About the Images and Inverse Images
Of Intuitionistic or Vague Fuzzy Subsets

G. Vasanti
Associate Professor, Department of Basic Sciences and Humanities
Aditya Institute of Technology and Management, Tekkali.

Keywords: Intuitionistic Fuzzy or Vague Subset, Intuitionistic Fuzzy (Inverse) Image.

Abstract. In this paper an exclusive study of some standard (lattice) algebraic properties for Intuitionistic fuzzy (inverse) images of Intuitionistic fuzzy subsets is done. Further as in crisp setup, characterizations for injectivity and surjectivity of maps in terms of some (lattice) algebraic properties of Intuitionistic fuzzy images and Intuitionistic fuzzy inverse images are performed.

1 Introduction

The traditional view in science, especially in mathematics, is to avoid uncertainty at all levels at any cost. Thus, 'being uncertain' is regarded as 'being unscientific'. But unfortunately in real life most of the information that we have to deal with is mostly uncertain. One of the paradigm shifts in science and mathematics in this century is to accept uncertainty as part of science and the desire to be able to deal with it, as there is very little left out in the practical real world for scientific and mathematical processing without this acceptance!

One of the earliest successful attempts in this direction is the development of the Theories of Probability and Statistics. However, both of them have their own natural limitations. Another successful attempt again in the same direction is the so called Fuzzy Set Theory, introduced by Lotfi Zadeh.

According to Zadeh, a fuzzy Subset of a set X is a function μ from X to the closed interval [0,1] of real numbers. The function μ, he called, the membership function which assigns to each member x of X its membership value, μx in [0, 1].


In 1983, Atanassov[1] generalized the notion of Zadeh fuzzy subset of a set further by introducing an additional function ν, which he called a non-membership function with some natural conditions on μ and ν, calling these new generalized fuzzy subsets of a set, Intuitionistic fuzzy subsets. Thus according to him an Intuitionistic fuzzy subset of a set X, is a pair A=(μA, νA), where μA, νA: X→ [0, 1] of real numbers such that for each x in X, μ(x) + ν (x) ≤ 1, where μA is called the membership function of A and μA is called the non -membership function of A.

Interestingly the same notion of Intuitionistic fuzzy subset of a set was also introduced by Gau and Buehrer[6] in 1993 under a different name called Vague subset. Thus whether we called Intuitionistic fuzzy subset of a set or if-subset of a set for short, or vague subset of a set, they are one and the same.

In stead of using long phrases like Intuitionistic fuzzy subset or vague subset, here onwards, we use the phrase if-subset. Obviously, if/v-subset only means Intuitionistic fuzzy/vague subset.
Ever since Atanassov [1] introduced the notion of Intuitionistic fuzzy subset of a set, several mathematicians started imposing and studying both algebraic and topological structures on Intuitionistic fuzzy subsets. Looking at several of the papers that are in print and online, one thing which becomes evident is that various (lattice) algebraic properties of images and inverse images of a Intuitionistic fuzzy subset which, incidentally, not only play a crucial role in the study of both Intuitionistic Fuzzy Algebra and Intuitionistic Fuzzy Topology but also are necessary for the individual/exclusive development of Intuitionistic Fuzzy Set Theory, are not yet studied, although these concepts were existing since long. In fact, these concepts of if/v-image and if/v inverse image are dealt in Ming [17], Thakur and Pandey [13], Davvaz, Dudek and Jun [4], Yon, Jun and Kim [16]. However, in this paper, we make an exclusive and somewhat detailed study of these (lattice) algebraic properties of Intuitionistic fuzzy images and Intuitionistic fuzzy inverse images under crisp maps. Further as in crisp setup, we characterize some injectivity and surjectivity of maps in terms of some (lattice) algebraic properties of Intuitionistic fuzzy images and Intuitionistic fuzzy inverse images.

A few of the results in this paper may be available in the literature elsewhere but scattered; however, we presented them here not only collectively but also in a suitable way for further research to the individual/exclusive development of Intuitionistic Fuzzy Set Theory.

For any set X, the set of all if-subsets of X be denoted by I(X). By defining, for any pair of if-subsets A = (μ_A, v_A) and B = (μ_B, v_B) of X, A ≤ B if μ_A ≤ μ_B and v_B ≤ v_A, I(X) becomes a complete infinitely distributive lattice. In this case for any family \( (A_i)_{i \in I} \) of if-subsets of X, \( V_{i \in I} A_i \) = \( \bigvee_{i \in I} \mu_{A_i} \land \bigwedge_{i \in I} v_{A_i} \) and \( \Lambda_{i \in I} A_i \) = \( \bigwedge_{i \in I} \mu_{A_i} \land V_{i \in I} v_{A_i} \), where for \( \lambda : X \rightarrow [0,1] \), \( V_{i \in I} \lambda_i x \) = \( \bigvee_{i \in I} \lambda_i \mu A_i x \) and \( \Lambda_{i \in I} \lambda_i x \) = \( \bigwedge_{i \in I} \lambda_i \mu A_i x \).

For any set X, one can naturally associate, with X, the if-subset \( (\mu_X, v_X) = (1_X, 0_X) \), where \( 1_X \) is the constant map assuming the value 1 for each \( x \in X \) and \( 0_X \) is the constant map assuming the value 0 of for each \( x \in X \), which turns out to be the largest element in \( I(X) \). Observe that then, the if-empty subset \( \Phi \) of \( X \) gets naturally associated with the if-subset \( (\mu_\emptyset, v_\emptyset) = (0,1) \), which turns out to be the least element in \( I(X) \).

Let A = (μ_A, v_A) be an if-subset of X. Then it turns out that (v_A, μ_A) is also an if-subset of X, thus for any if-subset A = (μ_A, v_A) the if-complement of A, denoted by A^c, is defined by (v_A, μ_A). Observe that A^c = X \( - A \) = X \land A^c. Further for any pair A, B of if/v-subsets of X, we define B \( - A \) to be B \land A^c. In other words, for if/v-subsets B = (μ_B, v_B) and A = (μ_A, v_A) of X, B \( - A \) = (μ_B \land v_A, v_B \land μ_A).

Throughout this paper the capital letters X, Y, Z stand for arbitrary but fixed (crisp) sets, the small letters f, g stand for arbitrary but fixed (crisp) maps f: X \( \rightarrow \) Y and g: Y \( \rightarrow \) Z, the capital letters A, B, C, D, E, F together with their suffixes stand for if/v-subsets and the capital letters I and J stand for the index sets. In general whenever P is an if/v-subset of a set X, always μ_P and v_P denote the membership and non membership function of the if/v-subset P respectively. Also we frequently use the standard convention that V\( \emptyset \) = 0 and A\( \emptyset \) = 1.

## 2 Main Results

In this section the notions and properties of Intuitionistic fuzzy/vague image and Intuitionistic fuzzy/ vague inverse image for an Intuitionistic fuzzy/vague subset of a set under a crisp map are recalled and are shown to be well defined.

### 2.1 if-Images and if-Inverse Images

Let X, Y be a pair of sets and let f: X \( \rightarrow \) Y be a map. Let A = (μ_A, v_A) and B = (μ_B, v_B) be if/v-subsets of X and Y respectively.
Definitions and Statements 2.1.1: Let $X, Y, f$ and $B$ be as above. Then the subset $C$ of $X$ defined as in (3) above, called the Intuitionistic fuzzy/vague inverse image of $B$ under $f$ or simply the if/v-inverse image of $B$ under $f$, is denoted by $f^{-1}B$.

2.2 Mapping Properties of if-Images and if-Inverse Images

In this section we show that several of the mapping properties that hold good for Zadeh fuzzy subsets are also held good for the Intuitionistic fuzzy subsets.

2.2.1 Theorem Let $X, Y$ be a pair of sets and let $f: X \rightarrow Y$ be a map. Let $A, A_i$ and $B, B_i$ be if-subsets of $X$ and $Y$ respectively. Then the following are true:

1. $A \subseteq C$ implies $fA \subseteq fC$
2. $B \subseteq D$ implies $f^{-1}B \subseteq f^{-1}D$
3. $A \subseteq f^{-1}fA$
4. $A = f^{-1}fA \iff f$ is 1-1
5. $f f^{-1}B \subseteq B$
6. $f f^{-1}B = B \iff f$ is onto
7. $f(V_{i=1}A_i) = V_{i=1}fA_i$
8. $f(A_i \cup A_i) \leq A_i \cup fA_i$ and the equality is true whenever $f$ is 1-1.
9. $f^{-1}(V_{i=1}B_i) = V_{i=1}f^{-1}B_i$
10. $f^{-1}(A_i \cup B_i) = A_i \cup f^{-1}B_i$
11. $fA = \varphi \iff A = \varphi$, in particular, $f\varphi = \varphi$, and when $f$ is 1-1, $f A = fX \iff A = X$
12. $f^{-1}B = X \iff B \subseteq fX$, in particular, $f^{-1}fX = X$ and $f^{-1}B = \varphi \iff B \subseteq Y - fX$, in particular, $f^{-1}\varphi = \varphi$
13. $fX - fA \subseteq (fX - A)$ and the equality holds whenever $f$ is 1-1
14. $f^{-1}(Y - B) = X - (f^{-1}B)$
15. $f f^{-1}B = B \Lambda fX$ and hence always $ff^{-1}B \subseteq B$
16. $f f^{-1}B = f^{-1}(B \Lambda fX)$
17. $fA \subseteq B \iff A \subseteq f^{-1}B$
18. (i) $ff^{-1}A = fA$ (ii) $f^{-1}f^{-1}B = f^{-1}B$

2.2.2 Theorem Let $X, Y$ and $Z$ be three sets and let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be a pair of maps. Then the following are true:

1. $(gf)(A) = g(f(A))$ for all $A \subseteq X$
2. $(g \circ f)^{-1}(C) = f^{-1}(g^{-1}C)$ for all $C \subseteq Z$

Acknowledgements:
The author would like to express her heart full thanks to the Dr.N.V.E.S.Murthy, Department of Computer Science and System Engineering, AUCE, Andhra University, for his constant
encouragement and help throughout the preparation of this paper. The author would like to express her thankfulness to the management of Aditya Institute of Technology and Management, Tekkali for their inspiration and support.

References