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Solution of Some problems in Hungarian mathematical competition. IV.

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ABSTRACT. In this work, we continue to present some interesting problems in the Transylvanian Hungarian Mathematical Competition held in 2012. In this article, one can find very interesting proofs of [1].

INTRODUCTION

I have seen the very interesting problems in [1], which is exclusively collected by Prof. Fang Chen, Department of Mathematics, Xinjiang Normal University, Urumchi 830054, China. These problems are very interesting for me, as the solutions are derived from my favorite branch, none other than the Theory of Numbers. However, one can make more simplest or elegant method to answer them. I have done my part as usual in the best possible way.

PROBLEMS

There are 6 problems are discussed in the article [1]. But, this article will answer only 5 out of them, as I said erlier, I am big fan of Number Theory…ha ha ha

D1st Problem: Solve in Z the following equation: $\frac{3}{\sqrt{x}}+\frac{2}{\sqrt{y}}=\frac{1}{\sqrt{2}}$. (Ferenc Kacs´o)

D2nd Problem: Let's consider set $M = \{a^2 - 2ab + 2b^2 \mid a, b \in Z\}$. Show that $2012 \notin M$. Prove that M is a closed subset of N in respect of the multiplication of integers. (B´ela B´ır´)

D3rd Problem: Solve in R equation $5x^3 - 18x^2 + 43x - 6 = 3.2^{x+2}$. (B´ela Kov´acs)

SOLUTIONS OF CITED ABOVE THREE

We give fairly full information towards 2nd and much less full information towards 1st. But, 3rd one, which mixes polynomial and exponential, has less structure. If you are looking for non-negative *integer* solutions *x,* we can use by trail and error method for x = 0, 1, 2, and so on to find some suitable solutions. Incase, if we go far enough in our computations, we can *establish* that we found them, by showing that after the numbers we have tested $3.2^{x+2} = 12(2^x)$ is bigger than the left side.

For closure, observe the following famous identity, which can be verified by expanding each side.

$$(s^2+t^2)(u^2+v^2)=(su+tv)^2+(sv-tu)^2.$$

Now, we show that 2012 is not expressible as sum of two squares, so in particular is not of the form described. The number 2012 is same as 4(503). It is easy to verify that if this is a sum of 2 squares, let $x^2 + y^2 = 2012$, then x and y should be even, which imples that 503 can be expressible as sum of two squares. Now we have to show that, this is not possible.

There are few such squares, so we could check with these squares or else we can find that $503 \equiv -1 (\text{mod } 4)$. Also it is easy for us, to vearify that a sum of two squares cannot be congruent to –

$1 \pmod 4$). For the square of an odd number is congruent to 1modulo 4, indeed modulo 8. For D1), there are two things to do, one of which takes a fair bit of work. It is plausible, and *should be proved*, that our numbers x and y are can be expressed as in the form of $2a^2$ and $2b^2$ respectively.

Let us assume for positive integers of a and b such that $\frac{3}{a}+\frac{2}{b}=1 \Rightarrow (a-3)(b-2)=6.$ The part about proving that solutions x and y must be of the shape we have described is a somewhat complicated variant of the proof that $\sqrt{2}$ is irrational. As a start, note that by squaring both sides we can conclude that xy is a perfect square.

D5th Problem: Uncle John has taken blood pressure drops for a long time according to he following rule: 1 drop for one day, 2 drops daily for two days, ...,10 drops daily for ten days, 9 drops daily for nine days, ..., 2 drops daily for twodays, 1 drop for one day, 2 drops daily for two days, ... One day he forgot how many drops he should take, finally he took 5 drops. What is the probability that he guessed right the daily dose? Later he remembered taking 5 drops previous day, so he calmed down that he guessed the dose correctly with high probability. What is this newer probability? (Agnes Mik´)

Solution: Clearly the probability would be $\frac{2\times 5}{1+2+...+10+...+2+1}=\frac{10}{99}$ and then the new probability would be $\frac{4}{5}$. Since Uncle John's pattern of taking blood pressuredrops repeats every 99 days, and 10 of these days are when he takes 5 drops.Therefore, the probability that he guesses right with no other information $\frac{10}{99}$. Next, if we know that the previous day he took 5 drops, then the day he forgot can only be one of 10 days, the last 4 days, of each of the two cycles of 5 drops, plus the first day of each of the two cycles of 6 drops. Only if it were one of the two 6drop days would Uncle John guess wrong, so his probability of guessing correctly has risen to $\frac{4}{5}$.

D6th Problem: a) At least how many elements must be selected from the group $(Z_{2k},+)$ such that among the selected elements surely there exist three (not necessarily distinct) with sum ˆ0?

b) The same question for $(Z_{15},+)$. (Szil´ard Andr´as)

Solution: For the part (a), the minimum number we are looking for is exactly k + 1. The proof below also shows that there is a unique optimal counterexample (the set of odd elements).

Lower bound The set of odd elements in Z_{2k} has cardinality k, and a sum of three odd numbers is odd, and therefore nonzero. Upper bound Let $A \subseteq Z_{2k}$ have the property that no sum of three (not necessarily distinct) elements of A is zero. Let A contains an even number, $2j$ for some $j \in Z_{2k}$. Then $-j \notin A$ and $k-j \notin A$, for otherwise we could write $x+x+2j=0$, where x is $-j$ or $k - j$. Let $Y = Z_{2k} \setminus \{-j, k-j\}$; we thus have $A \subseteq Y$. Let us define the map $i: X \to Y$ defined by $i(t)=-2j-t$. Then $i(t) \neq t$ for any $t \in Y$, and A cannot contain both t and $i(t)$, for otherwise we could write $t+i(t)+2j=0 \Rightarrow$ A contains at most half the elements of Y, and hence $A| \le k-1$. If, on the other hand, A does not contain any even element, then A consists only of odd elements, so $|A| \le k$ and we are done.

For the part (b), arguing similarly, for $2k = 15$ one sees that the minimal number is 7, and that there are two optimal counter examples : $\{1,4,6,9,11,14\}$ and $\{2,3,7,8,12,13\}$.

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[1] http://vixra.org/pdf/1206.0015v1.pdf