

Mathematical Models for Interrelation of Characteristics of the Developing Defects with the Parameters of Acoustic Emission Signals

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Abstract. Mathematical models and mechanisms of acoustic emission signal generation are presented. It is shown that the reasons of acoustic emission signal origin are related to the local alterations of microstructure of materials and processes of movement of distribution at formation of tensions in solids. It is proved that the origination of signals is based on the analysis of acoustic wave energy released per load cycle and the work of the external forces at elastic deflection.

Introduction

The method of acoustic emission (AE) allows to conduct researches of kinetics by volume structural alteration on the different stages of materials deformation in real time. Physical nature of acoustic origin emission in materials is related to the processes of deformation and destruction. Mathematical models of the data processing in the system of determining defects by AE methods are based on the concepts of continuum environment and the continuum theory of dislocations, where the acoustic emission signal is indicative for a number of processes of the defect structure [1, 2]. These processes are related to the movement of the dislocation [3, 4] the transition from elastic to plastic deformations [5-7], the extension of the dislocation loops [8, 9] and the annihilation of the separate sections of the structure [10, 11].

Since the AE method makes it possible to detect the early stages of cracking prior to catastrophic failure, many attempts have been made to determine the settlement of relations connecting the AE settings to the characteristics of micro- and macro-cracks in the materials [12-14]. This quite challengeable task is not resolved even nowadays.

Dislocations, generated by various sources increase the energy of the local distortions of the crystal lattice. Mechanisms of the internal restructuring of the materials are accompanied by the elastic tension stress wave emission. Growing defect produces a signal that can be remotely detected and it is possible to find its location by processing a difference of wave arrival time to the group of acoustic sensors [15, 16]. According to the intensity of the emission regulations conclusions about the danger of defects are considered [17]. The structural state of the material can appear an important factor influencing on properties and longevity of material other things being equal. Therefore, the basis for the study of the influence of the characteristics of developing defects on the parameters of AE signals in this paper is the energy of the elastic waves, revealed during the loading cycle and the work of the external forces in the elastic deformation of the body.

The main tasks of the paper are:

- development of the mathematical models of explaining mechanism of emission accumulation on the basis of the developing defects energy concepts;
- obtainment of the analytical expressions of interrelation of fracture parameters with the characteristics of AE signals through the functions of the applied tension;
- obtainment of the calculation formulas of the elastic waves energy amount revealed during the loading cycle.

Prognostication of the structural states of polycrystalline and composition materials on the basis of relation between the evolution of imperfect structure and kinetics of accumulation of the damages registered with the method of AE on the different stages of plastic flow and destruction

presents not only a scientific but also technical task and can be the basis for hardware facilities development, algorithms and methods of AE signals recording creation.

The aim of the work is development of the mathematical models of interrelation of characteristics of the developing defects with AE signals.

Materials and Methods

Theoretical foundations of achievement of the results are based on a central tenet of the theory of elasticity, general acoustics and mathematical analyses.

In the process of affiliation between the evolution of imperfect structure with kinetics of accumulation of the tensions, registered by the method of AE, verification the following hypotheses is implemented:

- the hypothesis of continuity, which assumes that material fully fills the volume occupied by it;
- the hypothesis about homogeneity and isotropy, which assumes that properties of material are identical in all directions;
- the hypothesis about the small deformations, change assuming that deformations are small as compared to the sizes of the deformed body. The location of external forces of the relatively separated parts of the body are ignored;
- the hypothesis about the linear dependence between deformations and load, assuming that for most materials the Hooke's law setting straight proportional dependence between deformations and loading is correct.

The basis of the relationship between the characteristics of the developing defects and AE signal parameters kinetics of natural changes in the defective structure at the accumulation of damage can be laid.

Methodological basis of the paper is mathematical description of the crack's model as AE radiating element on the basis of tension and deformation state.

The structure of material at a loading can be presented as a complex system of the possessing properties of non-linearity, nonequilibrium and irreversibility.

The reasons for the destruction of the structures of the production plants operating in aggressive environments, high values, alternating loads is the combined effect of tensile state, defects with sharp edges and a microstructure that is inclined to cracking.

When metal deformation distance between atoms under the influence of the external forces changes, lines and planes through the atom are distorted, due to that the crystal lattice is distorted. Under eliminating the external forces atoms resumed their places in the crystal lattice.

The mechanism of formation of plastic deformation is due to the motion of dislocations through the crystal lattice under shear tensions. At any stage of the structure deformation the total deformation will consist of elastic and plastic components. In both cases of elastic and plastic deformations independent structure-insensitive options are tension σ , strain ε , strain rate and time.

Possible mechanisms of AE generating under cyclic loading are:

- the accumulation of plastic strain;
- stepwise development of the fatigue cracks;
- promotion of the cracks;
- increase in the plastic deformation zone near the crack tip;
- mutual friction of the crack edges.

Total N_S acoustic emission is proportional to the volume of the region within which the tension exceeds some allowable values σ [18].

The mathematical description of a cracked the model as an AE source on the basis of tension-strain state can be described by the following equations [19]:

- the equation of motion:

$$\sigma_{ik} = \rho \cdot \ddot{u}_i ; \quad (1)$$

- communication deformations and displacements (equation Cauchy):

$$\varepsilon_{ik} = \frac{u_{i,k} + u_{k,i}}{2}; \quad (2)$$

- communication tension and strain (Hooke's law):

$$\sigma_{ik} = \frac{E}{1 + \mu} \left(\varepsilon_{ik} + \frac{\mu}{1 - 2\mu} \right). \quad (3)$$

where σ_{ik} – is a components of the tension tensor,
 $u_{i,k}, u_{k,i}$ – is a components of the displacement vector points of the body,
 ε_{ik} – is a components of the tensor of small deformations,
 μ – is a Poisson's coefficient,
 \ddot{u}_i – is an acceleration of the movement.
 ρ – density of materials

In structures with close packing of atoms violation of the correctness of their position creates in the center of dislocation the less dense packing, i.e. an increase of the specific volume of the part of the material takes place. The size and shape of the expansion zone of the motion of the dislocation with speed v vary with frequency ν :

$$v = \frac{v}{b}, \quad (4)$$

where b - is the parameter of lattice in the direction of the dislocation motion.

Higher energy of elastic waves is allowed by the movement of the dislocation cluster.

Based on the hypothesis of small deformations and linear nature of the relationship between tension and load, it is possible to obtain analytical expressions of their relationship with parameters of AE signals.

For a rectangular parallelepiped with the dimensions a, b, c relative to the main directions of deformation under the influence of tension $\sigma_1, \sigma_2, \sigma_3$ comprise:

$$\varepsilon_1 = \frac{\Delta a}{a}; \quad \varepsilon_2 = \frac{\Delta b}{b}; \quad \varepsilon_3 = \frac{\Delta c}{c}. \quad (5)$$

Volume deformation is the ε_V :

$$\varepsilon_V = \varepsilon_1 + \varepsilon_2 + \varepsilon_3. \quad (6)$$

Generalized Hooke's law for isotropic material is expressed by the relations [7]:

$$\varepsilon_1 = \frac{1}{E} [\sigma_1 - \mu(\sigma_2 + \sigma_3)]; \quad (7)$$

$$\varepsilon_2 = \frac{1}{E} [\sigma_2 - \mu(\sigma_1 + \sigma_3)]; \quad (8)$$

$$\varepsilon_3 = \frac{1}{E} [\sigma_3 - \mu(\sigma_1 + \sigma_2)]. \quad (9)$$

or accuracy of the second order:

$$\varepsilon_V = \frac{1 - 2\mu}{E} (\sigma_1 + \sigma_2 + \sigma_3). \quad (10)$$

In the present form Hooke's law is valid not only for the main direction of deformation of x , y , z , but also for any three mutually perpendicular directions.

The potential energy of elastic deformation W_{pot} equals to the external force A , expended on the deformation of the body. In the case of bulk tension state the total work the principal tension σ_1 , σ_2 , σ_3 on respective movements with the relative deformations ε_1 , ε_2 , ε_3 equals:

$$W_{pot} = A = \frac{\sigma_1 \varepsilon_1}{2} + \frac{\sigma_2 \varepsilon_2}{2} + \frac{\sigma_3 \varepsilon_3}{2}. \quad (11)$$

When a solid body deforms not only the volume but also its shape varies. However, in the mechanism of AE signals generation of, this component is not critical, and using the expression for ε_1 , ε_2 , ε_3 , can be gotten as follow:

$$W_{pot} = A = \frac{1}{2E} [\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1 \sigma_2 + \sigma_2 \sigma_3 + \sigma_3 \sigma_1)]. \quad (12)$$

This part of the elastic waves energy and performed work of the external forces in the solid-state strain carries the main burden on the occurrence of AE signals from developing defects.

Each relative deformation ε_1 , ε_2 , ε_3 is the result of the three tensions σ_1 , σ_2 , σ_3 . This fact forms the basis of the proposed model, the relationship of the applied tension to the parameters of AE signals.

Results and Discussion.

Results of the work is generation of analytic expressions which connects the characteristics of the developing defects with AE signal parameters based on the acoustic wave energy analyses released per load cycle and the work of the external forces at elastic deflection.

The presented model assumes that the tension field varies from σ_j to $\sigma_j + d\sigma_j$ and is the only independent parameter. Denoting by \bar{n} , the average value of the emissions savings in the volume V , while increasing the applied tension from θ to σ_0 , we obtain:

$$\bar{n} = \frac{N_\Sigma}{V}. \quad (13)$$

The AE changing density equals:

$$d\bar{n} = \frac{\partial \bar{n}}{\partial \sigma_1} d\sigma_1 + \frac{\partial \bar{n}}{\partial \sigma_2} d\sigma_2 + \frac{\partial \bar{n}}{\partial \sigma_3} d\sigma_3. \quad (14)$$

Values σ_1 , $d\sigma_2$, $d\sigma_3$ can be presented as:

$$d\sigma_j = \frac{\partial \sigma_j}{\partial x_1} dx_1 + \frac{\partial \sigma_j}{\partial x_2} dx_2 + \frac{\partial \sigma_j}{\partial x_3} dx_3, \quad (15)$$

whence

$$d\bar{n} = dx_1 \left(\frac{\partial \bar{n}}{\partial \sigma_1} \frac{\partial \sigma_1}{\partial x_1} + \frac{\partial \bar{n}}{\partial \sigma_2} \frac{\partial \sigma_2}{\partial x_1} + \frac{\partial \bar{n}}{\partial \sigma_3} \frac{\partial \sigma_3}{\partial x_1} \right) + \quad (16)$$

$$+ dx_2 \left(\frac{\partial \bar{n}}{\partial \sigma_1} \frac{\partial \sigma_1}{\partial x_2} + \frac{\partial \bar{n}}{\partial \sigma_2} \frac{\partial \sigma_2}{\partial x_2} + \frac{\partial \bar{n}}{\partial \sigma_3} \frac{\partial \sigma_3}{\partial x_2} \right) + dx_3 \left(\frac{\partial \bar{n}}{\partial \sigma_1} \frac{\partial \sigma_1}{\partial x_3} + \frac{\partial \bar{n}}{\partial \sigma_2} \frac{\partial \sigma_2}{\partial x_3} + \frac{\partial \bar{n}}{\partial \sigma_3} \frac{\partial \sigma_3}{\partial x_3} \right)$$

Taking into account the accumulation of the AE acts as a function of applied tension can be determined from the equation:

$$N(\sigma) = \iiint \frac{\partial \bar{n}}{\partial \sigma_j} \frac{\partial \sigma_j}{\partial x_i} dx_i. \quad (17)$$

The value $\frac{\partial \bar{n}}{\partial \sigma_j}$ depends on the material of controlled products and the tension field expressed in terms of ratio $\frac{\partial \sigma_j}{\partial x_i}$.

The process of fatigue crack propagation is very durable. It continues until the cross-section of the construction material would not be so small that the tension acting in to exceed destructive ones. Then there will be a rapid fragile destruction.

The loading occurs by cycles of a certain period. Within a few load cycles characterized by an initial period of inactivity, a high rate of AE events occurrence comes. At this moment, there is an avalanche-like appearance of AE signals.

The number of the loading cycle identical pulses equals:

$$N = \frac{\omega}{2\pi\beta} \ln \frac{CG}{U_t} \left(\sqrt{\frac{d}{E}} \right) \frac{\Delta k}{1-R} \sqrt{v}, \quad (18)$$

where

- ω – is an AE signal frequency,
- β – is a damping coefficient,
- C – is a proportionality coefficient,
- G – is a amplification coefficient,
- U_t – is a maximum tension needed to run counter AE signals,
- d – is a design thickness in the control zone,
- Δk – is a range of the variation of the tension intensity factor,
- v – is a velocity of crack grow,
- R – is a normalized value of the tension intensity factor.

$$R = \frac{k_{min}}{k_{max}}. \quad (19)$$

The formula show that the amount of energy liberated per cycle, in is defined as follow:

$$W = \left(\sqrt{\frac{d \cdot v}{E}} \right) \frac{\Delta k}{1-R}. \quad (20)$$

Under the complicated conditions of the products, operation fatigue crack arises in the surface layers and then develops deep into the material, forming a sharp incision. Theoretical consideration of the shift phenomenon can be performed under the following simplifications and approximations.

The relationship between the load and the deformation under the shift is similar to the tensions-strain diagram in tensions tests. The case of plane tensions state, when on the verge of a highlighted surface in the form of a square with a side only tangential tension act, is a pure shift. Hooke's law is expressed in pure shift by the relationship of absolute shift ΔS , transverse force Q , the cross-sectional area F and a shift modulus G :

$$\Delta S = \frac{Qa}{GF}. \quad (21)$$

In this setting, the potential energy of deformation under shift is:

$$W_{pot} = \frac{Q^2 a}{2GF}. \quad (22)$$

All the potential energy in the pure shift is consumed only for form changing. Changes in volume at shift deformation equals zero. With regard to the mechanism of the of AE signals generation the situation is quite different. A good correlation between the presence of welds in the longitudinal and transverse cracks with AE signals is detected in [9], but the samples containing voluminous defects hardly radiated AE signals. This indicates that AE signal parameters depend only on the number of the growing cracks. This dynamic development of the defects, but not their configuration is the basis of AE sources generation mechanism.

In pure bending cross sections remain plane and while turning, they become normal with respect to the axis of the curved cracks. Longitudinal deformation lines curve along a circular arc, and the contours of longitudinal lines intersect at right angles.

Changing the tension on the sectional height is subjected to the linear law, that is the maximum tension are in layers with coordinates y_{max} and the minimum are at $y = 0$.

The formula for determining the normal tension coordinate in any stratum of section at the distance y from the horizontal axis in the direction x is:

$$\sigma = \frac{M \cdot y}{J \cdot x}, \quad (23)$$

where M – is a bending moment,
 J – is an axial moment of inertia.

The normal tension at any section point is directly proportional to the bending moment, the distance from the neutral layer and inversely proportional to the axial moment of inertia.

The formula of the potential energy in bending can be applied in the case of developing cracks:

$$W_{pot} = \int \frac{M^2 dy}{2EJ_x}, \quad (24)$$

where J_x – is the moment of inertia concerning the neutral section line (axis x), passing through its center of gravity.

Using the superposition principle the generalized formula for the potential energy of a crack developing in view of shift strain and bending can be.

Reasons of the work significance are based on the establishing of the interrelation between the characteristics of the developing defects with AE signal parameters.

Transferring of findings and theoretical models of the provisions of the tension-deformation of the material on the occurrence of AE signals detects the need for information not only on the magnitude of the principal tensions in a developing crack, but their directions to determine the source of the AE as a tension concentrator.

The study of the mechanism of AE is of great practical interest, since the destruction of the structure of the material in these conditions is particularly dangerous because it occurs under much less tension and limits of stress.

The initial stage of the moving dislocations process develops slowly. Then there is its acceleration with the ending, when the last dislocation annihilates in the cluster. There is a rapid relaxation of the elastic field in homogeneity, which leads to the occurrence of AE.

If the number of elementary events leading to the emergence of elastic waves is large, and the energy released each time is small, AE signals are perceived as a weak noise. The energy state of the body varies slightly, and the probability of the next of the act is practically independent of the previous one.

If the solid body state is far from equilibrium it is possible the avalanche-type process in which during a small period of time a large number of events are involved. The energy of the arising elastic wave can on many orders to surpass the energy of the continuous AE. The number of the amplitude jumps will be smaller the influence of the each of the previous acts on the succeeding will be essential and the emerging AE will be discrete.

Conclusions.

The mechanisms of AE signals occurrence at a cycling loading allows to determine the quantitative interrelations between AE parameters and the defects of the material structure developing while loading. The common basis of such mechanisms is nonreversible character of changes concentrating in the material structure. This leads to the tension redistribution and creation of the terms for generation of AE new volumes of the material.

The presented mathematical models of interrelation characteristics of developing defects with the parameters of acoustic emission signals allow us to explain the mechanism of tension occurrence in the structure of the materials through the potential energy and the work of the external forces expended on the deformation of the body.

The obtained analytical expression of the interrelation parameters of the destruction of the characteristics of AE signals through a function of the applied tension allows to quantify the amount of the emission savings under the material cyclic loading.

The given formulas of the amount of the energy released during the loading cycles may be useful in the development of the algorithms and methods for detecting AE signals.

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