$$

$$\exists x; p - u'x^2 - (1-u')(1+x)^2 = b; \forall p; 0 < u' < 1; b \neq 0$$

$$a \neq 0$$

$$a^2 p' - u(ax)^2 - (1-u)(a(1+x))^2 = 0 = q' - u''(ax)^2 - (1-u'')(a(1+x))^2$$

$$a^2 p - u'(ax)^2 - (1-u')(a(1+x))^2 = a^2 b$$

$$= a^2 p - q' + (u'' - u')a^2(x^2 - (x+1)^2)$$

$$\Rightarrow -\frac{q'}{a^2} = b - p' + (u' - u'')(x^2 - (x+1)^2) \in Z; \forall a$$

$$a = q' \Rightarrow q' = 1$$

Impossible ! It means that for all x, there exists p old prime number for which b=0 and the conjecture is also true for the old definition !

**Conclusion**

We have generalized the definition of the primes and proved the Legendre conjecture for the generalization of the definition of primes, a reasoning which leaded to absurdity allowed to prove that this conjecture is true for the old definition too.