

The Beal's Conjecture (Disproved)

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ABSTRACT

In this paper, we disprove the Beal's conjecture by two different proofs.

INTRODUCTION

Mathematicians have long been intrigued by Pierre Fermat's famous assertion that $A^x + B^y = C^z$ is impossible (as stipulated) and the remark written in the margin of his book that he had a demonstration or "proof". This became known as Fermat's Last Theorem (FLT) despite the lack of a proof. Andrew Wiles proved the relationship in 1994, though everyone agrees that Fermat's proof could not possibly have been the proof discovered by Wiles. Number theorists remain divided when speculating over whether or not Fermat actually had a proof, or whether he was mistaken. This mystery remains unanswered though the prevailing wisdom is that Fermat was mistaken. This conclusion is based on the fact that thousands of mathematicians have cumulatively spent many millions of hours over the past 350 years searching unsuccessfully for such a proof.

It is easy to see that if $A^x + B^y = C^z$ then either A, B, and C are prime or, if not co-prime that any common factor could be divided out of each term until the equation existed with co-prime bases. (Co-prime is synonymous with pair-wise relatively prime and means that in a given set of numbers, no two of the numbers share a common factor.)

You could then restate FLT [2] by saying that $A^x + B^y = C^z$ is impossible with co-prime bases. (Yes, it is also impossible without co-prime bases, but non co-prime bases can only exist as a consequence of co-prime bases.) [1]

In the next section, we will conclude the existence of the theorem by two different approaches.

The Beal Conjecture states that

$$\text{"If } A^x + B^y = C^z \tag{1}$$

where A, B, C, x, y and z are all positive integers and x, y and z are greater than 2 then, A, B and C will have a common prime factor".

FIRST PROOF THAT THE BEAL CONJECTURE IS INCORRECT

Let us assume for a moment that the Conjecture is correct.

Then, if the common prime factor is P and if x, y and z can be expressed as (X +2), (Y +2) and (Z+2) respectively, since they are greater than 2, we can express (1) as follows:

$$(pA_1)^{X+2} = (pB_1)^{Y+2} = (pC_1)^{Z+2} \tag{2}$$

where $A = (pA_1)$; $B = (pB_1)$; $C = (pC_1)$

Expanding (2), we have

$$(p^{X+2})(A_1)^{X+2} + (p^{Y+2})(B_1)^{Y+2} = (p^{Z+2})(C_1)^{Z+2} \tag{3}$$

Since p^2 is a common factor throughout (3), we can rewrite the equation as follows

$$(p^X)(A_1)^{X+2} + (p^Y)(B_1)^{X+2} = (p^Z)(C_1)^{Z+2} \quad (4)$$

If $X = Y = Z$ as a special case, then (4) reduces to

$$(A_1)^{X+2} + (B_1)^{X+2} = (C_1)^{Z+2} \quad (5)$$

Equation (5) is such that it satisfies all the requirements of the Beal conjecture, namely, A_1, B_1, C_1 ($X+2$), ($Y+2$) and ($Z+2$) are all positive integers and the last three are greater than 2. So, A_1, B_1, C_1 have a common prime factor.

In other words, from (5), if the same logic were to be followed in “n” steps, assuming that X, Y, Z are equal as a special case, one arrives at the following equation

$$(A_n)^{X+2} + (B_n)^{X+2} = (C_n)^{Z+2} \quad (6)$$

in which A_n, B_n, C_n follows: (i) either do not have a common prime factor any longer or (ii) they are all equal to one another.

If case (i) is true, the Beal Conjecture is disproved.

If case (ii) is true, it follows that

$A_n = B_n = C_n$ say, equal to k , in which case, we have

$$k^{X+2} + k^{Y+2} = k^{Z+2} \quad (7)$$

If $X = Y = Z$, the (7) cannot be true, as the sum of the two terms on the LHS is twice the term on RHS. In other words, the original assumption that the Beal Conjecture is true is incorrect. There are exceptional cases in which it is not true.

SECOND PROOF THAT THE BEAL CONJECTURE IS INCORRECT

Let us take the conjecture with n, k instead of x, y, z

$$A^i + B^n = C^k \quad (1)$$

Let us consider that A, B and C has a common factor, namely ‘m’. Then, each value of A, B and C can be expressible as ma, mb and md respectively.

Now, (1) can be reduced to,

$$\Rightarrow (ma)^i + (mb)^n = (md)^k$$

$$\Rightarrow \left((ma)^{\frac{i}{n}} \right)^n + (mb)^n = \left((md)^{\frac{k}{n}} \right)^n$$

$$\Rightarrow m^n \left(m^{\frac{i-1}{n}} a^{\frac{i}{n}} \right)^n + m^n b^n = m^n \left(m^{\frac{k-1}{n}} d^{\frac{k}{n}} \right)^n$$

$$\Rightarrow m^n \left\{ \left(m^{\frac{i-1}{n}} a^{\frac{i}{n}} \right)^n + b^n \right\} = m^n \left(m^{\frac{k-1}{n}} d^{\frac{k}{n}} \right)^n$$

$$\Rightarrow \left(m^{\frac{i-1}{n}} a^{\frac{i}{n}} \right)^n + b^n = \left(m^{\frac{k-1}{n}} d^{\frac{k}{n}} \right)^n$$

$$\cong X^n + Y^n = Z^n \tag{2}$$

Where

$$X = \left(m^{\frac{i-1}{n}} a^{\frac{i}{n}} \right), Y = b \text{ and } Z = \left(m^{\frac{k-1}{n}} d^{\frac{k}{n}} \right)$$

The (2) is reduced FLT and FLT is true for $n > 2$.
Hence, Beal's Conjecture is disproved. \square

REFERENCES

[1] <http://www.bealconjecture.com/>

[2] Harold M. Edwards, Fermat's Last Theorem: A Genetic Introduction to Algebraic Number Theory, Springer-Verlag New York Berlin Heidelberg, 2000

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