

## Petrov classification of the Liénard–Wiechert field

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**Abstract.** Newman [1] showed analogies between the Liénard–Wiechert field [2-7] and the Robinson-Trautman (RT) solutions [8-10] of the Einstein gravitational equations. Here we present one more analogy: The Maxwell field produced by a point charge in arbitrary motion has the same algebraic type as the RT metrics. This result is possible because the Weert superpotential [3, 6, 7, 11-16] permits to construct an “electromagnetic Weyl tensor” which admits the Petrov classification [8, 17-24] as in general relativity.

### 1. - Introduction

In this work we shall use the notation of [3, 4, 12-16].

4.1. In Minkowski space the superpotential  $K_{ijr}$  of Weert [11, 14] for the bounded part  $T_{ar}$  of the Maxwell tensor associated to Liénard–Wiechert (LW) field, has [12, 13, 15, 16] the same algebraic and differential symmetries than the Lanczos potential  $L_{ijr}$  [9, 25-30] in Riemannian spacetimes. The Lanczos spintensor is a generator for the conformal tensor, thus it is natural to construct the “electromagnetic Weyl tensor”:

$$C_{ijbc} = K_{ijb,c} - K_{ijc,b} + K_{bci,j} - K_{bcj,i} + g_{ic}T_{bj} - g_{ib}T_{jc} + g_{jb}T_{ic} - g_{jc}T_{bi} \quad (1)$$

where  $(g_{ab}) = \text{Diag}(1, 1, 1, -1)$   $ab$  is the Minkowski’s metric.

We can apply to (1) the Petrov classification [8, 17, 23, 24] in its tensorial version [19, 22], thus it is possible to show that  $C_{ijbc}$  is algebraically special with types D and II of the Penrose diagram [18], which reinforce the analogies founded by Newman [1] between the LW field and RT solutions [8-10]. Therefore, we say that  $K_{ijr}$  is an “electromagnetic Minkowski spintensor”, which is very important [12, 16] to elucidate the physical meaning of the Weert potential: It is the density of the intrinsic angular momentum of the LW electromagnetic field. Similarly, we propose [28] that the Lanczos potential is connected with the angular momentum of the Kerr’s black hole in general relativity.

### 2.- Algebraic structure of the LW field

The Weert generator is given by [3, 6, 7, 11-16]:

$$K_{ijr} = \frac{q^2}{4} w^{-4} \left[ w^{-1} (4W - 3) (v_i K_j - v_j K_i) K_r + 4(a_j K_i - a_i K_j) K_r + (g_v K_i - g_n K_j) \right] \quad (2)$$

thus (1) permits to obtain the conditions:

$$(C_{ijbc} K_n - C_{ijbn} K_c) K^j K^b = 0 \quad (3)$$

that is

$$C_{ijbc} K^j K^b = -2q^2 w^{-4} (1+W) K_i K_c \quad , \quad {}^* C_{ijbc} K^j K^b = 0 \quad (4)$$

which implies [19, 22] the algebraically special structure of  $C_{ijbc}$ , being  $K_r$  a 2-degenerated Debever-Penrose vector [8, 31]. In particular, the relations (3) and (4) lead to Petrov types D or II as the RT metrics in general relativity, q.e.d. On the other hand, it is interesting to comment that the search [9, 10] of Lanczos potentials for RT spacetimes is similar to the construction [6, 32] of the superpotential  $\tilde{K}_{ij}$  for the radiative part  $\tilde{T}_{ab}$  of LW field, and it is unknown some possible physical meaning for this superpotential.

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