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A CONJECTURE ABOUT FUNCTIONS

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ABSTRACT. In [1] *Julien Laurendeau* stated the conjecture on functions. In this paper we will address the same and we conclude the conjecture.

1. INTRODUCTION

As we know that conjecture is a proposition that is unproven [2]. *Karl Popper* pioneered the use of the term conjecture in scientific philosophy. Let us state the *Julien Laurendeau's conjecture.*
There are no solutions to the following equation:

$$\left(f(x)+g(x)+h(x)\right)+\left(f(x)+g(x)\right)h(x)=2. \tag{1}$$

Knowing that all mentioned functions are non-linear and that:

$$\left.\begin{aligned} f(x) &= \frac{f(x)+\left(g(x)-h(x)\right)f(x)}{2} \\ g(x) &= \frac{g(x)+\left(h(x)-f(x)\right)g(x)}{4} \\ h(x) &= \frac{h(x)+\left(f(x)-g(x)\right)h(x)}{6} \end{aligned}\right\} \tag{2}$$

With $f(x)\neq 0, g(x)\neq 0, h(x)\neq 0. \forall x\in R.$
Let us move to the next section to conclude this conjecture.

2. CONCLUDING THE CONJECTURE

Let us prove the conjecture in case wise/ part wise.
Part-I: At least one has to be zero.

$$\text{Let}\quad \begin{aligned} K_A &= \{x\in R \mid f(x)\neq 0\} \\ K_B &= \{x\in R \mid g(x)\neq 0\} \\ K_C &= \{x\in R \mid h(x)\neq 0\} \end{aligned}$$

For time being consider *f(x) = A, g(x) = B* and *h(x) =C,* then our (1) and (2) becomes;

$$(A+B+C)+(A+B)C=2 \tag{3}$$

$$\left.\begin{aligned} &(i)2A = A+(B-C)A \\ &(ii)4B = B+(C-A)B \\ &(iii)6C = C+(A-B)C \end{aligned}\right\} \tag{4}$$

With A $\neq 0$, $B\neq 0$ and $C\neq 0$.

For condition-(i):

$$2A = A + (B - C)A; \forall x \in R$$
$$\Leftrightarrow A = (B - C)A; \forall x \in R$$
$$\Rightarrow 1 = B - C; \forall x \in K_A$$

For condition-(ii):

$$4B = B + (C - A)B; \forall x \in R$$
$$\Leftrightarrow 3B = (C - A)B; \forall x \in R$$
$$\Rightarrow 3 = C - A; \forall x \in K_B$$

For condition-(iii):

$$6C = C + (A - B)C; \forall x \in R$$
$$\Leftrightarrow 5C = (A - B)C; \forall x \in R$$
$$\Rightarrow 5 = A - B; \forall x \in K_C$$

$$\forall x \in (K_A \cap K_B \cap K_C) : C = B - 1$$

Clearly, $\Rightarrow 3 = (B - 1) - A$
$\Rightarrow 5 = A - B$
$B = 4 + A$
$\Rightarrow 5 = A - (4 + A) = -4,$
This is contradiction $\Rightarrow (K_A \cap K_B \cap K_C) = \emptyset$.
i.e., there is no $x \in R$ such that $f(x) \neq 0$, $g(x) \neq 0$ and $h(x) \neq 0$.

Part-II: Let us check the individual cases

Case – I:

$$\{x \in R \mid C(x) = 0\}$$

Condition is $A + B = 2$

$$2A = A + AB$$
$$\Leftrightarrow A = AB$$
$$4B = B - AB$$
$$\Leftrightarrow -3B = AB$$
$$A = -3B$$
$$A + B = -3B + B$$
$$-3B + B = -2B = 2$$
$$\Leftrightarrow B = -1$$
$$\Rightarrow A = 3$$

Case – II:

$$\{x \in R \mid B(x) = 0\}$$

Condition is $A + C + AC - 2 = 0$

$$2A = A - AC$$
$$\Leftrightarrow -A = AC$$
$$6C = C + AC$$
$$\Leftrightarrow 5C = AC$$
$$\Rightarrow -A = 5C$$
$$\Leftrightarrow A = -5C$$

Now we can simplify the conditions: A + C + AC -2 = 0

$$\Leftrightarrow -5C + C + (-5C)C - 2 = 0$$
$$\Leftrightarrow -5C^2 - 4C - 2 = 0$$
$$\Leftrightarrow C_{1,2} = \frac{1}{2(-5)}\left(-4 \pm \sqrt{16 - 4(-5)(-2)}\right)$$

i.e., C has no solution in R.
Hence $\{x \in R \mid B(x) = 0\} = \text{Ø}$.

Case – III:

$$\{x \in R \mid A(x) = 0\}$$

Condition is $B + C + BC - 2 = 0$

$$4B = B + BC$$
$$\Leftrightarrow 3B = BC$$
$$6C = C - BC$$
$$\Leftrightarrow 5C = -BC$$
$$\Leftrightarrow -5C = BC$$
$$3B = -5C$$
$$\Leftrightarrow B = -5/3\ C$$

Now by simplifying the condition, we get;

$$\frac{-5}{3}C + C + \left(\frac{-5}{3}C\right)C - 2 = 0$$
$$\frac{-5}{3}C^2 + \left(\frac{-2}{3}C\right) - 2 = 0$$
$$\frac{5}{3}C^2 + \left(\frac{2}{3}C\right) + 2 = 0$$
$$\Leftrightarrow C_{1,2} = \frac{1}{2\left(\frac{5}{3}\right)}\left(-\frac{2}{3} \pm \sqrt{\frac{4}{9} - 4\left(\frac{5}{3}\right)2}\right)$$
$$= \frac{3}{10}\left(-\frac{2}{3} \pm \sqrt{\frac{4}{9} - \frac{40}{3}}\right)$$

i.e., C has no solution in R.
Hence $\{x \in R \mid A(x) = 0\} = \text{Ø}$

Finally…

$$\forall x \in R, f(x) = 0 \text{ or } g(x) = 0 \text{ or } h(x) = 0$$
$$\forall x \in R : f(x) \neq 0 \text{ and } g(x) \neq 0$$
$$\forall x \in R : h(x) = 0$$
$$\Rightarrow h = 0 \Leftrightarrow C = 0$$
$$\Rightarrow \mathit{contradiction}$$

Therefore, there is no such real functions f, g, h □

3. SHORT PROOF

We have from all deductions, the following:
Form (i), we have; $2A = A + (B-C)A; \forall x \in R$

$$\Rightarrow 2A - A \overline{+ (B-C)A} = 0$$
$$\Rightarrow A(2-(1+(B-C)) = 0$$
$$\Rightarrow A(1-(B-C)) = 0 \updownarrow$$

similarly we can find the others as shown below:

$$\left.\begin{array}{l} A(1-(B-C)) = 0 \\ B(3-(C-A)) = 0 \\ C(5-(A-B)) = 0 \end{array}\right\} \ldots (*)$$

Since $A \neq 0$, $B \neq 0$ & $C \neq 0$,
The (*) becomes by adding;

$$\left.\begin{array}{l} 1-(B-C) = 0 \\ 3-(C-A) = 0 \\ 5-(A-B) = 0 \end{array}\right\} or \left\{\begin{array}{l} 1 = (B-C) \\ 3 = (C-A) \ldots (+) \\ 5 = (A-B) \end{array}\right.$$

i.e., 9 = 0, this is contradiction. □

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REFERENCES

[1] http://rxiv.org/pdf/1307.0159v1.pdf

[2] http://en.wikipedia.org/wiki/Conjecture