

AN INVENTORY MODEL WITH POWER DEMAND PATTERN, WEIBULL DISTRIBUTION DETERIORATION AND WITHOUT SHORTAGES

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Abstract. An inventory problem can be solved by using several methods starting from trial and error methods to mathematical and simulation methods. Mathematical methods help in deriving certain rule and which may suggest how to minimize the total inventory cost in case of deterministic demand. Here an attempt has been made for obtaining a deterministic inventory model for power demand pattern incorporating two-parameter Weibull distribution deterioration and without shortages.

Introduction

A number of researchers have worked on inventory with constant demand rate, time varying demand patterns. A few of the researchers have considered the demand of the items as power demand pattern.

Datta and Pal [3] have developed an order level inventory system with power demand pattern, assuming the deterioration of items governed by a special form of Weibull density function

$$\theta(t) = \theta_0 t, 0 < \theta_0 < (1, t) 0.$$

They used special form of Weibull density function to sidetrack the mathematical complications in deriving a compact EOQ model. Gupta and Jauhari [4] has developed an EOQ model for deteriorating items with power demand pattern with an additional feature of permissible delay in payments. They also used special form of Weibull density function for deterioration of items.

A step forward to special form of Weibull density function, here we shall develop an EOQ model with power demand pattern and using actual form of Weibull density function $Z(t) = \alpha \beta t^{\beta-1}$, where $\alpha(0 < \alpha \leq 1), \beta > 0$ for deterioration of items. Despite of all mathematical intricacies, expressions for various inventory parameters are obtained.

Here we shall develop the same problem with power demand pattern without shortages.

Assumptions and Notations

Inventory model is developed under the following assumptions and notations.

Assumptions

Replenishment rate is infinite.

The lead-time is zero.

Shortages in inventory are not allowed.

The demand is given by the power demand pattern for which, demand upto time t is assumed to be

$$d \left(\frac{t}{T} \right)^{\frac{1}{n}}$$

Where d is the demand size during the fixed cycle time T, $(0 < n < \infty)$ is the pattern index and

$$\left(\frac{dt^{\frac{(1-n)}{n}}}{nT^{\frac{1}{n}}} \right)$$

is the demand rate at time t

The rate of deterioration at any time $t > 0$ follows the two-parameter Weibull distribution $Z(t) = \alpha\beta t^{\beta-1}$, where $\alpha(0 < \alpha \leq 1)$ is the scale parameter and $\beta(0)$ is the shape parameter.

Notations:

T – The fixed length of each ordering/production cycle.

C_1 - The holding cost, per unit time.

C_2 - The cost of each deteriorated unit.

Development of the Model

Let S be the number of items produced or purchased at the beginning of the cycle. It will be the initial inventory at time $t = 0$ and d be the demand during period T . Now, the inventory level S gradually falls during time period $(0, T)$, due to demand and deterioration. At time $t = T$ inventory level becomes zero and shortages are not allowed.

Let $I(t)$ be the on-hand inventory, then the various states of the system are governed by the following differential equations:

$$\frac{dI(t)}{dt} + \alpha\beta t^{\beta-1}I(t) = -\frac{dt^{\frac{(1-n)}{n}}}{nT^{\frac{1}{n}}} \quad (1)$$

$$\text{Where } Z(t) = \alpha\beta t^{\beta-1} \quad (2)$$

Using 2 in 1, the Solution of $I(t)$ is,

$$\begin{aligned} e^{\int \alpha\beta t^{\beta-1}} &= e^{\alpha t^\beta} \\ I(t) e^{\alpha t^\beta} &= \int -\frac{dt^{\frac{(1-n)}{n}}}{nT^{\frac{1}{n}}} e^{\alpha t^\beta} dt \\ I(t) &= -\frac{de^{-\alpha t^\beta}}{nT^{\frac{1}{n}}} \int_0^t t^{\frac{(1-n)}{n}} (1 + \alpha t^\beta) dt \\ &= -\frac{de^{-\alpha t^\beta}}{nT^{\frac{1}{n}}} \left[\frac{t^{\frac{(1-n)}{n}+1}}{\frac{(1-n)}{n}+1} \right] - \frac{d\alpha e^{-\alpha t^\beta}}{nT^{\frac{1}{n}}} \int_0^t t^{\frac{(1-n)}{n}} e^{-\alpha t^\beta} dt \end{aligned}$$

Solving further, on expanding $e^{\alpha t^\beta} = 1 + \alpha t^\beta$, as $(\alpha \leq 1)$ gives

$$I(t) = S e^{-\alpha t^\beta} - \frac{d}{T^{\frac{1}{n}}} e^{-\alpha t^\beta} \frac{1}{t^{\frac{1}{n}}} - \frac{d\alpha}{T^{\frac{1}{n}}} e^{-\alpha t^\beta} \frac{t^{\frac{(1+n\beta)}{n}}}{(1+n\beta)} \quad (3)$$

Using $I(T) = 0$, in 3 gives,

$$S = d + \frac{d\alpha}{(1+n\beta)} T^\beta \tag{4}$$

The total amount of deteriorated units,

$$= S - \int_0^T \frac{dt}{n T^{\frac{1}{n}}}$$

Using 4 in the above, the total amount of deteriorated units in $[0, T]$

$$\begin{aligned} &= d + \frac{d\alpha}{(1+n\beta)} T^\beta - \int_0^T \frac{dt}{n T^{\frac{1}{n}}} \\ &= d + \frac{d\alpha}{(1+n\beta)} T^\beta - \frac{dT^{\frac{1+n-n}{n}}}{T^{\frac{1}{n}}} \\ &= \frac{d\alpha}{(1+n\beta)} T^\beta \end{aligned}$$

Average total cost per unit is given by

$$C(S, T) = \frac{C_3 d\alpha}{(1+n\beta)} T^\beta + \frac{C_1}{T} \int_0^T I(t) dt$$

Substituting the values of $I(t)$ and eliminating S using 4 and integrating, yields,

$$\begin{aligned} C(S, T) &= \frac{C_3 d\alpha}{(1+n\beta)} T^\beta + \frac{C_1}{T} \int_0^T \left(S e^{-\alpha t^\beta} - \frac{d}{T^{\frac{1}{n}}} e^{-\alpha t^\beta} t^{\frac{1}{n}} - \frac{d\alpha}{T^{\frac{1}{n}}} e^{-\alpha t^\beta} \frac{t^{\frac{(1+n\beta)}{n}}}{(1+n\beta)} \right) \\ &= \frac{C_3 d\alpha}{(1+n\beta)} T^\beta + \frac{C_1}{T} \int_0^T \left[\left(\left\{ d + \frac{d\alpha}{(1+n\beta)} T^\beta \right\} e^{-\alpha t^\beta} \right. \right. \\ &\quad \left. \left. - d(1-\alpha t^\beta) t^{\frac{1}{n}} - \frac{d\alpha}{(1+n\beta) T^{\frac{1}{n}}} (1-\alpha t^\beta) t^{\frac{(1+n\beta)}{n}} \right) dt \right] \\ &= \frac{C_3 d\alpha}{(1+n\beta)} T^\beta + \frac{C_1}{T} \int_0^T \left[\left(\left\{ d + \frac{d\alpha}{(1+n\beta)} T^\beta \right\} (1-\alpha t^\beta) \right. \right. \\ &\quad \left. \left. - \frac{d}{T^{\frac{1}{n}}} t^{\frac{1}{n}} (1-\alpha t^\beta) - \frac{d\alpha}{T^{\frac{1}{n}} (1+n\beta)} t^{\frac{(1+n\beta)}{n}} (1-\alpha t^\beta) \right) dt \right] \end{aligned}$$

$$= \frac{C_3 d\alpha}{(1+n\beta)} T^\beta + \frac{C_1}{T} \left[dt - \frac{d\alpha t^{\beta+1}}{(\beta+1)} + \frac{d\alpha T^\beta t}{(1+n\beta)} - \frac{dt^{\frac{1}{n}+1} n}{T^{\frac{1}{n}}(1+n)} - \frac{\frac{d\alpha t^{\frac{1}{n}+\beta+1}}{n}}{\frac{1}{T^n(1+n(\beta+1))}} - \frac{\frac{d\alpha n}{t^{\frac{1+n\beta}{n}}}}{(1+n\beta)T^n} \right]_0^T dt$$

(neglecting higher powers of $\alpha^2, \alpha^3, \dots$)

$$C(S,T) = \frac{C_3 d\alpha}{(1+n\beta)} T^\beta + \frac{C_1}{T} \left[dT - \frac{C_1 d\alpha T^\beta}{(\beta+1)} + \frac{C_1 d\alpha T^\beta}{(1+n\beta)} - \frac{dT^{\frac{1}{n}+1} n}{T^{\frac{1}{n}}(1+n)} + \frac{\frac{d\alpha T^{\frac{1}{n}+\beta+1}}{n}}{\frac{1}{T^n(1+n(\beta+1))}} - \frac{\frac{d\alpha n}{T^{\frac{1+n\beta}{n}}}}{(1+n\beta)T^n} \right]$$

$$C(S,T) = \left[\frac{C_3 d\alpha}{(1+n\beta)} T^\beta + C_1 d - \frac{C_1 d\alpha T^\beta}{(\beta+1)} + \frac{C_1 d\alpha T^\beta}{(1+n\beta)} - \frac{C_1 dT n}{T(1+n)} + \frac{C_1 d\alpha T^\beta n}{(1+n(\beta+1))} - \frac{C_1 d\alpha n}{(1+n\beta)(1+n(\beta+1))} T^\beta \right]$$

On simplification,

$$C(S,T) = \left[\frac{C_3 d\alpha}{(1+n\beta)} T^\beta + \frac{C_1 d}{(1+n)} + \frac{C_1 d\alpha\beta T^\beta}{(1+n(\beta+1))(\beta+1)} \right]$$

For the minimization of cost, we set

$$\frac{dC(T)}{dT} = 0$$

$$\Rightarrow \frac{C_3 d\alpha\beta}{(1+n\beta)} T^{\beta-1} + \frac{C_1 d\alpha\beta^2 T^{\beta-1}}{(1+n(\beta+1))(\beta+1)} = 0,$$

$$\Rightarrow \frac{C_3 d}{(1+n\beta)} T^\beta + \frac{C_1 d\beta T^\beta}{(1+n(\beta+1))(\beta+1)} = 0$$

On solving it, we obtain value of T and let this value of T be the optimum value T^* (say), Substituting the optimum value. T^* in 4, the optimum value of S is,

$$S = d + \frac{d\alpha}{(1+n\beta)} T^\beta$$

Conclusion

From the above work, An inventory model is studied using Weibull distribution deterioration without shortages. An EOQ model with Weibull distribution deterioration with shortages are studied by many authors. Here without shortage case been studied. Further, we can extend this problem using reserve inventory cases.

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