

A CONJECTURE ABOUT FUNCTIONS

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ABSTRACT. In [1] *Julien Laurendeau* stated the conjecture on functions. In this paper we will address the same and we conclude the conjecture.

1. INTRODUCTION

As we know that conjecture is a proposition that is unproven [2]. *Karl Popper* pioneered the use of the term conjecture in scientific philosophy. Let us state the *Julien Laurendeau's conjecture*.

There are no solutions to the following equation:

$$(f(x) + g(x) + h(x)) + (f(x) + g(x))h(x) = 2. \quad (1)$$

Knowing that all mentioned functions are non-linear and that:

$$\left. \begin{aligned} f(x) &= \frac{f(x) + (g(x) - h(x))f(x)}{2} \\ g(x) &= \frac{g(x) + (h(x) - f(x))g(x)}{4} \\ h(x) &= \frac{h(x) + (f(x) - g(x))h(x)}{6} \end{aligned} \right\} \quad (2)$$

With $f(x) \neq 0, g(x) \neq 0, h(x) \neq 0, \forall x \in R$.

Let us move to the next section to conclude this conjecture.

2. CONCLUDING THE CONJECTURE

Let us prove the conjecture in case wise/ part wise.

Part-I: At least one has to be zero.

$$K_A = \{x \in R \mid f(x) \neq 0\}$$

$$\text{Let } K_B = \{x \in R \mid g(x) \neq 0\}$$

$$K_C = \{x \in R \mid h(x) \neq 0\}$$

For time being consider $f(x) = A, g(x) = B$ and $h(x) = C$, then our (1) and (2) becomes;

$$(A + B + C) + (A + B)C = 2 \quad (3)$$

$$\left. \begin{aligned} (i) 2A &= A + (B - C)A \\ (ii) 4B &= B + (C - A)B \\ (iii) 6C &= C + (A - B)C \end{aligned} \right\} \quad (4)$$

With $A \neq 0, B \neq 0$ and $C \neq 0$.

For condition-(i):

$$\begin{aligned} 2A &= A + (B - C)A; \forall x \in R \\ \Leftrightarrow A &= (B - C)A; \forall x \in R \\ \Rightarrow 1 &= B - C; \forall x \in K_A \end{aligned}$$

For condition-(ii):

$$\begin{aligned} 4B &= B + (C - A)B; \forall x \in R \\ \Leftrightarrow 3B &= (C - A)B; \forall x \in R \\ \Rightarrow 3 &= C - A; \forall x \in K_B \end{aligned}$$

For condition-(iii):

$$\begin{aligned} 6C &= C + (A - B)C; \forall x \in R \\ \Leftrightarrow 5C &= (A - B)C; \forall x \in R \\ \Rightarrow 5 &= A - B; \forall x \in K_C \end{aligned}$$

$$\forall x \in (K_A \cap K_B \cap K_C): C = B - 1$$

Clearly, $\Rightarrow 3 = (B - 1) - A$
 $\Rightarrow 5 = A - B$

$$B = 4 + A$$

$$\Rightarrow 5 = A - (4 + A) = -4,$$

This is contradiction $\Rightarrow (K_A \cap K_B \cap K_C) = \emptyset$.

i.e., there is no $x \in R$ such that $f(x) \neq 0$, $g(x) \neq 0$ and $h(x) \neq 0$.

Part-III: Let us check the individual cases

Case – I:

$$\{x \in R \mid C(x) = 0\}$$

Condition is $A + B = 2$

$$\begin{aligned} 2A &= A + AB \\ \Leftrightarrow A &= AB \\ 4B &= B - AB \\ \Leftrightarrow -3B &= AB \\ A &= -3B \\ A + B &= -3B + B \\ -3B + B &= -2B = 2 \\ \Leftrightarrow B &= -1 \\ \Rightarrow A &= 3 \end{aligned}$$

Case – II:

$$\{x \in R \mid B(x) = 0\}$$

Condition is $A + C + AC - 2 = 0$

$$\begin{aligned} 2A &= A - AC \\ \Leftrightarrow -A &= AC \\ 6C &= C + AC \\ \Leftrightarrow 5C &= AC \\ \Rightarrow -A &= 5C \\ \Leftrightarrow A &= -5C \end{aligned}$$

Now we can simplify the conditions: $A + C + AC - 2 = 0$

$$\Leftrightarrow -5C + C + (-5C)C - 2 = 0$$

$$\Leftrightarrow -5C^2 - 4C - 2 = 0$$

$$\Leftrightarrow C_{1,2} = \frac{1}{2(-5)} \left(-4 \pm \sqrt{16 - 4(-5)(-2)} \right)$$

i.e., C has no solution in R .

Hence $\{x \in R \mid B(x) = 0\} = \emptyset$.

Case – III:

$$\{x \in R \mid A(x) = 0\}$$

Condition is $B + C + BC - 2 = 0$

$$4B = B + BC$$

$$\Leftrightarrow 3B = BC$$

$$6C = C - BC$$

$$\Leftrightarrow 5C = -BC$$

$$\Leftrightarrow -5C = BC$$

$$3B = -5C$$

$$\Leftrightarrow B = -5/3 C$$

Now by simplifying the condition, we get;

$$\frac{-5}{3}C + C + \left(\frac{-5}{3}C\right)C - 2 = 0$$

$$\frac{-5}{3}C^2 + \left(\frac{-2}{3}C\right) - 2 = 0$$

$$\frac{5}{3}C^2 + \left(\frac{2}{3}C\right) + 2 = 0$$

$$\Leftrightarrow C_{1,2} = \frac{1}{2\left(\frac{5}{3}\right)} \left(-\frac{2}{3} \pm \sqrt{\frac{4}{9} - 4\left(\frac{5}{3}\right)2} \right)$$

$$= \frac{3}{10} \left(-\frac{2}{3} \pm \sqrt{\frac{4}{9} - \frac{40}{3}} \right)$$

i.e., C has no solution in R .

Hence $\{x \in R \mid A(x) = 0\} = \emptyset$

Finally...

$$\forall x \in R, f(x) = 0 \text{ or } g(x) = 0 \text{ or } h(x) = 0$$

$$\forall x \in R : f(x) \neq 0 \text{ and } g(x) \neq 0$$

$$\forall x \in R : h(x) = 0$$

$$\Rightarrow h = 0 \Leftrightarrow C = 0$$

$$\Rightarrow \text{contradiction}$$

Therefore, there is no such real functions f, g, h \square

3. SHORT PROOF

We have from all deductions, the following:

Form (i), we have; $2A = A + (B - C)A; \forall x \in R$

$$\begin{aligned} &\Rightarrow 2A - A + \overline{(B - C)A} = 0 \\ &\Rightarrow A(2 - (1 + (B - C))) = 0 \\ &\Rightarrow A(1 - (B - C)) = 0 \downarrow \end{aligned}$$

similarly we can find the others as shown below:

$$\left. \begin{aligned} A(1 - (B - C)) &= 0 \\ B(3 - (C - A)) &= 0 \\ C(5 - (A - B)) &= 0 \end{aligned} \right\} \dots (*)$$

Since $A \neq 0, B \neq 0$ & $C \neq 0$,

The (*) becomes by adding;

$$\left. \begin{aligned} 1 - (B - C) &= 0 \\ 3 - (C - A) &= 0 \\ 5 - (A - B) &= 0 \end{aligned} \right\} \text{or} \left\{ \begin{aligned} 1 &= (B - C) \\ 3 &= (C - A) \dots (+) \\ 5 &= (A - B) \end{aligned} \right.$$

i.e., $9 = 0$, this is contradiction. \square

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REFERENCES

- [1] <http://rxiv.org/pdf/1307.0159v1.pdf>
- [2] <http://en.wikipedia.org/wiki/Conjecture>