THE GENERALIZATION OF THE CONCEPT OF PRIME NUMBER AND ITS APPLICATION

Jameli Ghanouchi Dr. K. Raja Rama Gandhi

RIME, département de Mathématiques, Tanisia

Professor in Mathematics, BITS-Vizag, India

Abstract. In this paper, we generalize the concept of prime number and define the real primes. It allows applying the new concept to cryptology.

Definition

A real number is compound if it can be written as $\prod_j p_j^{n_j}$ where p_j are primes and n_j are rationals. This decomposition in prime factors is unique. A prime real number or R-prime can be written only as $p=p.1$. Thus we define other real prime numbers like π , e , $\ln(2)$. Of course, it is a convention, because, we can consider π^2 as prime and π will be no more prime. It is equivalent in what will follow.

Thus $\sqrt[q]{p} = p^{\frac{1}{q}}$ is compound. Also $\sqrt[q]{p} + 1 = p^{\frac{1}{q}} + 1$ is prime when p is prime and we have $\sqrt[2]{p} - 1 = (p-1)(\sqrt[2]{p} + 1)^{-1}(\sqrt[2]{p} - 1)^{-1} \dots (\sqrt[2]{p} + 1)^{-1}$ compound for p prime, for example.

Another example: $\sqrt[3]{p^2} - \sqrt[3]{p} + 1 = (p+1)(\sqrt[3]{p} + 1)^{-1}$
 It is $5/2$ that divides 5 not the contrary!

Division of a real by a real
 The GCD of two numbers
 p and q are prime numbers :

$$p \neq q \Rightarrow GCD(p, q) = 1$$

$$nm < 0 \Rightarrow GCD(p^n, p^m) = 1$$

$$mn > 0; m > 0; GCD(p^n, p^m) = p^{\min(m, n)}$$

$$mn < 0; m < 0; GCD(p^n, p^m) = p^{\max(m, n)}$$

$$i \geq n_i \geq 1; GCD\left(\prod_{n=1}^i p_n^{m_n}, \prod_{l=1}^j p_l^{q_l}\right) = \prod_{l=1}^j GCD(p_{n_l}^{m_{n_l}}, p_{n_l}^{q_{n_l}})$$

So a real number y divides a real number x if $GCD(x, y)$ is different of 1.

Theorem

p is prime then

$$\forall a \in R, \exists k \in R; a^p = a + kp$$

Proof of the theorem

$$a.10^{-u} = \sum_{m=0}^{m=\infty} a_m.10^{-m}; a_m \in N$$

$$\exists k, k'; a^p.10^{-pu} = \sum_{m=0}^{m=\infty} a_m^p.10^{-m} + kp = \sum_{m=0}^{m=\infty} (a_m + k'p).10^{-m} + kp = \sum_{m=0}^{m=\infty} a_m.10^{-m} + k''p$$

The probabilities

What the probability that a number between $x+dx$ and x is prime? It is

$$p(x' \in [x, x+dx]) = \frac{d \log(x)}{x} = \frac{dx}{x^2}$$

Effectively

$$\log\left(1 + \frac{dx}{x}\right) = \log(x+dx) - \log(x) = \frac{dx}{x} = d \log(x)$$

And

$$\begin{aligned} p(x' \in [x, x+dx]) &= p(x' \in [0, x+dx]) - p(x' \in [0, x]) = \frac{\log(x+dx)}{x+dx} - \frac{\log(x)}{x} \\ &= \frac{\log(x+dx)}{x} - \frac{\log(x)}{x} = \frac{d \log(x)}{x} \end{aligned}$$

How many primes are there between x and $x+dx$? There are

$$\pi(x) = \int \frac{dx}{d \log(x)} = \infty$$

Let us build real primes P and Q . We have p_i a prime and u_n a sequence.

We know that $p_n = 1 + \sqrt[n]{p_{n-1}}$ is prime. With N enough great, $P = p_N$. Also with another prime q_i and another sequence v_n , we have another real prime with M enough great, $Q = q_M$. As $1 + \sqrt{P}$ is prime and $1 + \sqrt{Q}$ is prime, let $n = \sqrt{P} + \sqrt{Q}$. Let e coprime with n and let $d = kn - e$,

If we have n and e public keys, we crypt M by $M = C + e + kn$ and decrypt it by d and n with $C = M - e + k'n = M + d + k'n$.

Another possibility is to take $n = (P-1)(Q-1)$ and e coprime with n then n and e are public keys and $M = C^e + kn$ then $C = M^d + k'n$.

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