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The ways of finding uncountable set solutions for equations of $pA^a \pm qB^b \equiv rD^c$. (elementary aspect)

PROF. DR. K. RAJA RAMA GANDHI[1], REUVEN TINT[1], MICHAEL TINT[2]

Resource person in Math for Oxford University Press and Professor at BITS-Vizag[1]
Number Theorist, Israel[1]
Software Engineer, Israel[2]
Email: editor126@gmail.com, reuven.tint@gmail.com, tintmisha@gmail.com

Abstract. Now we shall give the variants of uncountable set solutions that are a prime numbers and also a not coprime numbers of

$$pA^a \pm qB^b \equiv rD^c$$

, as well as, the version of anti-solution for the Pillai's conjecture.

1.1. Algorithm for finding uncountable set solutions in positive integers of the equation

$$u^{a_1}A^a + \vartheta^{b_1}B^b \equiv r^{c_1}D^c \ [\mathbf{1}]$$

for arbitrary of pairwise coprime a, b, c and
arbitrary natural $a_1, b_1, c_1, u, \vartheta, r$.

1.1.1. We have the identity

$$u_0(\vartheta_0 - r_0) + \vartheta_0(r_0 - u_0) \equiv r_0(\vartheta_0 - u_0) \quad [\mathbf{2}],$$

where $u_0, \vartheta_0, r_0 -$ are arbitrary coprime$(u_0, \vartheta_0, r_0) = 1$
natural numbers, such that

$$u_0 = u^{a_1}; \quad \vartheta_0 = \vartheta^{b_1}; \ r_0 = r^{c_1}$$

and

$$\vartheta^{b_1} > r^{c_1}; \ r^{c_1} > u^{a_1} \ [\mathbf{3}];$$

$$(x = \vartheta^{b_1} - r^{c_1}) + (y = r^{c_1} - u^{a_1}) \equiv (z = \vartheta^{b_1} - u^{a_1});$$

$$u^{a_1}(\vartheta^{b_1} - r^{c_1}) + \vartheta^{b_1}(r^{c_1} - u^{a_1}) \equiv$$

$$\equiv r^{c_1}(\vartheta^{b_1} - u^{a_1}) \quad [\mathbf{4}];$$

1.1.2 . In addition, we have identity

$$[A^a = (x^\alpha y^{qc} z^{mb})^a] + [B^b = (x^{pc} y^\beta z^{ma})^b] \equiv$$
$$\equiv [D^c = (x^{pb} y^{qa} z^\gamma)^c],$$

if $x + y = z$.

The values of the parameters of all the exponents found from the equations:

$$\alpha a - pbc = 1$$
$$\beta b - qac = 1$$
$$\gamma c - mab = 1$$

All infinite set of solutions of these equations, supplemented by one

$(a = b, c) = 1$ and

$$x + y = z,$$

where x, y -are arbitrary natural numbers, give solutions of the equations $[\mathbf{1}]$ are not coprime:

It follows that,

$$u^{a_1}A^a + \vartheta^{b_1}B^b \equiv r^{c_1}D^c \quad [\mathbf{5}],$$

1.2.2. If

$$u_0 = u^n;\ \vartheta_0 = \vartheta^n;\ r_0 = r^n,$$

where $n -$ is arbitrary natural number,

$$u^n(\vartheta^n - r^n) + \vartheta^n(r^n - u^n) \equiv r^n(\vartheta^n - u^n),$$

$$(x_1 = \vartheta^n - r^n) + (y_1 = r^n - u^n) \equiv (z_1 = \vartheta^n - u^n),$$

$$A_1 = x_1^{\alpha} y_1^{gc} z_1^{mb};\ B_1 = x_1^{pc} y_1^{\beta} z_1^{ma};$$

$$D_1 = x_1^{pb} y_1^{qa} z_1^{\gamma}$$

$(x_1 + y_1 = z_1)$, then

$$u^n A_1^a + \vartheta^n B_1^b \equiv r^n D_1^c \quad [\mathbf{6}]$$

1.2.3. <u>Example.</u>

1)

$$(x_1^{\alpha} y_1^{qc} z_1^{mb})^a + (x_1^{pc} y_1^{\beta} z_1^{ma})^b \equiv \left(x_1^{pb} y_1^{qa} z_1^{\gamma}\right)^c.$$

Suppose,

$$a = 4; b = 5; c = 7; (4,5,7) = 1.$$

Then,

$$\alpha \times 4 - p \times 5 \times 7 = 1$$

$$p = 1;\ \alpha = 9$$
$$\beta \times 5 - q \times 4 \times 7 = 1$$
$$q = 3;\ \beta = 17.$$

$$\gamma \times 7 - m \times 4 \times 5 = 1$$
$$m = 1; \gamma = 3$$

$$(x_1^9 y_1^{21} z_1^5)^4 + (x_1^7 y_1^{17} z_1^4)^5 = (x_1^5 y_1^{12} z_1^3)^7$$

,if

$$x_1 + y_1 = z_1$$

2) Similarly, $u = 2;\ \vartheta = 5; r = 3, n = 3.$

$$x_1 = \vartheta^n - r^n = 5^3 - 3^3 = 98$$
$$y_1 = r^n - u^n = 3^3 - 2^3 = 19$$
$$z_1 = \vartheta^n - u^n = 5^3 - 2^3 = 117$$

$$2^3(98^9 \times 19^{21} \times 117^5)^4 + 5^3(98^7 \times 19^{17} \times 117^4)^5 =$$

$$= 3^3(98^5 \times 19^{12} \times 117^3)^7$$

3)

$$\cancel{98^{35} \times 19^{84} \times 117^{20}} \times (2^3 \times 98 + 5^3 \times 19) =$$

$$= \cancel{98^{35} \times 19^{84} \times 117^{20}} \times 3^3 \times 117$$

$$784 + 2375 = 3159$$

4)

$$98 + 19 = 117$$

5)

$$2^3 \times 98 + 5^3 \times 19 = 3^3 \times 117$$

P.S. Using the [**3**] it follows that

1)
$\vartheta^{b_1} > r^{c_1};\ b_1 \times Ln\,\vartheta > c_1 Ln\, r$ and $b_1 > c_1 \times \frac{Ln\, r}{Ln\,\vartheta};$

2)
$r^{c_1} > u^{a_1};\ c_1 \times Ln\, r > a_1 Ln\, u$ and $c_1 > a_1 \times \frac{Ln\, u}{Ln\, r};$

§ 2

Version of anti-solution for Pillai's conjecture.

2.1. Pillai's conjecture concerns: "For fixed positive integers A, B, C the equation

$$Ax^m + By^n = C$$

has only finitely many solutions (x,y,m,n –are positive integers)". Anti-solution presented in other notations.

2.1.1. Let

$$Mx - Ny = z\ [\mathbf{7}],$$

M, N, x, y - are arbitrary natural numbers.

Then,

$$Mx^{\alpha a - pb} - Ny^{\beta b - qa} = z\ [\mathbf{8}].$$

Here,

$$\alpha a - pb = 1 \;\; (\alpha a, pb) = 1$$
$$\beta b - qa = 1 \; (\beta b, qa) = 1.$$

Multiplying $[\mathbf{8}]$ by

$t = x^{pb} y^{qa}$,

we get

$$M(x^{\alpha} y^{q})^{a} - N\left(x^{p} y^{\beta}\right)^{b} = z x^{pb} y^{qa} \; [\mathbf{9}].$$

2.1.2. We fix the parameters $M, N, x, y, q, a, p, \beta, b$.
Indeed,

$$a = a_1 \times a_2 \times \ldots k \times \ldots a_i < \infty$$
$$b = b_1 \times b_2 \times \ldots t \times \ldots b_j < \infty$$

2.1.3. The result is

$$M[(x^{\alpha} y^{q})^{\prod_{i=1, i \neq k}^{i<\infty} a_i}]^{a_k} - N[(x^{p} y^{\beta})^{\prod_{j=1 j \neq t}^{j<\infty} bj}]^{b_t} \equiv$$
$$\equiv z x^{pb} y^{qa} \;\; [\mathbf{10}],$$
$$1 \leq k < \infty; 1 \leq t < \infty$$

2.1.4. Therefore, from $[\mathbf{10}]$ it follows that, equation

$$Ax^{m} - By^{n} = c$$

has an infinite set of solutions for given values

$$A = M; B = N$$

, fixed value

$$C = z x^{pb} y^{qa}$$

and arbitrary $\left(m = a_i, n = b_j\right) = 1$.
Defined in this case "C" does not always coincide with the fixed values, but may coincide with the fixed set.

§ 3

3.1.
Variants of finding uncountable set coprime solutions (all in each case) equations

$$mA^{a} \pm qB^{b} = rD^{c}$$

(elementary aspect)
3.1.1. With respect to

$$B^{b} \pm A^{a} = D^{2}$$

$(2^3 + 1 = 3^2;\; 3^4 - 2^5 = 7^2; 2^9 - 7^3 = 13^2;$
$7^3 - 3^5 = 10^2;\; 15^3 + 7^4 = 76^2$ and etc.
,then

$$A^{a}(B^{b}k \pm N) - B^{b}(A^{a}k - N) \equiv ND^{2}$$

3.1.2. If

$$N = rD^{c-2}$$

where k, r -are arbitrary natural numbers, including $1, c > 2$,

$$A^a(B^bk \pm rD^{c-2}) - B^b(A^ak - rD^{c-2}) \equiv rD^c\ [\mathbf{11}].$$

Compare with the identity

$$x^a(y^bk - z^c) + y^b(z^c - x^ak) \equiv z^c(y^b - x^a)$$

,where x, y, z, a, b, c, k -are arbitrary positive integers, including 1.

Example:

$$15^3 + 7^4 = 76^2$$

$$B^b = 15^3;\ A^a = 7^4;\ D^2 = 76^2$$

$$k = 2; c = 3; r = 3.$$

Therefore,

$$7^4 \times (15^3 \times 2 + 3 \times 76) - 15^3(7^4 \times 2 - 3 \times 76) = 3 \times 76^3.$$

$$7^4 \times 6978 + 15^3 \times 4574 = 3 \times 76^3$$

3.1.3. Let using $[\mathbf{11}]$

$$B^bk + rD^{c-2} = P\ [\mathbf{12}]$$

,where $(B, D) = 1$ –are coprime,

$P -$ is arbitrary positive integer.

Then, all the solutions $[\mathbf{12}]$ give

$$k = P(B^b)^{\varphi(D^{c-2})-1} + D^{c-2}t$$

$$r = p\frac{1 - (B^b)^{\varphi(D^{c-2})}}{D^{c-2}} - B^bt,$$

where $\varphi(D^{c-2}) -$ is Euler function, equal to the number of positive integers coprime to D^{c-2} and less D^{c-2};

$t -$is any integer

Then,

$$k = 2(15^3)^{\varphi(76)-1} + 76 \times 3$$

$$r = 2 \times \frac{1 - (15^3)^{\varphi(76)}}{76} - 15^3 \times 3$$

$$\varphi(76) = \varphi(2^2 \times 19) = (2^2 - 2)(19 - 1) = 36.$$

$$\varphi(76) = \varphi(37) = 36.$$

Under the condition of

$$10^3 \div 37$$

gives a remainder 1 , 36 can be replaced by a factor equal to 3 . Therefore, if $p = 2$ then

$$15^3[2 \times (15^3)^{3-1} + 76 \times 3] + 76 \times \left[2 \times \frac{1-(15^3)^3}{76} - 15^3 \times 3\right] = 2.$$

3.1.3.1. If

$$P = A^{n-a}m$$

$$m \times A^n + qB^b = rD^c$$

,where $n > a, c > 2, m$ are arbitrary positive integers.

Thus we have,

$$m \times 7^n + q \times 15^3 = r76^c$$

, where as an example,

$q = 7^4k - r76^{c-2}$; and etc.

References:

[1] SOME ANTI-SOLUTIONS OF THE PILLAI'S CONJECTURE AND PROOF OF FERMAT'S LAST THEOREM. Bulletin of Mathematical Sciences & Applications ISSN: 2278-9634 Vol. 2 No. 4 (2013), p. 47-64 p. 1, § 1, i.1.1.