

## NOTES ON BIPOLAR-VALUED FUZZY SUBGROUPS OF A GROUP

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**Abstract:** In this paper, we study some of the properties of bipolar-valued fuzzy subgroup and prove some results on these.

### Introduction

In 1965, Zadeh [12] introduced the notion of a fuzzy subset of a set, fuzzy sets are a kind of useful mathematical structure to represent a collection of objects whose boundary is vague. Since then it has become a vigorous area of research in different domains, there have been a number of generalizations of this fundamental concept such as intuitionistic fuzzy sets, interval-valued fuzzy sets, vague sets, soft sets etc [5]. Lee [7] introduced the notion of bipolar-valued fuzzy sets. Bipolar-valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval  $[0, 1]$  to  $[-1, 1]$ . In a bipolar-valued fuzzy set, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degree  $(0, 1]$  indicates that elements somewhat satisfy the property and the membership degree  $[-1, 0)$  indicates that elements somewhat satisfy the implicit counter property. Bipolar-valued fuzzy sets and intuitionistic fuzzy sets look similar each other. However, they are different each other [7, 8]. We introduce the concept of bipolar-valued fuzzy subgroup and established some results.

### 1. Preliminaries

**1.1 Definition:** A bipolar-valued fuzzy set (BVFS)  $A$  in  $X$  is defined as an object of the form  $A = \{ \langle x, A^+(x), A^-(x) \rangle / x \in X \}$ , where  $A^+ : X \rightarrow [0, 1]$  and  $A^- : X \rightarrow [-1, 0]$ . The positive membership degree  $A^+(x)$  denotes the satisfaction degree of an element  $x$  to the property corresponding to a bipolar-valued fuzzy set  $A$  and the negative membership degree  $A^-(x)$  denotes the satisfaction degree of an element  $x$  to some implicit counter-property corresponding to a bipolar-valued fuzzy set  $A$ . If  $A^+(x) \neq 0$  and  $A^-(x) = 0$ , it is the situation that  $x$  is regarded as having only positive satisfaction for  $A$  and if  $A^+(x) = 0$  and  $A^-(x) \neq 0$ , it is the situation that  $x$  does not satisfy the property of  $A$ , but somewhat satisfies the counter property of  $A$ . It is possible for an element  $x$  to be such that  $A^+(x) \neq 0$  and  $A^-(x) \neq 0$  when the membership function of the property overlaps that of its counter property over some portion of  $X$ .

**1.1 Example:**  $A = \{ \langle a, 0.5, -0.3 \rangle, \langle b, 0.1, -0.7 \rangle, \langle c, 0.5, -0.4 \rangle \}$  is a bipolar-valued fuzzy subset of  $X = \{a, b, c\}$ .

**1.2 Definition:** Let  $G$  be a group. A bipolar-valued fuzzy subset  $A$  of  $G$  is said to be a bipolar-valued fuzzy subgroup of  $G$  (BVFSG) if the following conditions are satisfied,

- (i)  $A^+(xy) \geq \min\{A^+(x), A^+(y)\}$ ,
- (ii)  $A^+(x^{-1}) \geq A^+(x)$ ,
- (iii)  $A^-(xy) \leq \max\{A^-(x), A^-(y)\}$ ,

(iv)  $A^-(x^{-1}) \leq A^-(x)$ , for all  $x$  and  $y$  in  $G$ .

**1.2 Example:** Let  $G = \{1, -1, i, -i\}$  be a group with respect to the ordinary multiplication. Then  $A = \{ \langle 1, 0.5, -0.6 \rangle, \langle -1, 0.4, -0.5 \rangle, \langle i, 0.2, -0.4 \rangle, \langle -i, 0.2, -0.4 \rangle \}$  is a bipolar-valued fuzzy subgroup of  $G$ .

**1.3 Definition:** Let  $A = \langle A^+, A^- \rangle$  and  $B = \langle B^+, B^- \rangle$  be any two bipolar-valued fuzzy subsets of sets  $G$  and  $H$ , respectively. The product of  $A$  and  $B$ , denoted by  $A \times B$ , is defined as  $A \times B = \{ \langle (x, y), (A \times B)^+(x, y), (A \times B)^-(x, y) \rangle / \text{for all } x \text{ in } G \text{ and } y \text{ in } H \}$ , where  $(A \times B)^+(x, y) = \min \{ A^+(x), B^+(y) \}$  and  $(A \times B)^-(x, y) = \max \{ A^-(x), B^-(y) \}$ , for all  $x$  in  $G$  and  $y$  in  $H$ .

**1.4 Definition:** Let  $G$  be a group. A bipolar-valued fuzzy subgroup  $A$  of  $G$  is said to be a bipolar-valued fuzzy normal subgroup of  $G$  if

- (i)  $A^+(xy) = A^+(yx)$ ,
- (ii)  $A^-(xy) = A^-(yx)$ , for all  $x$  and  $y$  in  $G$ .

**1.5 Definition:** Let  $A$  be a bipolar-valued fuzzy subgroup of a group  $G$ . For any  $a \in G$ ,  $aA$  defined by  $(aA^+)(x) = A^+(a^{-1}x)$  and  $(aA^-)(x) = A^-(a^{-1}x)$ , for every  $x \in G$  is called the bipolar-valued fuzzy coset of the group  $G$ .

**1.6 Definition:** Let  $A$  be a bipolar-valued fuzzy subgroup of a group  $G$  and  $H = \{x \in G / A^+(x) = A^+(e) \text{ and } A^-(x) = A^-(e)\}$ , then  $o(A)$ , order of  $A$  is defined as  $o(A) = o(H)$ .

**1.7 Definition:** Let  $A$  and  $B$  be two bipolar-valued fuzzy subgroups of a group  $G$ . Then  $A$  and  $B$  are said to be conjugate bipolar-valued fuzzy subgroup of  $G$  if for some  $g \in G$ ,  $A^+(x) = B^+(g^{-1}xg)$  and  $A^-(x) = B^-(g^{-1}xg)$ , for every  $x \in G$ .

**1.8 Definition:** Let  $A$  be a bipolar-valued fuzzy subgroup of a group  $G$ . Then for any  $a$  and  $b$  in  $G$ , a bipolar-valued fuzzy middle coset  $aAb$  of  $G$  is defined by  $(aA^+b)(x) = A^+(a^{-1}xb^{-1})$  and  $(aA^-b)(x) = A^-(a^{-1}xb^{-1})$ , for every  $x \in G$ .

## 2. PROPERTIES:

**2.1 Theorem:** Let  $A = \langle A^+, A^- \rangle$  be a bipolar-valued fuzzy subgroup of a group  $G$ . If  $A^+(x) < A^+(y)$  and  $A^-(x) > A^-(y)$ , for some  $x$  and  $y$  in  $G$ , then

- (i)  $A^+(xy) = A^+(x) = A^+(yx)$  and (ii)  $A^-(xy) = A^-(x) = A^-(yx)$ .

**proof:** Let  $A = \langle A^+, A^- \rangle$  be a bipolar-valued fuzzy subgroup of a group  $G$ .

Let  $A^+(x) < A^+(y)$  and  $A^-(x) > A^-(y)$ , for some  $x$  and  $y$  in  $G$ .

Now,  $A^+(xy) \geq \min \{ A^+(x), A^+(y) \} = A^+(x)$ ; and  $A^+(x) = A^+(xyy^{-1}) \geq \min \{ A^+(xy), A^+(y) \} = A^+(xy)$ .

Also,  $A^+(yx) \geq \min \{ A^+(y), A^+(x) \} = A^+(x)$ ; and  $A^+(x) = A^+(y^{-1}yx) \geq \min \{ A^+(y), A^+(yx) \} = A^+(yx)$ .

Therefore  $A^+(xy) = A^+(x) = A^+(yx)$ . Hence (i) is proved.

Now,  $A^-(xy) \leq \max \{ A^-(x), A^-(y) \} = A^-(x)$ ; and  $A^-(x) = A^-(xyy^{-1}) \leq \max \{ A^-(xy), A^-(y) \} = A^-(xy)$ .

Also,  $A^-(yx) \leq \max \{ A^-(y), A^-(x) \} = A^-(x)$ ; and  $A^-(x) = A^-(y^{-1}yx) \leq \max \{ A^-(y), A^-(yx) \} = A^-(yx)$ .

Therefore  $A^-(xy) = A^-(x) = A^-(yx)$ . Hence (ii) is proved.

**2.2 Theorem:** Let  $A = \langle A^+, A^- \rangle$  be a bipolar-valued fuzzy subgroup of a group  $G$ . If  $A^+(x) < A^+(y)$  and  $A^-(x) < A^-(y)$ , for some  $x$  and  $y$  in  $G$ , then

$$(i) \quad A^+(xy) = A^+(x) = A^+(yx) \text{ and } (ii) \quad A^-(xy) = A^-(y) = A^-(yx).$$

**proof:** Let  $A = \langle A^+, A^- \rangle$  be a bipolar-valued fuzzy subgroup of a group  $G$ .

Let  $A^+(x) < A^+(y)$  and  $A^-(x) < A^-(y)$ , for some  $x$  and  $y$  in  $G$ .

Now,  $A^+(xy) \geq \min\{A^+(x), A^+(y)\} = A^+(x)$ ; and

$$A^+(x) = A^+(xyy^{-1}) \geq \min\{A^+(xy), A^+(y)\} = A^+(xy).$$

And,  $A^+(yx) \geq \min\{A^+(y), A^+(x)\} = A^+(x)$ ; and

$$A^+(x) = A^+(y^{-1}yx) \geq \min\{A^+(y), A^+(yx)\} = A^+(yx).$$

Therefore  $A^+(xy) = A^+(x) = A^+(yx)$ . Hence (i) is proved.

Now,  $A^-(xy) \leq \max\{A^-(x), A^-(y)\} = A^-(y)$ ; and

$$A^-(y) = A^-(x^{-1}xy) \leq \max\{A^-(x), A^-(xy)\} = A^-(xy).$$

And,  $A^-(yx) \leq \max\{A^-(y), A^-(x)\} = A^-(y)$ ; and

$$A^-(y) = A^-(yxx^{-1}) \leq \max\{A^-(yx), A^-(x)\} = A^-(yx).$$

Therefore  $A^-(xy) = A^-(y) = A^-(yx)$ . Hence (ii) is proved.

**2.3 Theorem:** Let  $A = \langle A^+, A^- \rangle$  be a bipolar-valued fuzzy subgroup of a group  $G$ . If

$A^+(x) > A^+(y)$  and  $A^-(x) > A^-(y)$ , for some  $x$  and  $y$  in  $G$ , then

$$(i) \quad A^+(xy) = A^+(y) = A^+(yx) \text{ and } (ii) \quad A^-(xy) = A^-(x) = A^-(yx). \text{ Proof: It is trivial.}$$

**2.4 Theorem:** Let  $A = \langle A^+, A^- \rangle$  be a bipolar-valued fuzzy subgroup of a group  $G$ . If  $A^+(x) > A^+(y)$  and  $A^-(x) < A^-(y)$ , for some  $x$  and  $y$  in  $G$ , then

$$(i) \quad A^+(xy) = A^+(y) = A^+(yx) \text{ and } (ii) \quad A^-(xy) = A^-(y) = A^-(yx).$$

**Proof:** It is trivial.

**2.5 Theorem:** Let  $A = \langle A^+, A^- \rangle$  be a bipolar-valued fuzzy subgroup of a finite group  $G$ , then  $o(A)$  divides  $o(G)$ .

**Proof:** Let  $A$  be a bipolar-valued fuzzy subgroup of a finite group  $G$  with  $e$  as its identity element. Clearly  $H = \{x \in G / A^+(x) = A^+(e) \text{ and } A^-(x) = A^-(e)\}$  is a subgroup of the group  $G$ . By Lagrange's theorem  $o(H) \mid o(G)$ .

Hence by the definition of the order of the bipolar-valued fuzzy subgroup of the group  $G$ , we have  $o(A) \mid o(G)$ .

**2.6 Theorem:** Let  $A = \langle A^+, A^- \rangle$  and  $B = \langle B^+, B^- \rangle$  be two bipolar-valued fuzzy subsets of an abelian group  $G$ . Then  $A$  and  $B$  are conjugate bipolar-valued fuzzy subsets of the group  $G$  if and only if  $A = B$ .

**Proof:** Let  $A$  and  $B$  be conjugate bipolar-valued fuzzy subsets of group  $G$ , then for some  $y \in G$ , we have  $A^+(x) = B^+(y^{-1}xy) = B^+(y^{-1}yx) = B^+(ex) = B^+(x)$ .

Therefore  $A^+(x) = B^+(x)$ .

And,  $A^-(x) = B^-(y^{-1}xy) = B^-(y^{-1}yx) = B^-(ex) = B^-(x)$ .

Therefore  $A^-(x) = B^-(x)$ . Hence  $A = B$ .

Conversely if  $A = B$  then for the identity element  $e$  of group  $G$ ,

we have  $A^+(x) = B^+(e^{-1}xe)$  and  $A^-(x) = B^-(e^{-1}xe)$  for every  $x \in G$ .

Hence  $A$  and  $B$  are conjugate bipolar-valued fuzzy subsets of the group  $G$ .

**2.7 Theorem:** If  $A = \langle A^+, A^- \rangle$  and  $B = \langle B^+, B^- \rangle$  are conjugate bipolar-valued fuzzy subgroups of the group  $G$ , then  $o(A) = o(B)$ .

**Proof:** Let  $A$  and  $B$  are conjugate bipolar-valued fuzzy subgroups of the group  $G$ .

$$\begin{aligned} \text{Now, } o(A) &= \text{order of } \{ x \in G / A^+(x) = A^+(e) \text{ and } A^-(x) = A^-(e) \} \\ &= \text{order of } \{ x \in G / B^+(y^{-1}xy) = B^+(y^{-1}ey) \text{ and } B^-(y^{-1}xy) = B^-(y^{-1}ey) \} \\ &= \text{order of } \{ x \in G / B^+(x) = B^+(e) \text{ and } B^-(x) = B^-(e) \} \\ &= o(B). \text{ Hence } o(A) = o(B). \end{aligned}$$

**2.8 Theorem:** Let  $A = \langle A^+, A^- \rangle$  be a bipolar-valued fuzzy normal subgroup of a group  $G$ . Then for any  $y$  in  $G$  we have  $A^+(yxy^{-1}) = A^+(y^{-1}xy)$  and  $A^-(yxy^{-1}) = A^-(y^{-1}xy)$ , for every  $x \in G$ .

**Proof:** Let  $A$  be a bipolar-valued fuzzy normal subgroup of a group  $G$ .

For any  $y$  in  $G$ . Then we have,  $A^+(yxy^{-1}) = A^+(x) = A^+(xyy^{-1}) = A^+(y^{-1}xy)$ .

Therefore  $A^+(yxy^{-1}) = A^+(y^{-1}xy)$ .

And,  $A^-(yxy^{-1}) = A^-(x) = A^-(xyy^{-1}) = A^-(y^{-1}xy)$ .

Therefore  $A^-(yxy^{-1}) = A^-(y^{-1}xy)$ .

**2.9 Theorem:** A bipolar-valued fuzzy subgroup  $A = \langle A^+, A^- \rangle$  of a group  $G$  is normalized if and only if  $A^+(e) = 1$  and  $A^-(e) = 0$ , where  $e$  is the identity element of the group  $G$ .

**Proof:** If  $A$  is normalized then there exists  $x \in G$  such that  $A^+(x) = 1$  and  $A^-(x) = 0$ , but by properties of a bipolar-valued fuzzy subgroup  $A$  of the group  $G$ ,  $A^+(x) \leq A^+(e)$  and  $A^-(x) \geq A^-(e)$  for every  $x \in G$ .

since  $A^+(x) = 1$  and  $A^-(x) = 0$  and  $A^+(x) \leq A^+(e)$  and  $A^-(x) \geq A^-(e)$ .

Therefore  $1 \leq A^+(e)$  and  $0 \geq A^-(e)$ . But  $1 \geq A^+(e)$  and  $0 \leq A^-(e)$ .

Hence  $A^+(e) = 1$  and  $A^-(e) = 0$ .

Conversely if  $A^+(e) = 1$  and  $A^-(e) = 0$ , then by the definition of normalized bipolar-valued fuzzy subset  $A$  is normalized.

**2.10 Theorem:** If  $A = \langle A^+, A^- \rangle$  is a bipolar-valued fuzzy subgroup of a group  $G$ , then for any  $a$  in  $G$  the bipolar-valued fuzzy middle coset  $aAa^{-1}$  of  $G$  is also a bipolar-valued fuzzy subgroup of a group  $G$ .

**Proof:** Let  $A$  is a bipolar-valued fuzzy subgroup of a group  $G$  and  $a$  in  $G$ . To prove  $aAa^{-1} = \langle aA^+a^{-1}, aA^-a^{-1} \rangle$  is a bipolar-valued fuzzy subgroup of  $G$ . Let  $x$  and  $y$  in  $G$ .

$$\begin{aligned} \text{Then } (aA^+a^{-1})(xy^{-1}) &= A^+(a^{-1}xy^{-1}a) \\ &= A^+(a^{-1}xaa^{-1}y^{-1}a) \\ &= A^+(a^{-1}xa(a^{-1}ya)^{-1}) \\ &\geq \min \{ A^+(a^{-1}xa), A^+(a^{-1}ya) \} \\ &= \min \{ (aA^+a^{-1})(x), (aA^+a^{-1})(y) \}. \end{aligned}$$

Therefore  $(aA^+a^{-1})(xy^{-1}) \geq \min \{ (aA^+a^{-1})(x), (aA^+a^{-1})(y) \}$ .

$$\begin{aligned} \text{And } (aA^-a^{-1})(xy^{-1}) &= A^-(a^{-1}xy^{-1}a) \\ &= A^-(a^{-1}xaa^{-1}y^{-1}a) \end{aligned}$$

$$\begin{aligned}
&= A^-(a^{-1}xa(a^{-1}ya)^{-1}) \\
&\leq \max \{ A^-(a^{-1}xa), A^-(a^{-1}ya) \} \\
&= \max \{ (a A^-a^{-1})(x), (a A^-a^{-1})(y) \}.
\end{aligned}$$

Therefore  $(a A^-a^{-1})(xy^{-1}) \leq \max \{ (a A^-a^{-1})(x), (a A^-a^{-1})(y) \}$ . Hence  $aAa^{-1}$  is a bipolar-valued fuzzy subgroup of a group  $G$ .

**2.11 Theorem:** Let  $A = \langle A^+, A^- \rangle$  be a bipolar-valued fuzzy subgroup of a group  $G$  and  $aAa^{-1}$  be a bipolar-valued fuzzy middle coset of the group  $G$ , then  $o(aAa^{-1}) = o(A)$ , for any  $a \in G$ .

**Proof:** Let  $A$  be a bipolar-valued fuzzy subgroup of a group  $G$  and  $a \in G$ . By Theorem 2.10, the bipolar-valued fuzzy middle coset  $aAa^{-1}$  is a bipolar-valued fuzzy subgroup of a group  $G$ . Further by the definition of a bipolar-valued fuzzy middle coset of the group  $G$  we have  $(a A^+a^{-1})(x) = A^+(a^{-1}xa)$  and  $(a A^-a^{-1})(x) = A^-(a^{-1}xa)$ , for every  $x$  in  $G$ .

Hence for any  $a$  in  $G$ ,  $A$  and  $aAa^{-1}$  are conjugate bipolar-valued fuzzy subgroup of the group  $G$  as there exists  $a \in G$  such that  $(a A^+a^{-1})(x) = A^+(a^{-1}xa)$  and  $(a A^-a^{-1})(x) = A^-(a^{-1}xa)$  for every  $x$  in  $G$ . By Theorem 2.6,  $o(aAa^{-1}) = o(A)$  for any  $a$  in  $G$ .

**2.12 Theorem:** Let  $A = \langle A^+, A^- \rangle$  be a bipolar-valued fuzzy subgroup of a group  $G$  and  $B = \langle B^+, B^- \rangle$  be a bipolar-valued fuzzy subset of a group  $G$ . If  $A$  and  $B$  are conjugate bipolar-valued fuzzy subsets of the group  $G$  then  $B$  is a bipolar-valued fuzzy subgroup of a group  $G$ .

**Proof:** Let  $A$  be a bipolar-valued fuzzy subgroup of a group  $G$  and  $B$  be a bipolar-valued fuzzy subset of a group  $G$ . And let  $A$  and  $B$  are conjugate bipolar-valued fuzzy subsets of the group  $G$ . To prove  $B$  is a bipolar-valued fuzzy subgroup of the group  $G$ .

Let  $x$  and  $y$  in  $G$ . Then  $xy^{-1}$  in  $G$ .

Now,  $B^+(xy^{-1}) = A^+(g^{-1}xy^{-1}g) = A^+(g^{-1}xgg^{-1}y^{-1}g) = A^+(g^{-1}xg(g^{-1}yg)^{-1}) \geq \min \{ A^+(g^{-1}xg), A^+(g^{-1}yg) \} = \min \{ B^+(x), B^+(y) \}$ . Therefore  $B^+(xy^{-1}) \geq \min \{ B^+(x), B^+(y) \}$ .

And  $B^-(xy^{-1}) = A^-(g^{-1}xy^{-1}g) = A^-(g^{-1}xgg^{-1}y^{-1}g) = A^-(g^{-1}xg(g^{-1}yg)^{-1}) \leq \max \{ A^-(g^{-1}xg), A^-(g^{-1}yg) \} = \max \{ B^-(x), B^-(y) \}$ .

Therefore  $B^-(xy^{-1}) \leq \max \{ B^-(x), B^-(y) \}$ . Hence  $B$  is a bipolar-valued fuzzy subgroup of the group  $G$ .

**2.13 Theorem:** Let a bipolar-valued fuzzy subgroup  $A = \langle A^+, A^- \rangle$  of a group  $G$  be conjugate to a bipolar-valued fuzzy subgroup  $M = \langle M^+, M^- \rangle$  of  $G$  and a bipolar-valued fuzzy subgroup  $B = \langle B^+, B^- \rangle$  of a group  $H$  be conjugate to a bipolar-valued fuzzy subgroup  $N = \langle N^+, N^- \rangle$  of  $H$ . Then a bipolar-valued fuzzy subgroup  $A \times B = \langle (A \times B)^+, (A \times B)^- \rangle$  of a group  $G \times H$  is conjugate to a bipolar-valued fuzzy subgroup  $M \times N = \langle (M \times N)^+, (M \times N)^- \rangle$  of  $G \times H$ .

**Proof:** Let  $A$  and  $B$  be bipolar-valued fuzzy subgroups of the groups  $G$  and  $H$ . Let  $x, x^{-1}$  and  $f$  be in  $G$  and  $y, y^{-1}$  and  $g$  be in  $H$ . Then  $(x, y), (x^{-1}, y^{-1})$  and  $(f, g)$  are in  $G \times H$ .

$$\begin{aligned}
\text{Now, } (A \times B)^+(f, g) &= \min \{ A^+(f), B^+(g) \} = \min \{ M^+(xfx^{-1}), N^+(ygy^{-1}) \} \\
&= (M \times N)^+(xfx^{-1}, ygy^{-1}) = (M \times N)^+[(x, y)(f, g)(x^{-1}, y^{-1})] \\
&= (M \times N)^+[(x, y)(f, g)(x, y)^{-1}].
\end{aligned}$$

Therefore,  $(A \times B)^+(f, g) = (M \times N)^+[(x, y)(f, g)(x, y)^{-1}]$ .

$$\begin{aligned}
\text{And, } (A \times B)^-(f, g) &= \max \{ A^-(f), B^-(g) \} = \max \{ M^-(xfx^{-1}), N^-(ygy^{-1}) \} \\
&= (M \times N)^-(xfx^{-1}, ygy^{-1}) = (M \times N)^-[(x, y)(f, g)(x^{-1}, y^{-1})] \\
&= (M \times N)^-[(x, y)(f, g)(x, y)^{-1}].
\end{aligned}$$

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Therefore,  $(A \times B)^-(f, g) = (M \times N)^-[(x, y)(f, g)(x, y)^{-1}]$ .

Hence a bipolar-valued fuzzy subgroup  $A \times B$  of a group  $G \times H$  is conjugate to a bipolar-valued fuzzy subgroup  $M \times N$  of  $G \times H$ .

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