Intuitionistic Fuzzy semiboundary and intuitionistic Fuzzy Product Related spaces

¹V. Thiripurasundari and ²S. Murugesan

Department of Mathematics

^{1,2}Sri S.Ramasamy Naidu Memorial college, Sattur-626 203, India

E-mail: ¹thiripurasund@gmail.com, ² satturmuruges@gmail.com

Keywords: Intuitionistic fuzzy topology, intuitionistic fuzzy topological spaces, intuitionistic fuzzy semi open set, intuitionistic fuzzy semi closed set, intuitionistic fuzzy semi boundary, intuitionistic fuzzy product related spaces, intuitionistic fuzzy semi continuous function.

Abstract: In this paper intuitionistic fuzzy semi boundary is introduced and its properties are investigated. Intuitionistic fuzzy semi continuous functions are characterized via intuitionistic fuzzy semi boundary.

1. Introduction

In the year 1965, Zadeh[10] introduced the concept of fuzzy sets. After that there have been a number of generalizations of this fundamental concepts. In the year 1986 Atanassov[1] introduced intuitionistic fuzzy set as a generalization of fuzzy set. Using the concept of intuitionistic fuzzy set, Coker[5] extended the concept of fuzzy topological spaces introduced by C.L.Chang[4] to intuitionistic fuzzy topological spaces. Pu and Liu[8] defined the notion of fuzzy boundary in fuzzy topological spaces in 1980. Properties of fuzzy boundary were investigated by Ahmed and Athar[2].

In this paper we introduce intuitionistic fuzzy semi boundary and investigate some of their properties. Further intuitionistic fuzzy semi boundary in product related spaces is analysed. Finally necessary conditions for intuitionistic fuzzy semi continuous functions are obtained via intuitionistic fuzzy semi boundary. Throughout this paper X, Y are non-empty sets.

2. Preliminaries

Definition 2.1: [1] An intuitionistic fuzzy set (IFS) A in X is an object having the form $A = \{(x, \mu_A(x), \gamma_A(x)) | x \in X\}$ where the functions $\mu_A: X \rightarrow [0,1]$ and $\gamma_A: X \rightarrow [0,1]$ denote the degree of membership (namely, $\mu_A(x)$) and the degree of non-membership (namely, $\gamma_A(x)$) of each element $x \in X$ to the set A respectively, and

$$0 \leq \mu_A(x) + \gamma_A(x) \leq 1 \text{ for each } x \in X$$

Definition 2.2: [1] Let A and B be IFS's of the forms

$A = \{(x, \mu_A(x), \gamma_A(x)) | x \in X\}$ and $B = \{(x, \mu_B(x), \gamma_B(x)) | x \in X\}$. Then,

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$

(b) $A=B$ if and only if $A \subseteq B$ and $B \subseteq A$

(c) The complement of A is denoted by $\bar{A}$ and is defined by

$$\bar{A} = \{(x, \gamma_A(x), \mu_A(x)) | x \in X\},$$

(d) $A \cap B = \{(x, \mu_A(x) \wedge \mu_B(x), \gamma_A(x) \vee \gamma_B(x)) | x \in X\},$

(e) $A \cup B = \{(x, \mu_A(x) \vee \mu_B(x), \gamma_A(x) \wedge \gamma_B(x)) | x \in X\}.$

The IFS's $0_\sim = \{ \langle x, 0, 1 \rangle | x \in X \}$ and $1_\sim = \{ \langle x, 1, 0 \rangle | x \in X \}$ are respectively the empty set and the whole set of X . For the sake of simplicity, we will use the notation $A = \langle x, \mu_A, \gamma_A \rangle$ instead of $A = \{(x, \mu_A(x), \gamma_A(x)) | x \in X\}$.

Definition 2.3: [1] Let X and Y be IFTS's and $f: X \rightarrow Y$ be a function. If

$B = \{\langle y, \mu_B(y), \gamma_B(y) \rangle | y \in Y\}$ is an IFS in Y , then the pre image of B under f , denoted by $f^{-1}(B)$ is the IFS in X defined by

$$f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\gamma_B)(x) \rangle | x \in X\}$$

and the image of A under f , denoted by $f(A) = \langle y, f(\mu_A), f(\gamma_A) \rangle$ is an IFS of Y , where for each $y \in Y$

$$f(\mu_A)(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \mu_A(x) & \text{if } f^{-1}(y) \neq \emptyset, \\ 0 & \text{otherwise.} \end{cases}$$

$$f(\gamma_A)(y) = \begin{cases} \inf_{x \in f^{-1}(y)} \gamma_A(x) & \text{if } f^{-1}(y) \neq \emptyset, \\ 1 & \text{otherwise.} \end{cases}$$

It is noted that $\overline{f^{-1}(B)} = f^{-1}(\bar{B})$

Definition 2.4: [5] An intuitionistic fuzzy topology (IFT) on X is a family τ of IFS's in X satisfying the following axioms:

- (1) $0_\sim, 1_\sim \in \tau$,
- (2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
- (3) $\bigcap G_i \in \tau$ for any family $\{G_i / i \in J\} \subseteq \tau$.

In this case, the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS) in X . The complement $\bar{A}$ of an IFOS A in IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS) in X .

Definition 2.5: [5] Let (X, τ) be an IFTS and let $A = \langle x, \mu_A, \gamma_A \rangle$ be an IFS in X . Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure of A are defined by

$$\text{Int}(A) = \bigcup \{G | G \text{ is an IFOS in } X \text{ and } G \subseteq A\},$$

$$\text{Cl}(A) = \bigcap \{K | K \text{ is an IFCS in } X \text{ and } A \subseteq K\}.$$

Properties of closure and interior of a intuitionistic fuzzy set which are needed in the sequel, are summarized in the following lemma.

Lemma 2.6: For any IFS's A and B in IFTS (X, τ) ,

- (1) $\text{Cl}(\bar{A}) = \overline{\text{Int}(A)}$;
- (2) $\text{Int}(\bar{A}) = \overline{\text{Cl}(A)}$;
- (3) A is an intuitionistic fuzzy closed (IFC) $\Leftrightarrow \text{Cl}(A) = A$ (resp. intuitionistic fuzzy open (IFO) $\Leftrightarrow \text{Int}(A) = A$)
- (4) $A \leq B \Rightarrow \text{Cl}(A) \leq \text{Cl}(B)$; ($\text{Int}(A) \leq \text{Int}(B)$);
- (5) $\text{Cl}(\text{Cl}(A)) = \text{Cl}(A)$; ($\text{Int}(\text{Int}(A)) = \text{Int}(A)$);
- (6) $\text{Cl}(A) \vee \text{Cl}(B) = \text{Cl}(A \vee B)$;
- (7) $\text{Cl}(A) \wedge \text{Cl}(B) \geq \text{Cl}(A \wedge B)$;
- (8) $\text{Int}(A) \vee \text{Int}(B) \leq \text{Int}(A \vee B)$;
- (9) $\text{Int}(A) \wedge \text{Int}(B) = \text{Int}(A \wedge B)$.

Definition 2.7: [9] Let A be a IFS in an IFTS (X, τ) . Then the intuitionistic fuzzy boundary of A is defined as $\text{IBd } A = \text{Cl } A \wedge \overline{\text{Cl } A}$. $\text{IBd } A$ is a intuitionistic fuzzy closed set (IFCS).

3. Intuitionistic Fuzzy Semiboundary

Definition 3.1: [6] An IFS A in an IFTS (X, τ) is called an intuitionistic fuzzy semi open set (IFSOS) if $A \subseteq_{cl} (int(A))$ (or) if there exists $V \in \tau$ such that $V \leq A \leq CIV$.

An IFS A is called intuitionistic fuzzy semi-closed set, if the complement of $\bar{A}$ is an IFSOS.

Definition 3.2: [6] Let A be a IFS in an IFTS (X, τ) . Then Semi interior (SInt) and semi closure (SCl) of A are given as

$$SInt A = \bigcup \{G / G \text{ is an IFSOS in } X, G \subseteq A\}$$

$$SCl A = \bigcap \{K / K \text{ is an IFSCS in } X, A \subseteq K\}$$

Theorem 3.3: For IFSs A and B in an IFTS (X, τ) . Then

- (1) $SInt(A \vee B) \geq SInt(A) \vee (SInt B)$;
- (2) $SInt(A \wedge B) = SInt(A) \wedge (SInt B)$;
- (3) $SCl(A \vee B) = SCl(A) \vee SCl(B)$
- (4) $SCl(A \wedge B) \leq SCl(A) \wedge SCl(B)$;
- (5) $SCl SCl(A) = SCl(A)$
- (6) $SInt SInt(A) = SInt(A)$
- (7) $\overline{SCl(A)} = SInt(\bar{A})$;
- (8) $\overline{SInt(A)} = SCl(\bar{A})$.

Proof: (1) $SInt(A)$ and $SInt(B)$ are both intuitionistic fuzzy semiopen and $A \leq A \vee B$, $B \leq A \vee B \Rightarrow SInt(A) \leq SInt(A \vee B)$ and $SInt(B) \leq SInt(A \vee B)$. Hence, $SInt(A) \vee SInt(B) \leq SInt(A \vee B)$.

(2) $A \wedge B \leq A$ and $A \wedge B \leq B \Rightarrow SInt(A \wedge B) \leq SInt(A) \wedge SInt(B)$.

$SInt(A \wedge B) \leq SInt(B)$. Therefore $SInt(A \wedge B) \leq SInt(A) \wedge SInt(B)$.

Conversely, $SInt(A) \leq A$ and $SInt(B) \leq B \Rightarrow SInt(A) \wedge SInt(B) \leq A \wedge B$ and $SInt(A) \wedge SInt(B)$ is intuitionistic fuzzy semiopen. But $SInt(A \wedge B)$ is the largest intuitionistic fuzzy semiopen set contained in $A \wedge B$. Therefore $SInt(A) \wedge SInt(B) \leq SInt(A \wedge B)$ Hence $SInt(A) \wedge SInt(B) = SInt(A \wedge B)$

(3) It follows easily from (2).

(4) Since $A \wedge B \leq A$, $A \wedge B \leq B \Rightarrow SCl(A \wedge B) \leq SCl(A)$,

$SCl(A \wedge B) \leq SCl(B)$ Hence, $SCl(A \wedge B) \leq SCl(A) \wedge SCl(B)$

(5) – (8) proofs are straight forward.

In theorem 3.3 (1 and 4), the equality may not holds as seen in the following example.

Example 3.4: Let $X = \{a, b\}$ and let $A = \langle x, \frac{0.6}{a} + \frac{0.5}{b}, \frac{0.4}{a} + \frac{0.3}{b} \rangle$. Then

$\tau = \{0_\sim, 1_\sim, A\}$ is an IFT on X . $\tau' = \{0_\sim, 1_\sim, \langle x, \frac{0.4}{a} + \frac{0.3}{b}, \frac{0.6}{a} + \frac{0.5}{b} \rangle\}$.

$IFSOS(X) = \{0_\sim, 1_\sim, \langle x, \frac{l_1}{a} + \frac{l_2}{b}, \frac{m_1}{a} + \frac{m_2}{b} \rangle \text{ where } 0.6 \leq l_1 \leq 1, 0.5 \leq l_2 \leq 1,$

$0 \leq m_1 \leq 0.4, 0 \leq m_2 \leq 0.3\}$.

$IFSCS(X) = \{0_\sim, 1_\sim, \langle x, \frac{\alpha_1}{a} + \frac{\alpha_2}{b}, \frac{\beta_1}{a} + \frac{\beta_2}{b} \rangle \text{ where } 0 \leq \alpha_1 \leq 0.4, 0 \leq \alpha_2 \leq 0.3,$

$0.6 \leq \beta_1 \leq 1, 0.5 \leq \beta_2 \leq 1\}$.

$B = \langle x, \frac{0.6}{a} + \frac{0.2}{b}, \frac{0.4}{a} + \frac{0.5}{b} \rangle$ and $C = \langle x, \frac{0.3}{a} + \frac{0.5}{b}, \frac{0.6}{a} + \frac{0.3}{b} \rangle$.

Let

Then, $B \wedge C = \langle x, \frac{0.3}{a} + \frac{0.2}{b}, \frac{0.6}{a} + \frac{0.5}{b} \rangle, B \vee C = \langle x, \frac{0.6}{a} + \frac{0.5}{b}, \frac{0.4}{a} + \frac{0.3}{b} \rangle, SCl B = 1_{\sim},$
 $SCl C = 1_{\sim}, SInt B = 0_{\sim}, SInt C = 0_{\sim}, SCl(B \wedge C) = \langle x, \frac{0.3}{a} + \frac{0.2}{b}, \frac{0.6}{a} + \frac{0.5}{b} \rangle,$

$$SInt(B \vee C) = \langle x, \frac{0.6}{a} + \frac{0.5}{b}, \frac{0.4}{a} + \frac{0.3}{b} \rangle.$$

Hence, $SInt(B \vee C) = \langle x, \frac{0.6}{a} + \frac{0.5}{b}, \frac{0.4}{a} + \frac{0.3}{b} \rangle \neq 0_{\sim} = SInt(B) \vee SInt(C),$

$$SCl(B \wedge C) = \langle x, \frac{0.3}{a} + \frac{0.2}{b}, \frac{0.6}{a} + \frac{0.5}{b} \rangle \neq 1_{\sim} = SCl(B) \wedge SCl(C).$$

Definition 3.5: Let A be a IFS in an IFTS (X, τ) . Then the intuitionistic fuzzy semiboundary of A is defined as $ISBd A = SCl A \wedge SCl \bar{A}$. $ISBd A$ is a intuitionistic fuzzy semiclosed set(IFSCS).

Remark 3.6: In classical topology, for an arbitrary set A of a topological space X, $A \cup SBd A = SCl A$, but $A \vee ISBd A \leq SCl A$, for an arbitrary intuitionistic fuzzy set A in an IFTS (X, τ) , where the equality may not hold as is seen in the following example.

Example 3.7: Let $X = \{a, b\}$ and let $A = \langle x, \frac{0.2}{a} + \frac{0.3}{b}, \frac{0.4}{a} + \frac{0.6}{b} \rangle$. Then $\tau = \{0_{\sim}, 1_{\sim}, A\}$ is an IFTS on X,
 $\tau' = \{0_{\sim}, 1_{\sim}, \langle x, \frac{0.4}{a} + \frac{0.6}{b}, \frac{0.2}{a} + \frac{0.3}{b} \rangle\}$. $IFSOS(X) = \{0_{\sim}, 1_{\sim}, \langle x, \frac{l_1}{a} + \frac{l_2}{b}, \frac{m_1}{a} + \frac{m_2}{b} \rangle \text{ where } 0.2 \leq l_1 \leq 0.4, 0.3 \leq l_2 \leq 0.6, 0.2 \leq m_1 \leq 0.4, 0.3 \leq m_2 \leq 0.6\}$

$$IFSCS(X) = \{0_{\sim}, 1_{\sim}, \langle x, \frac{\alpha_1}{a} + \frac{\alpha_2}{b}, \frac{\beta_1}{a} + \frac{\beta_2}{b} \rangle \text{ where } 0.2 \leq \alpha_1 \leq 0.4, 0.3 \leq \alpha_2 \leq 0.6,$$

$$0.2 \leq \beta_1 \leq 0.4, 0.3 \leq \beta_2 \leq 0.6\}.$$

Choose, $P = \langle x, \frac{0.5}{a} + \frac{0.6}{b}, \frac{0.4}{a} + \frac{0.3}{b} \rangle$ be IFTS on X.

$$\text{Then } SCl P = 1_{\sim}, SCl \bar{P} = \langle x, \frac{0.4}{a} + \frac{0.3}{b}, \frac{0.4}{a} + \frac{0.6}{b} \rangle,$$

$$ISBd P = SCl P \wedge SCl \bar{P} = \langle x, \frac{0.4}{a} + \frac{0.3}{b}, \frac{0.4}{a} + \frac{0.6}{b} \rangle.$$

Hence, $P \vee ISBd P = \langle x, \frac{0.5}{a} + \frac{0.6}{b}, \frac{0.4}{a} + \frac{0.3}{b} \rangle \neq 1_{\sim} = SCl P,$

Proposition 3.8: Let A and B be IFS's in an IFTS (X, τ) . Then the following conditions hold.

- (1) $ISBd A = ISBd \bar{A}$
- (2) If A is IFSC, then $ISBd A \leq A$
- (3) If A is IFSO, then $ISBd A \leq \bar{A}$
- (4) Let $A \leq B$ and $B \in IFSCS(X)$ (resp., $B \in IFSCS(X)$). Then, $ISBd A \leq B$ (resp., $ISBd A \leq \bar{B}$);
- (5) $(\overline{ISBd A}) = SInt A \vee SInt \bar{A}$;
- (6) $ISBd A \leq IBd A$;
- (7) $SCl(ISBd A) \leq IBd A$

Proof: (1) $ISBd A = SCl A \wedge SCl \bar{A} = SCl \bar{A} \wedge SCl A$

$$= SCl \bar{A} \wedge SCl \bar{\bar{A}} = ISBd \bar{A}$$

(2) $ISBd A = SCl A \wedge SCl \bar{A} \leq SCl A = A$ Hence $ISBd A \leq A$.

(3) A is IFSO implies $\bar{A}$ IFSC. By (1), (2) $ISBd \bar{A} \leq \bar{A}$ Hence $ISBd A \leq \bar{A}$.

(4) Since $A \leq B$ implies $SCl A \leq SCl B$

$ISBd A = SCl A \wedge SCl \bar{A} \leq SCl B \wedge SCl \bar{A} \leq SCl B = B$. Hence, $ISBd A \leq B$.

(5) $\overline{ISBd A} = \overline{(SCl A \wedge SCl \bar{A})} = (\overline{SCl A}) \vee (\overline{SCl \bar{A}}) = SInt \bar{A} \vee SInt \bar{\bar{A}}$

$= SInt \bar{A} \vee SInt A$. Hence, $\overline{ISBd A} = SInt A \vee SInt \bar{A}$.

(6) Since $SCl A \leq Cl A$ and $SCl \bar{A} \leq Cl \bar{A}$.

Then $ISBd A = SCl A \wedge SCl \bar{A} \leq Cl A \wedge Cl \bar{A} \leq IBd A$. Hence, $ISBd A \leq IBd A$.

(7) $SCl(ISBd A) = SCl(SCl A \wedge SCl \bar{A}) \leq SCl SCl A \wedge SCl SCl \bar{A} = SCl A \wedge SCl \bar{A}$

$= ISBd A \leq IBd A$ Hence, $SCl(ISBd A) \leq IBd A$.

The converse of (2), (3) and the equality may hold in (6), (7) of Proposition 3.8 is not true as seen in the following example.

Example 3.9: Choose IFSs $P = \langle x, \frac{0.5}{a} + \frac{0.6}{b}, \frac{0.4}{a} + \frac{0.3}{b} \rangle$

$Q = \langle x, \frac{0.3}{a} + \frac{0.4}{b}, \frac{0.6}{a} + \frac{0.4}{b} \rangle$ in the IFTS (X, τ) defined in example 3.7.

Then (1) $SCl P = 1_{\sim}, SCl \bar{P} = \langle x, \frac{0.4}{a} + \frac{0.3}{b}, \frac{0.4}{a} + \frac{0.6}{b} \rangle$,

$ISBd P = \langle x, \frac{0.4}{a} + \frac{0.3}{b}, \frac{0.4}{a} + \frac{0.6}{b} \rangle \leq P$. but P is not intuitionistic fuzzy semi closed.

(2) $SCl Q = \langle x, \frac{0.3}{a} + \frac{0.4}{b}, \frac{0.4}{a} + \frac{0.4}{b} \rangle$, $SCl \bar{Q} = 1_{\sim}$,

$ISBd Q = \langle x, \frac{0.3}{a} + \frac{0.4}{b}, \frac{0.4}{a} + \frac{0.4}{b} \rangle \leq \bar{Q}$.

but Q is not intuitionistic fuzzy semi open.

(3) $Cl P = 1_{\sim}, Cl \bar{P} = \langle x, \frac{0.4}{a} + \frac{0.6}{b}, \frac{0.2}{a} + \frac{0.3}{b} \rangle$, $IBd P = \langle x, \frac{0.4}{a} + \frac{0.6}{b}, \frac{0.2}{a} + \frac{0.3}{b} \rangle$.

Hence, $ISBd P \not\leq IBd P$

(4) $SCl(ISBd P) = \langle x, \frac{0.4}{a} + \frac{0.3}{b}, \frac{0.4}{a} + \frac{0.6}{b} \rangle \not\leq IBd P$.

Proposition 3.10: Let A be an intuitionistic fuzzy set in an IFTS (X, τ) . Then,

(1) $ISBd A = SCl A \wedge (\overline{SInt A})$;

(2) $ISBd (SInt A) \leq ISBd A$;

(3) $ISBd (SCl A) \leq ISBd A$;

(4) $SInt A \leq A \wedge (\overline{ISBd A})$.

The following example shows that the equality may not hold in (2), (3) and (4) of Proposition 3.10.

Example 3.11: Choose IFS $M = \langle x, \frac{0.5}{a} + \frac{0.2}{b}, \frac{0.5}{a} + \frac{0.6}{b} \rangle$ in the IFTS (X, τ) defined in example 3.7.

Then, (1) $SCl M = 1_{\sim}$, $SCl \bar{M} = 1_{\sim}$, $SInt M = 0_{\sim}$, $SInt \bar{M} = 0_{\sim}$

$ISBd M = 1_{\sim}$, $\overline{ISBd M} = 1_{\sim}$.

$ISBd (SInt M) = SCl (SInt M) \wedge SCl (\overline{SInt M}) = SCl (0_{\sim}) \wedge SCl (1_{\sim}) = 0_{\sim}$.

Hence, $\text{ISBd}(\text{SInt } M) = 0_{\sim} \not\leq \text{ISBd } M = 1_{\sim}$

$$(2) \text{ISBd}(\text{Scl } M) = \text{Scl}(\text{Scl } M) \wedge \text{Scl}(\overline{\text{Scl } M}) = \text{Scl}(1_{\sim}) \wedge \text{Scl}(0_{\sim}) = 0_{\sim}.$$

Hence, $\text{ISBd}(\text{Scl } M) \not\leq \text{ISBd } M$

$$(3) \text{SInt } M = 0_{\sim} \not\leq M \wedge \overline{\text{ISBd } M} = \langle x, \frac{0.5}{a} + \frac{0.2}{b}, \frac{0.5}{a} + \frac{0.6}{b} \rangle.$$

Remark 3.12: In general topology, the following conditions hold.

$$\text{SBd } A \cap \text{SInt } A = \emptyset ; \text{SInt } A \cup \text{SBd } A = \text{Scl } A; \text{SInt } A \cup \text{SInt } A^c \cup \text{SBd } A = X.$$

In intuitionistic fuzzy topology, these may not hold as seen in the following example.

Choose IFS

Example 3.13: Choose IFS $P = \langle x, \frac{0.5}{a} + \frac{0.6}{b}, \frac{0.4}{a} + \frac{0.3}{b} \rangle$ in the IFTS $(X\tau)$ defined in example 3.7.

$$(1) \text{ISBd } P = \langle x, \frac{0.4}{a} + \frac{0.3}{b}, \frac{0.4}{a} + \frac{0.6}{b} \rangle, \text{SInt } P = \langle x, \frac{0.4}{a} + \frac{0.6}{b}, \frac{0.4}{a} + \frac{0.3}{b} \rangle$$

Hence, $\text{ISBd } P \wedge \text{SInt } P \neq 0_{\sim}$.

$$(2) \text{SInt } P \vee \text{ISBd } P = \langle x, \frac{0.4}{a} + \frac{0.6}{b}, \frac{0.4}{a} + \frac{0.3}{b} \rangle \neq 1_{\sim} = \text{Scl } P$$

$$(3) \text{SInt } P \vee \text{SInt } \bar{P} \vee \text{ISBd } P = \langle x, \frac{0.4}{a} + \frac{0.6}{b}, \frac{0.4}{a} + \frac{0.3}{b} \rangle \neq 1_{\sim}$$

Theorem 3.14: Let A and B be IFS's in an IFTS $(X\tau)$. Then,
 $\text{ISBd}(A \vee B) \leq \text{ISBd } A \vee \text{ISBd } B$.

Proof: $\text{ISBd}(A \vee B) = \text{Scl}(A \vee B) \wedge \text{Scl}(\overline{A \vee B})$

$$\leq (\text{Scl } A \vee \text{Scl } B) \wedge (\text{Scl } \bar{A} \wedge \text{Scl } \bar{B})$$

$$= [\text{Scl } A \wedge (\text{Scl } \bar{A} \wedge \text{Scl } \bar{B})] \vee [\text{Scl } B \wedge (\text{Scl } \bar{A} \wedge \text{Scl } \bar{B})]$$

$$= [\text{ISBd } A \wedge \text{Scl } \bar{B}] \vee [\text{ISBd } B \wedge \text{Scl } \bar{A}] \leq \text{ISBd } A \vee \text{ISBd } B$$

Hence, $\text{ISBd}(A \vee B) \leq \text{ISBd } A \vee \text{ISBd } B$

In theorem 3.14, the equality may not hold as seen in the following example.

Example 3.15: Choose IFS's $P = \langle x, \frac{0.5}{a} + \frac{0.6}{b}, \frac{0.4}{a} + \frac{0.3}{b} \rangle$

$Q = \langle x, \frac{0.5}{a} + \frac{0.2}{b}, \frac{0.5}{a} + \frac{0.6}{b} \rangle$ in the IFTS $(X\tau)$ defined in example 3.7.

Then $\text{Scl } P = 1_{\sim}, \text{Scl } \bar{P} = \langle x, \frac{0.4}{a} + \frac{0.3}{b}, \frac{0.4}{a} + \frac{0.6}{b} \rangle,$

$$\text{ISBd } P = \langle x, \frac{0.4}{a} + \frac{0.3}{b}, \frac{0.4}{a} + \frac{0.6}{b} \rangle. \text{Scl } Q = 1_{\sim}, \text{Scl } \bar{Q} = 1_{\sim}, \text{ISBd } Q = 1_{\sim}.$$

$$P \vee Q = \langle x, \frac{0.5}{a} + \frac{0.6}{b}, \frac{0.4}{a} + \frac{0.3}{b} \rangle, \text{Scl } P \vee Q = 1_{\sim}, \text{Scl } (\overline{P \vee Q}) = \langle x, \frac{0.4}{a} + \frac{0.3}{b}, \frac{0.4}{a} + \frac{0.6}{b} \rangle.$$

$$\text{ISBd } (P \vee Q) = \langle x, \frac{0.4}{a} + \frac{0.3}{b}, \frac{0.4}{a} + \frac{0.6}{b} \rangle \neq 1_{\sim} = \text{ISBd } P \vee \text{ISBd } Q.$$

Hence, $\text{ISBd}(P \vee Q) \not\leq \text{ISBd } P \vee \text{ISBd } Q$.

Remark 3.16: Let P, Q be IFS's in an IFTS $(X\tau)$, then the IFS's

$\text{ISBd}(P \wedge Q)$ and $\text{ISBd } P \wedge \text{ISBd } Q$ behave arbitrarily is seen in the following example.

Example 3.17: Choose IFS's $P = \langle x, \frac{0.5}{a} + \frac{0.6}{b}, \frac{0.4}{a} + \frac{0.3}{b} \rangle$

$$Q = \langle x, \frac{0.5}{a} + \frac{0.2}{b}, \frac{0.5}{a} + \frac{0.6}{b} \rangle, \quad R = \langle x, \frac{0.4}{a} + \frac{0.8}{b}, \frac{0.6}{a} + \frac{0.2}{b} \rangle$$

and

$S = \langle x, \frac{0.6}{a} + \frac{0.3}{b}, \frac{0.4}{a} + \frac{0.5}{b} \rangle$ in the IFTS $(X\tau)$ of example 3.7. Then,

$$(i) \text{ ISBd } P = \langle x, \frac{0.4}{a} + \frac{0.3}{b}, \frac{0.4}{a} + \frac{0.6}{b} \rangle, \text{ ISBd } Q = 1_{\sim}, \text{ ISBd } (P \wedge Q) = 1_{\sim}$$

$$\text{ISBd } P \wedge \text{ISBd } Q = \langle x, \frac{0.4}{a} + \frac{0.3}{b}, \frac{0.4}{a} + \frac{0.6}{b} \rangle$$

Hence, $\text{ISBd}(P \wedge Q) > \text{ISBd } P \wedge \text{ISBd } Q$.

$$(ii) \text{ ISBd } R = 1_{\sim}, \text{ ISBd } S = \langle x, \frac{0.4}{a} + \frac{0.5}{b}, \frac{0.4}{a} + \frac{0.3}{b} \rangle$$

$$\text{ISBd } R \wedge \text{ISBd } S = \langle x, \frac{0.4}{a} + \frac{0.5}{b}, \frac{0.4}{a} + \frac{0.3}{b} \rangle$$

$$\text{ISBd}(R \wedge S) = \langle x, \frac{0.4}{a} + \frac{0.3}{b}, \frac{0.4}{a} + \frac{0.5}{b} \rangle$$

Hence, $\text{ISBd } R \wedge \text{ISBd } S > \text{ISBd } (R \wedge S)$.

Theorem 3.18: For any intuitionistic fuzzy sets A and B in an IFTS $(X\tau)$,

$$\text{ISBd } (A \wedge B) \leq (\text{ISBd } A \wedge \text{Scl } B) \vee (\text{ISBd } B \wedge \text{Scl } A)$$

Proof:

$$\text{ISBd } (A \wedge B) = \text{Scl}(A \wedge B) \wedge \overline{\text{Scl}(A \wedge B)}$$

$$\leq (\text{Scl } A \wedge \text{Scl } B) \wedge (\text{Scl } \bar{A} \vee \text{Scl } \bar{B})$$

$$= [(\text{Scl } A \wedge \text{Scl } B) \wedge \text{Scl } \bar{A}] \vee [(\text{Scl } A \wedge \text{Scl } B) \wedge \text{Scl } \bar{B}]$$

$$= [(\text{Scl } A \wedge \text{Scl } \bar{A}) \wedge \text{Scl } B] \vee [(\text{Scl } B \wedge \text{Scl } \bar{B}) \wedge \text{Scl } A]$$

Hence, $\text{ISBd } (A \wedge B) \leq (\text{ISBd } A \wedge \text{Scl } B) \vee (\text{ISBd } B \wedge \text{Scl } A)$.

Corollary 3.19: For any intuitionistic fuzzy sets A and B in an IFTS $(X\tau)$, then

$$\text{ISBd } (A \wedge B) \leq \text{ISBd } A \vee \text{ISBd } B.$$

The equality in the above theorem may not hold as seen in the following example.

Example 3.20: Choose IFS's $R = \langle x, \frac{0.4}{a} + \frac{0.8}{b}, \frac{0.6}{a} + \frac{0.2}{b} \rangle$ and

$S = \langle x, \frac{0.6}{a} + \frac{0.3}{b}, \frac{0.4}{a} + \frac{0.5}{b} \rangle$ in the IFTS $(X\tau)$ of example 3.7.

Then, $\text{ISBd } R = 1_{\sim}, \text{ ISBd } S = \langle x, \frac{0.4}{a} + \frac{0.5}{b}, \frac{0.4}{a} + \frac{0.3}{b} \rangle$

$$(\text{ISBd } R \wedge \text{Scl } S) \vee (\text{ISBd } S \wedge \text{Scl } R) = 1_{\sim} \vee \langle x, \frac{0.4}{a} + \frac{0.5}{b}, \frac{0.4}{a} + \frac{0.3}{b} \rangle = 1_{\sim}.$$

$$\text{ISBd } (R \wedge S) = \langle x, \frac{0.4}{a} + \frac{0.3}{b}, \frac{0.4}{a} + \frac{0.5}{b} \rangle$$

Hence,

$$\text{ISBd } (R \wedge S) \not\leq (\text{ISBd } R \wedge \text{Scl } S) \vee (\text{ISBd } S \wedge \text{Scl } R).$$

Proposition 3.21: For any IFS A in an IFTS X, then,

$$(1) \text{ ISBd } (\text{ISBd } A) \leq \text{ISBd } A,$$

$$(2) \text{ ISBd } (\text{ISBd } (\text{ISBd } A)) \leq \text{ISBd } (\text{ISBd } A)$$

Proof:

$$(1) \text{ ISBd (ISBd A) = SCl (ISBd A) } \wedge \text{ SCl } (\overline{\text{ISBd A}})$$

$$\leq \text{ SCl (ISBd A) } \leq \text{ ISBd A } \text{ of proposition 3.8}$$

Hence, $\text{ISBd (ISBd A)} \leq \text{ISBd A}$.

$$(2) \text{ ISBd (ISBd (ISBd A)) = SCl (ISBd (ISBd A)) } \wedge \text{ SCl } (\overline{\text{ISBd (ISBd A)}})$$

$$\leq \text{ ISBd (ISBd A) } \wedge \text{ SCl } (\overline{\text{ISBd (ISBd A)}}) \leq \text{ ISBd (ISBd A)}$$

Hence, $\text{ISBd (ISBd (ISBd A))} \leq \text{ISBd (ISBd A)}$.

Remark 3.22: The equality may not hold in proposition 3.21 (1) as seen in the following example. We could not find an example to show that equality in proposition 3.21 (2) may not hold.

Example 3.23: Choose IFS $M = \langle x, \frac{0.5}{a} + \frac{0.2}{b}, \frac{0.5}{a} + \frac{0.6}{b} \rangle$ in the IFTS (X, τ) defined in example 3.7.

Then, (1) $\text{SCl } M = 1_{\sim}$, $\text{SCl } \bar{M} = 1_{\sim}$, $\text{ISBd } M = 1_{\sim}$, $\overline{\text{ISBd } M} = 0_{\sim}$

$$\text{ISBd (ISBd } M) = \text{SCl (ISBd } M) \wedge \text{ SCl } (\overline{\text{ISBd } M}) = \text{SCl } 1_{\sim} \wedge \text{ SCl } 0_{\sim} = 1_{\sim} \wedge 0_{\sim} = 0_{\sim}$$

Hence, $\text{ISBd (ISBd } M) \neq \text{ISBd } M$.

4. Intuitionistic Fuzzy product related spaces

K.K.Azad[3] introduced the concept of fuzzy product related spaces. H.M.Hanafy[7] extend this concept to intuitionistic fuzzy topological spaces as follows:

Definition 4.1: [7] If $A = \langle x, \mu_A(x), \gamma_A(x) \rangle$ and $B = \langle y, \mu_B(y), \gamma_B(y) \rangle$ be IFSs of X and Y respectively. Then the product of intuitionistic fuzzy sets A and B be defined by,

$A \times B = \langle (x, y), \mu_{A \times B}(x, y), \gamma_{A \times B}(x, y) \rangle$, is IFS in $X \times Y$;

where $\mu_{(A \times B)}(x, y) = \min\{\mu_A(x), \mu_B(y)\}$ for each $(x, y) \in X \times Y$;

$$\gamma_{(A \times B)}(x, y) = \max\{\gamma_A(x), \gamma_B(y)\} \text{ for each } (x, y) \in X \times Y$$

and $0 \leq \mu_{A \times B}(x, y) + \gamma_{A \times B}(x, y) \leq 1$. It may be notice that

$\overline{A \times B} = \langle (x, y), \gamma_{A \times B}(x, y), \mu_{A \times B}(x, y) \rangle$. The following lemma is needed in sequel.

Lemma 4.2: Let A, B, C and D be a IFS in IFTS X , then

$$(A \wedge B) \times (C \wedge D) = (A \times D) \wedge (B \times C)$$

Proof: $\mu_{(A \wedge B) \times (C \wedge D)}(x, y) = \min\{\mu_{(A \wedge B)}(x), \mu_{(C \wedge D)}(y)\}$

$$= \min\{\min\{\mu_A(x), \mu_B(x)\}, \min\{\mu_C(y), \mu_D(y)\}\}$$

$$= \min\{\min\{\mu_A(x), \mu_D(y)\}, \min\{\mu_B(x), \mu_C(y)\}\}$$

$$= \min\{\mu_{(A \times D)}(x, y), \mu_{(B \times C)}(x, y)\}$$

$$\mu_{(A \wedge B) \times (C \wedge D)}(x, y) = \mu_{(A \times D) \wedge (B \times C)}(x, y)$$

$$\gamma_{(A \wedge B) \times (C \wedge D)}(x, y) = \max\{\gamma_{(A \wedge B)}(x), \gamma_{(C \wedge D)}(y)\}$$

$$= \max\{\max\{\gamma_A(x), \gamma_B(x)\}, \max\{\gamma_C(y), \gamma_D(y)\}\}$$

$$= \max\{\max\{\gamma_A(x), \gamma_D(y)\}, \max\{\gamma_B(x), \gamma_C(y)\}\}$$

$$= \max\{\gamma_{(A \times D)}(x, y), \gamma_{(B \times C)}(x, y)\}$$

$$\gamma_{(A \wedge B) \times (C \wedge D)}(x, y) = \gamma_{(A \times D) \wedge (B \times C)}(x, y).$$

Hence, $(A \wedge B) \times (C \wedge D) = (A \times D) \wedge (B \times C)$

Definition 4.3: [7] An IFTS (X, τ_X) is product related to another IFTS (Y, τ_Y) if for any intuitionistic fuzzy sets U of X and V of Y , whenever $\bar{A} \not\geq U$ and $\bar{B} \not\geq V$ imply $\bar{A} \times 1_{\sim} \vee 1_{\sim} \times \bar{B} \geq U \times V$, where $A \in \tau_X$ and $B \in \tau_Y$ there exist $A_1 \in \tau_X$ and $B_1 \in \tau_Y$ such that $\bar{A}_1 \geq U$ or $\bar{B}_1 \geq V$ and $\bar{A}_1 \times 1_{\sim} \vee 1_{\sim} \times \bar{B}_1 = \bar{A} \times 1_{\sim} \vee 1_{\sim} \times \bar{B}$.

Lemma 4.4: [7] If A is an IFS of X and B is an IFS of Y , then

$$(1) (A \times 1_{\sim}) \cap (1_{\sim} \times B) = A \times B;$$

$$(2) \overline{A \times B} = (\bar{A} \times 1_{\sim}) \cup (1_{\sim} \times \bar{B});$$

$$(3) (A \times 1_{\sim}) \cup (1_{\sim} \times B) = \overline{\bar{A} \times \bar{B}};$$

Proof: Let $A = \langle x, \mu_A(x), \gamma_A(x) \rangle$, $B = \langle y, \mu_B(y), \gamma_B(y) \rangle$.

$$(1) \text{ Since } A \times 1_{\sim} = \langle x, \min(\mu_A, 1_{\sim}), \max(\gamma_A, 0_{\sim}) \rangle = \langle x, \mu_A(x), \gamma_A(x) \rangle = A.$$

$$\text{Similarly } 1_{\sim} \times B = \langle y, \min(1_{\sim}, \mu_B), \max(0_{\sim}, \gamma_B) \rangle = \langle y, \mu_B(x), \gamma_B(x) \rangle = B.$$

We have

$$(A \times 1_{\sim}) \cap (1_{\sim} \times B) = A(x) \cap B(y) = \langle (x, y), \mu_A(x) \wedge \mu_B(y), \gamma_A(x) \vee \gamma_B(y) \rangle$$

$$\text{Hence, } (A \times 1_{\sim}) \cap (1_{\sim} \times B) = A \times B.$$

$$(2) \text{ Since } \bar{A} = \langle x, \gamma_A(x), \mu_A(x) \rangle, \bar{B} = \langle y, \gamma_B(y), \mu_B(y) \rangle$$

$$\bar{A} \times 1_{\sim} = \langle (x, y), \min(\gamma_A(x), 1_{\sim}(y)), \max(\mu_A(x), 0_{\sim}(y)) \rangle = \langle (x, y), \gamma_A(x), \mu_A(x) \rangle$$

$$1_{\sim} \times \bar{B} = \langle (x, y), \min(1_{\sim}(x), \gamma_B(y)), \max(0_{\sim}(x), \mu_B(y)) \rangle = \langle (x, y), \gamma_B(y), \mu_B(y) \rangle$$

$$(\bar{A} \times 1_{\sim}) \cup (1_{\sim} \times \bar{B}) = \langle (x, y), \max(\gamma_A(x), \gamma_B(y)), \min(\mu_A(x), \mu_B(y)) \rangle = \overline{\bar{A} \times \bar{B}}$$

(3) Follows from (1) and (2).

Lemma 4.5: Let A and B be IFSCS's in IFTS's X and Y respectively. Then $A \times B$ is an IFSCS in the intuitionistic fuzzy product topological space (IFPTS) of $X \times Y$.

Proof: Let $A = \langle x, \mu_A(x), \gamma_A(x) \rangle$, $B = \langle y, \mu_B(y), \gamma_B(y) \rangle$. From Lemma 4.4 we have $\overline{(A \times B)}(x, y) = ((\bar{A} \times 1_\sim) \cup (1_\sim \times \bar{B}))(x, y)$. Since $\bar{A} \times 1_\sim$ and $1_\sim \times \bar{B}$ are IFSOS's in X and Y respectively, hence $(\bar{A} \times 1_\sim) \cup (1_\sim \times \bar{B})$ is IFSOS of $X \times Y$. Hence $\overline{A \times B}$ is an IFSOS of $X \times Y$ and consequently $A \times B$ is an IFSCS of $X \times Y$.

Theorem 4.6: If A and B are IFS's of IFTS's X and Y respectively, then

$$(1) \text{ } SCl \text{ } A \times SCl \text{ } B \geq SCl(A \times B);$$

$$(2) \text{ } SInt \text{ } A \times SInt \text{ } B \leq SInt(A \times B).$$

Proof: (1) Since $A \leq SCl(A)$ and $B \leq SCl(B)$, hence $A \times B \leq SCl(A) \times SCl(B)$ implies $SCl(A \times B) \leq SCl(SCl(A) \times SCl(B))$ and from Lemma 4.5 we have $SCl(A \times B) \leq SCl(A) \times SCl(B)$.

(2) Follows from (1) and using the facts that $\overline{SCl(C)} = SInt(\bar{C})$ and $\overline{SInt(C)} = SCl(\bar{C})$.

Theorem 4.7: Let (X, τ) and (Y, δ) be product related IFTSs. Then for a IFS A of X and a IFS B of Y, then

$$(1) \text{ } SCl(A \times B) = SCl \text{ } A \times SCl \text{ } B;$$

$$(2) \text{ } SInt(A \times B) = SInt \text{ } A \times SInt \text{ } B.$$

Proof: (1) For intuitionistic fuzzy sets A_i' of X and B_j' of Y then note that

$$(i) \text{ } \inf\{A_i, B_j\} = \min(\inf A_i, \inf B_j);$$

$$(ii) \text{ } \inf\{A_i \times 1_\sim\} = (\inf A_i) \times 1_\sim;$$

$$(iii) \text{ } \inf\{1_\sim \times B_j\} = 1_\sim \times (\inf B_j)$$

By theorem 4.6, sufficient to show that $SCl(A \times B) \geq SCl \text{ } A \times SCl \text{ } B$.

Let $A_i \in IFSO(X)$ and $B_j \in IFSO(Y)$. Then,

$$\begin{aligned} SCl(A \times B) &= \inf\{(\overline{A_i \times B_j}) : (\overline{A_i \times B_j}) \geq A \times B\} \\ &= \inf\{\bar{A}_i \times 1_\sim \vee 1_\sim \times \bar{B}_j : \bar{A}_i \times 1_\sim \vee 1_\sim \times \bar{B}_j \geq A \times B\} \\ &= \inf\{\bar{A}_i \times 1_\sim \vee 1_\sim \times \bar{B}_j : \bar{A}_i \geq A \text{ (or) } \bar{B}_j \geq B\} \\ &= \min(\inf\{\bar{A}_i \times 1_\sim \vee 1_\sim \times \bar{B}_j : \bar{A}_i \geq A\}, \inf\{\bar{A}_i \times 1_\sim \vee 1_\sim \times \bar{B}_j : \bar{B}_j \geq B\}) \end{aligned}$$

$$\begin{aligned} \text{Since } \inf\{\bar{A}_1 \times 1_{\sim} \vee 1_{\sim} \times \bar{B}_1 : \bar{A}_1 \geq A\} &\geq \inf\{\bar{A}_1 \times 1_{\sim} : \bar{A}_1 \geq A\} \\ &= \inf\{\bar{A}_1 : \bar{A}_1 \geq A\} \times 1_{\sim} = (Scl A) \times 1_{\sim} \end{aligned}$$

$$\begin{aligned} \inf\{\bar{A}_1 \times 1_{\sim} \vee 1_{\sim} \times \bar{B}_1 : \bar{B}_1 \geq B\} &\geq \inf\{\bar{B}_1 \times 1_{\sim} : \bar{B}_1 \geq B\} \\ &= 1_{\sim} \times \inf\{\bar{B}_1 : \bar{B}_1 \geq B\} = 1_{\sim} \times (Scl B) \end{aligned}$$

Hence, $Scl(A \times B) \geq \min((Scl A) \times 1_{\sim}, 1_{\sim} \times (Scl B)) = Scl A \times Scl B$.

(2) This follows from (1) and using the facts that $\overline{Scl(C)} = SInt(\bar{C})$ and $\overline{SInt(C)} = Scl(\bar{C})$.

Theorem 4.8: Let $X_i, i = 1, 2, 3, \dots, n$, be a family of product related intuitionistic fuzzy topological spaces. If each A_i is a IFS in X_i , then

$$\begin{aligned} ISBd \prod_{i=1}^n A_i &= [ISBd A_1 \times Scl A_2 \times \dots \times Scl A_n] \\ &\vee [Scl A_1 \times ISBd A_2 \times Scl A_3 \times \dots \times Scl A_n] \\ &\vee \dots \vee [Scl A_1 \times Scl A_2 \times \dots \times ISBd A_n] \end{aligned}$$

Proof: It suffices to prove this for $n=2$.

$$\begin{aligned} ISBd (A_1 \times A_2) &= Scl(A_1 \times A_2) \wedge Scl(\overline{A_1 \times A_2}) \\ &= Scl(A_1 \times A_2) \wedge (\overline{SInt(A_1 \times A_2)}) \\ &= (Scl A_1 \times Scl A_2) \wedge (\overline{SInt A_1 \times SInt A_2}) \\ &= (Scl A_1 \times Scl A_2) \wedge [(\overline{SInt A_1 \wedge Scl A_1}) \times (\overline{SInt A_2 \wedge Scl A_2})] \\ &= (Scl A_1 \times Scl A_2) \wedge [(\overline{SInt A_1 \wedge Scl A_1}) \times 1 \vee 1 \times (\overline{SInt A_2 \wedge Scl A_2})] \\ &\quad \text{by Lemma 4.4 of (2)} \\ &= (Scl A_1 \times Scl A_2) \wedge [(Scl \bar{A}_1 \vee SInt \bar{A}_1) \times 1 \vee 1 \times (Scl \bar{A}_2 \vee SInt \bar{A}_2)] \\ &= (Scl A_1 \times Scl A_2) \wedge [(Scl \bar{A}_1 \times 1) \vee (1 \times Scl \bar{A}_2)] \quad \text{by Lemma 4.2} \\ &= [(Scl A_1 \times Scl A_2) \wedge (Scl \bar{A}_1 \times 1)] \vee [(Scl A_1 \times Scl A_2) \wedge (1 \times Scl \bar{A}_2)] \\ &= [(Scl A_1 \wedge Scl \bar{A}_1) \times (1 \wedge Scl A_2)] \vee [(Scl A_1 \wedge 1) \times (Scl A_2 \wedge Scl \bar{A}_2)] \end{aligned}$$

Hence, $ISBd (A_1 \times A_2) = (ISBd A_1 \times Scl A_2) \vee (Scl A_1 \times ISBd A_2)$.

5. Intuitionistic Fuzzy continuous function and Irresolute function:

Definition 5.1: [6] Let (X, τ) and (Y, δ) be two IFTS's. A function $f: (X, \tau) \rightarrow (Y, \delta)$ is said to be intuitionistic fuzzy semi continuous, if the pre image of each IFOS in Y is IFSOS in X .

Theorem 5.2:[11] Let (X, τ) and (Y, δ) be two IFTS's and let $f: X \rightarrow Y$ be a mapping. Then, the following conditions are equivalent

- (1) f is intuitionistic fuzzy semi continuous,
- (2) $f^I(B)$ is an IFSCS in X for each IFCS B in Y ,
- (3) $(SCL(f^I(B))) \leq (f^I(B))$, for each intuitionistic fuzzy set B in Y ,
- (4) $f^I(Int(B)) \leq SInt(f^I(B))$, for each intuitionistic fuzzy set B in Y ,
- (5) $f^I(B) \leq Cl.(SInt(f^I(B)))$, for each intuitionistic fuzzy set B in Y .

Theorem 5.3: Let X be a IFTS and $f: X \rightarrow Y$ be a intuitionistic fuzzy semi continuous function. Then, $ISBd f^I(B) \leq (IBd B)$, for any IFS B in Y .

Proof: Let f be intuitionistic fuzzy semi continuous and B be a IFS in Y . Then, $Cl B$ is intuitionistic fuzzy closed in Y . Which implies that $f^I(Cl B)$ is intuitionistic fuzzy semi closed in X .

$$\begin{aligned}
 \text{Therefore, } ISBd f^{-1}(B) &= SCL(f^{-1}(B)) \wedge SCL(\overline{f^{-1}(B)}) \\
 &\leq f^{-1}(Cl B) \wedge SCL f^{-1}(\overline{B}) \leq f^{-1}(Cl B) \wedge f^{-1}(Cl \overline{B}) \\
 &\leq f^{-1}(Cl B \wedge Cl \overline{B}) = f^{-1}(IBd B)
 \end{aligned}$$

Hence, $ISBd f^{-1}(B) \leq f^{-1}(IBd B)$.

Definition 5.4: [11] Let (X, τ) and (Y, δ) be two IFTS's. A function $f: (X, \tau) \rightarrow (Y, \delta)$ is said to be intuitionistic fuzzy irresolute if and only if the pre image of each IFSOS in Y is and IFSOS in X .

Theorem 5.5: [11] Let (X, τ) and (Y, δ) be two IFTS's and let $f: X \rightarrow Y$ be a mapping. Then, the following conditions are equivalent,

- (1) f is intuitionistic fuzzy irresolute,
- (2) $f^I(B)$ is an IFSCS in X for each IFSCS B in Y ,
- (3) $f^I(SCL A) \leq SCL(f^I(A))$ for each intuitionistic fuzzy set A in X ,
- (4) $SCL(f^I(B)) \leq f^I(SCL B)$ for each intuitionistic fuzzy set B in Y ,
- (5) $f^I(SInt B) \leq Int(f^I(B))$ for each intuitionistic fuzzy set B in Y .

The following theorem gives a necessary condition of intuitionistic fuzzy irresolute functions in terms of intuitionistic fuzzy semi boundary.

Theorem 5.6: Let $f: X \rightarrow Y$ be a intuitionistic fuzzy irresolute function.

Then, $ISBd f^{-1}(B) \leq f^{-1}(ISBd B)$ for any IFS B in Y .

Proof: Let B be a IFS in Y . Then, SCL is intuitionistic fuzzy closed in Y . Which implies that $f^I(SCL B)$ is intuitionistic fuzzy semi closed in X .

$$\begin{aligned}
 \text{Therefore, } ISBd f^{-1}(B) &= SCL(f^{-1}(B)) \wedge SCL(\overline{f^{-1}(B)}) \\
 &\leq f^{-1}(SCL(B)) \wedge SCL(f^{-1}(\overline{B})) \leq f^{-1}(SCL(B)) \wedge f^{-1}(SCL(\overline{B})) \\
 &\leq f^{-1}(SCL B \wedge SCL \overline{B}) = f^{-1}(ISBd B)
 \end{aligned}$$

Hence, $ISBd f^{-1}(B) \leq f^{-1}(ISBd B)$.

Acknowledgement: The authors are highly grateful to referees for valuable comments and suggestions for improving the paper.

REFERENCES

- [1] K. T. Atanassov, Intuitionistic fuzzy sets, Fuzzy sets and system 20 (1986), no.1, 87-96.
- [2] M. Athar and B. Ahmad, Fuzzy boundary and Fuzzy semi boundary, Advances in Fuzzy systems, Article ID.586893 (2008) 1-9 pages.
- [3] K. K. Azad, On fuzzy continuity, fuzzy almost continuity and fuzzy weakly continuity, J. Math. Anal. Appl. 82(1981), 14-32
- [4] C. L. Chang, Fuzzy topological spaces, J. Math. Anal. Appl. 24(1968), no.1, 182-190.
- [5] D. Coker, An introduction to intuitionistic fuzzy topological spaces, Fuzzy sets and system 88 (1997), no.1, 81-89.
- [6] H. G. rcay, D. Coker and A. H. Es, On fuzzy continuity in intuitionistic fuzzy topological spaces, J. Fuzzy Math. 5(1997), no.2, 365-378.
- [7] I. M. Hanafy, completely continuous functions in intuitionistic fuzzy topological spaces, Czechoslovak Math. J. 53(4) (2003), 793-803.
- [8] P. M. Pu and Y. M. Liu, Fuzzy topology I-Neighborhood structure of a fuzzy point and Moore-smith convergence, J. Math. Anal. Appl., Vol. 76, No. 2 (1980), 571-599.
- [9] V. Thiripurasundari and S. Murugesan, Intuitionistic fuzzy boundary and intuitionistic fuzzy continuous functions, (accepted in "The Journal of the Indian Academy of Mathematics").
- [10] L. A. Zadeh, Fuzzy sets, Information and control 8(1965), 338-353.
- [11] Zhen-Guoxu and Fu-Gui Shi, Some Weakly Mappings on Intuitionistic Fuzzy Topological Spaces, TAMKANG Journal of Mathematics, Vol 39, No 1, 25-32, Spring 2008.