

BEAL CONJECTURE

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Abstract

Beal Conjecture was formulated in 1997 and presented as a generalization of Fermat's Last Theorem, within the field of number theory.

It states that, if:

$$-A^x + B^y = C^z$$

-A, B, C, x, y, and z are positive integers, and x, y, and z > 2

-Then A, B and C must have a common prime factor.

This article presents solution and the proof for the Beal Conjecture.

Introduction

The aim of this paper is to provide proof of Beal Conjecture with the use of basic mathematics.

This article is divided into three parts:

The first part presents two theorems:

Theorem 1: Any two of exponents x, y, and z must be the same.

Theorem 2: The factors A, B and C must have a common prime factor.

The second part presents the solutions and proof of Beal Conjecture.

The third part includes analysis and discussion on topic of the solutions of Beal Conjecture including some examples.

1. THEOREMS

Theorem 1

If $A^x + B^y = C^z$

A, B, C, x, y, and z are positive integers and x, y, z > 2

A, B and C must have a common prime factor.

Then any two of exponents x, y, or z must be the same.

Proof

The exponents x, y and z we can express as:

The exponents x , y and z we can express as:

$$\begin{aligned} x &= f_1(n) ; x = n & x, y \text{ and } z > 2 \\ y &= f_2(n) ; y = n+u & n > 2 \text{ or } n \geq 3 \\ z &= f_3(n) ; z = n+v & 0 \leq u, v < n \end{aligned}$$

Then:

$$\begin{aligned} A^x + B^y &= C^z \\ A^n + B^{n+u} &= C^{n+v} \end{aligned}$$

Let us consider following cases:

1. $u = v = 0$
 $x = y = z = n$
2. $u \neq v$
 $x \neq y \neq z$
 $0 \leq u, v < n$,
(u or v or both ≥ 2)
3. $u \neq v$
 $0 \leq u, v < 2$

Case 1. For $u = v = 0$; $x = y = z = n$

We get the Fermat's Last Theorem

$$A^n + B^n = C^n$$

It has no integer solutions for $n > 2$.

Case 2. For $u \neq v$; $x \neq y \neq z$; $0 \leq u, v < n$, in case (u or v or both ≥ 2)

we get:

$$A^n + B^{n+u} = C^{n+v}$$

If A, B and C have a common prime factor N , then we can express them as:

$$A = KN$$

$$B = PN$$

$$C = SN$$

$K, P,$ and $S \geq 1$ and positive integers

N Common prime factor or Common divisor, then we can write:

$$(KN)^n + (PN)^{n+u} = (SN)^{n+v}$$

$$K^n + P^{n+u} N^u = S^{n+v} N^{n+v}$$

$$S^{n+v} N^v - P^{n+u} N^u - K^n = 0 \quad \text{or:}$$

$$\frac{K^n}{N^v} + \frac{P^{n+u}}{N^v} - \frac{S^{n+v}}{N^v} = 0$$

$$K^n N^{n-v} + P^{n+u} N^{n-v} - S^{n+v} = 0$$

This is our equation. For $K, P,$ and $S \geq 1$ and $n \geq 3$,

than in case $0 \leq u, v < n$, (u or v or both ≥ 2 ; $u \neq v$) it is not possible that common prime factor N and $A, B,$ and C are all integers simultaneously, except for the particular case where:

$$v=2 ; u=0 ; n=3$$

$$K=2 ; P=1 ; S=1$$

$$A^x + B^y = C^z$$

$$(KN)^n + (PN)^{n+u} = (SN)^{n+v}$$

$$K^n N^n + N^n = N^{n+v}$$

$$K^3 N^3 + N^3 = N^{3+2} \quad /:N^3$$

$$K^3 + 1 = N^2$$

$$2^3 + 1 = N^2$$

$$N^2 = 9 ;$$

$$N = 3 - \text{common prime factor}$$

$$K^3 N^3 + N^3 = N^{3+2}$$

$$2^3 3^3 + 3^3 = 3^{3+2}$$

$$6^3 + 3^3 = 3^5$$

Case 3: $u \neq v ; 0 \leq u, v < 2$

Then: $u, v = \{0, 1\}$

a. $u = 0 ; x = y = n$

$v = 1 ; z = n + 1$

b. $u = 1 ; x = z = n$

$v = 0 ; y = n + 1$

If we substitute in equation:

$$A^x + B^y = C^z$$

$$A^n + B^{n+u} = C^{n+v}$$

We then have:

(a) $A^n + B^{n+1} = C^{n+1}$

(b) $A^n + B^{n+1} = C^n$

Based on the equations (a), (b) and Beal Conjecture:

If $A^x + B^y = C^z$

A, B, C, x, y and z are positive integers and x, y, and z > 2.

A, B and C must have a common prime factor,

Then x, y, and z can never be all equal or different simultaneously. Any two of x, y, and z must be the same.

Theorem 2

If $A^x + B^y = C^z$

A, B, C, x, y and z are positive integers and x, y, z > 2.

Then A, B and C must have a common prime factor “Ni” or common factor N.

Proof

If we start from the opposite assumption for Beal Conjecture $A^x + B^y = C^z$ that:

A, B and C have no common prime factor.

1. A, B and C are all different prime factors $A \neq B \neq C$. E.g. 2, 3, 5, 7, 11...
2. Or different common factors with no common prime factors.
E.g. $A = 2 \times 11$, $B = 3 \times 7$, and $C = 5 \times 13 \times 17$.

Then following Theorem 1

$$\begin{aligned} \text{(a)} \quad & A^n + B^n = C^{n+1} \\ \text{(b)} \quad & A^n + B^{n+1} = C^n \end{aligned}$$

We have:

$$\begin{aligned} \text{(a)} \quad & A^n + B^n = C^n C \quad /:C^n \\ \text{(b)} \quad & A^n + B^n B = C^n \quad /:B^n \end{aligned}$$

$$\begin{aligned} \text{(a)} \quad & \frac{A^n}{C^n} + \frac{B^n}{C^n} = C \\ \text{(b)} \quad & \frac{A^n}{B^n} + \frac{B^n}{B^n} = \frac{C^n}{B^n} \end{aligned}$$

1. If A, B and C are all different prime factors $A \neq B \neq C$, then any one factor of A, B or C will be irrational or rational but can't be positive integer. If a prime factor is divisible only by 1 and themselves, then any one factor of A, B or C will be irrational or rational but can't be positive integer.

2. If any two of A, B, and C are coprime (each other and positive integers), then third one can't be a positive integer. In that case A, B and C can't be positive integers simultaneously.

3. From equation (a), C will be positive integer if: $\frac{A^n}{C^n}$ and $\frac{B^n}{C^n}$ are positive integer.

$\frac{A^n}{C^n}$ and $\frac{B^n}{C^n}$ will be positive integers.

A and B is divisible by C with no remainder. From that it follows:

$$A = KC ; B = PC$$

Where K and P are positive integer ≥ 1 ;

$$\begin{aligned} \frac{A}{C} &= \frac{KC}{C} = K \quad \text{or} \\ \frac{B}{C} &= \frac{PC}{C} = P \end{aligned}$$

If two of A, B and C are positive integers and have common prime factor, then third one must have common prime factor too.

Then equation (a) $A^n + B^n = C^n C$ for: $A = KC$ and $B = PC$ becomes:

$$\text{(a)} \quad (KC)^n + (PC)^n = C^{n+1}$$

Where $C = N$ is a common factor

K and P are positive integers ≥ 1

We can show the same for equation (b) $A^n + B^n = C^n$ for $A = KB$ and $C = SB$

(b) $(KB)^n + B^{n+1} = (SB)^n$

Where $B = N$ is common factor
 K and S are positive integers ≥ 1

Then equation (a) and (b) becomes:

- (1) $(KN)^n + (PN)^n = N^{n+1}$
- (2) $(KN)^n + N^{n+1} = (SN)^n$

From the above we can conclude that any one of A, B and C must be the common factor itself.

Based on the equations (1) and (2) and Beal Conjecture if: $A^x + B^y = C^z$

A, B, C, x, y and z are positive integers, and x, y, z > 2 we have proved that the factors of A, B and C must have a common prime factor Ni or common factor N for every value $n \geq 3$, and K, P, S positive integers ≥ 1

2. PROOF THE BEAL CONJECTURE

Based on the theorems 1 and 2 we have:

From equation (1):

(1) $K^n N^n + P^n N^n = N^{n+1}$
 $N^n (K^n + P^n) = N^n N$
 (1.1) $N = K^n + P^n$

And from equation (2):

(2) $K^n N^n + N^{n+1} = S^n N^n$
 $N^n (K^n + N) = S^n N^n$
 (2.1) $N = S^n - K^n$ ($S > K$)

These are solutions and proof of the Beal Conjecture for any value of:

$K, P, S \geq 1$; $K, P, S = \{1, 2, 3, 4, \dots\}$
 $n \geq 3$; $n = \{3, 4, 5, 6, \dots\}$

1- Substituted in equation (1), (1.1), (2), (2.1) any value of K,P,S ≥ 1 and $n \geq 3$ we obtain infinite number of solutions of the Beal Conjecture.

E.g. 1 $n = 4$; $K = 3$; $P = 2$
 $N = K^n + P^n = 3^4 + 2^4 = 97$
 $K^n N^n + P^n N^n = N^{n+1}$
 $3^4 97^4 + 2^4 97^4 = 97^5$
 $291^4 + 194^4 = 97^5$

E.g. 2 $n = 3$; $S = 5$; $K = 3$
 $N = S^n - K^n = 5^3 - 3^3 = 98$
 $K^n N^n + N^{n+1} = S^n N^n$
 $3^3 98^3 + 98^4 = 5^3 98^3$
 $294^3 + 98^4 = 490^3$

2- Also if we multiple with common factor N^{nt} equations (1) and (2), $t = \{1,2,3,\dots\}$, or multiple or divide with common prime factor N_i^n , where

$$N = N_1 \times N_2 \times \dots, \text{ and } N_1 = N_2 = \dots,$$

$$\text{E.g } N = 8 = 2 \times 2 \times 2 = N_1 N_2 N_3 ; N_1 = N_2 = N_3, \text{ or}$$

$$N = 18 = 3 \times 3 \times 2 = N_1 N_2 N_3$$

We get a different form of the same equations.

$$\text{E.g } K = 2 \quad P = 1 \quad n = 3$$

$$K^n N^n + P^n N^n = N^{n+1}$$

$$N = K^n + P^n$$

$$N = 2^3 + 1^3$$

$$N = 9$$

$$2^3 9^3 + 9^3 = 9^4$$

$$\text{E.g 1 } 2^3 9^3 + 9^3 = 9^4 \quad / 9^3$$

$$2^3 9^6 + 9^6 = 9^7$$

$$162^3 + 81^3 = 9^7$$

$$\text{E.g 2 } 2^3 9^3 + 9^3 = 9^4 \quad \text{or}$$

$$18^3 + 9^3 = 9^4 \quad / 3^3$$

$$2^3 3^3 3^3 + 3^3 3^3 = 3^4 3^4 \quad / 3^3$$

$$54^3 + 27^3 = 3^{11}$$

$$\text{E.g 3 } 18^3 + 9^3 = 9^4 \quad / : 3^3$$

$$2^3 3^3 3^3 + 3^3 3^3 = 3^4 3^4 \quad / : 3^3$$

$$6^3 + 3^3 = 3^5$$

3- If we substitute the value of any factor of A, B or C in a different form

$$\text{E.g: } 3^9 = 27^3 \text{ or } 3^6 = 9^3 \text{ or } 2^8 = 4^4 \text{ or } 2^9 = 8^3$$

Then we get a different form of the same equations.

$$\text{E.g 1 } 54^3 + 27^3 = 3^{11} ; 27^3 = 3^9$$

$$54^3 + 3^9 = 3^{11}$$

$$\text{E.g 2 } 18^3 + 9^3 = 9^4 ; 9^3 = 3^6$$

$$18^3 + 3^6 = 9^4$$

$$18^3 + 3^6 = 3^8 ; 9^4 = 3^8$$

$$\text{E.g 3 } 2^9 + 2^8 = 2^9 ; 2^8 = 4^4$$

$$4^4 + 2^8 = 8^3 ; 2^9 = 8^3$$

$$4^4 + 4^4 = 8^3$$

1. Following above and equations (1), (1.1), (2), and (2.1), for $K, P,$ and $S \geq 1$ and $n \geq 3$, we obtain infinite number of solutions of the Beal Conjecture.
2. If we multiple equations (1) and (2), with common factor N^{tn} or multiply or divide with common prime factor N_i^{tn} , as shown, $t = \{1,2,3,\dots\}$, we get a different form of the same equations.
3. If we substitute the value of any factor of A, B or C in different form, as shown, Eg. ($2^9 = 8^3$); we get a different form of the same equations. This is a solution and proof of Beal Conjecture.

3. ANALYSIS AND DISCUSSION

A. ANALYSIS WHEN “N” IS COMMON FACTOR $N = N_1 \times N_2 \times \dots$

If $Ax + By = Cz$

Where A, B and C are positive integers and have a common factor N, we can write:

$$A = KN \quad N = N_1 \times N_2 \times \dots$$

$$B = PN \quad N_i = N_1 \text{ or } N_2 \text{ or } \dots$$

$$C = SN$$

Where N_i is:

1. Common prime factor such as:

$$N_i = \{2,3,5,7,11,13,17,19,\dots\}$$

Where N is:

2. Common factor which contains common prime factor such as:

$$N = 65 = 5 \times 13 = N_1 \times N_2 \text{ or,}$$

$$N = 175 = 5 \times 5 \times 7 = N_1 \times N_2 \times N_3$$

3. K, P, S positive integers ≥ 1

$$K, P, S = \{1,2,3,4,\dots\}$$

Then we can write:

$$(KN)X + (PN)Y = (SN)Z$$

Where $N = N_1 \times N_2$ is common factor and N_1 and N_2 are common prime factors

We then have:

$$(KN_1N_2)^X + (PN_1N_2)^Y = (SN_1N_2)^Z$$

$$K_1 = KN_2$$

$$P_1 = PN_2$$

$$S_1 = SN_2$$

$$(K_1N_1)^X + (P_1N_1)^Y = (S_1N_1)^Z$$

$$N_1 = f(K_1, P_1, S_1), \text{ or}$$

$$K_2 = K_1$$

$$P_2 = P_1$$

$$S_2 = S_1$$

$$(K_2N_2)^X + (P_2N_2)^Y = (S_2N_2)^Z$$

$$N_2 = f(K_2, P_2, S_2)$$

If the exponents x, y and z are positive integers, all greater than 2, we can express them as:

$$x = f_1(n), \quad y = f_2(n) \quad \text{and} \quad z = f_3(n)$$

$$\begin{aligned} x &= n & x, y \text{ and } z &> 2 \\ y &= n+u & n &\geq 3 \text{ or } n > 2 \\ z &= n+v & 0 &\leq u, v < n \end{aligned}$$

If we substitute the expression for A, B, C, x, y and z in Beal conjecture we obtain the following form of the Beal conjecture.

$$(KiNi)^n + (PiNi)^{n+u} = (SiNi)^{n+v}$$

From this equation we can calculate a common prime factor Ni for every value of the indicated conditions.

$$\begin{aligned} n &\geq 3 \\ Ki, Pi, Si &\geq 1 \text{ and} \\ u, v &= \{0, 1\} \quad ; \quad u \neq v \end{aligned}$$

$$\text{For: } \begin{aligned} u &\neq v \\ u, v &= \{0, 1\} \end{aligned}$$

$$\begin{aligned} \text{a) } u &= 0 & ; & \quad x = y = n \\ v &= 1 & ; & \quad z = n + 1 \end{aligned}$$

We get:

$$(3) \quad (KiNi)^n + (PiNi)^n = (SiNi)^{n+1}$$

$$(3.1) \quad Ni = \frac{Ki^n + Pi^n}{Si^{n+1}}$$

$$\begin{aligned} \text{b) } u &= 1 & ; & \quad x = z = n \\ v &= 0 & ; & \quad y = n + 1 \end{aligned}$$

We get:

$$(4) \quad (KiNi)^n + (PiNi)^{n-1} = (SiNi)^n$$

$$(4.1) \quad Ni = \frac{Si^n - Ki^n}{Pi^{n-1}} \quad Si > Ki$$

$$\begin{aligned} \text{E.g } n &= 3; \quad K = 3; \quad P = 1 \\ N &= K^n + P^n = 3^3 + 1^3 = 28 \\ K^n N^n + P^n N^n &= N^{n+1} \\ 3^3 28^3 + 28^3 &= 28^4 \\ 84^3 + 28^3 &= 28^4 \end{aligned}$$

N = 28 is common factor

$$N = 2 \times 2 \times 7 = N_1 \times N_2 \times N_3$$

$N_1 = 2, N_2 = 2, N_3 = 7$ is common prime factor. And we get the same expression if we write:

$$3^3 N_1^3 N_2^3 N_3^3 + N_1^3 N_2^3 N_3^3 = N_1^4 N_2^4 N_3^4$$

$$3^3 2^3 2^3 7^3 + 2^3 2^3 7^3 = 2^4 2^4 7^4$$

$$N_1 = 2 \ ; \ N_2 = 2 \ ; \ N_3 = 7$$

$$K_1 = 3N_1 N_2 = 3 \times 2 \times 2 = 12$$

$$P_1 = N_1 N_2 = 2 \times 2 = 4$$

$$S_1 = N_1 N_2 = 2 \times 2 = 4$$

$$(KiNi)^n + (PiNi)^n = (SiNi)^{n+1}$$

$$K_1^3 N_1^3 + P_1^3 N_1^3 = S_1^4 N_1^4$$

$$12^3 N_1^3 + 4^3 N_1^3 = 4^4 N_1^4$$

$$N_1 = \frac{K_1^n + P_1^n}{S_1^{n+1}}$$

$$N_1 = \frac{12^3 + 4^3}{4^4} = 7$$

$$12^3 7^3 + 4^3 7^3 = 4^4 7^4$$

$$84^3 + 28^3 = 28^4$$

B. ANALYSIS FOR THE CASE WHEN

$$0 \leq u, v < n$$

$$K^n N^n + P^{n+u} N^{n+u} = S^{n+v} N^{n+v}$$

Case 1. $u \neq v \ 0 \leq u, v < 2$
 $u, v = \{0, 1\}$
 a) $u = 0 \ v = 1$ b) $u = 1 \ v = 0$

Case 2. $u \neq v \ 0 \leq u, v < n$
 (for u or v or both ≥ 2)

Case 3. $u = v = m$
 $0 \leq m \leq n-1$

ANALYSIS CASE 1

a) $u = 0$
 $v = 1$

$$K^n N^n + P^{n+u} N^{n+u} = N^{n+v}$$

$$K^n N^n + P^n N^n = N^{n+1}$$

$$N^n (K^n + P^n) = N^n N$$

$$N = K^n + P^n \quad ; \quad n \geq 3$$

N will be positive integer if
K and P positive integer ≥ 1

$$\text{b) } \quad \begin{array}{l} u = 1 \\ v = 0 \end{array}$$

$$K^n N^n + P^{n+u} N^{n+u} = N^{n+v}$$

$$K^n N^n + N^{n+1} = S^n N^n$$

$$N^n (K^n + N) = S^n N^n$$

$$N = S^n - K^n \quad ; \quad n \geq 3$$

N will be positive integer if $S > K$
and S and K positive integer ≥ 1

This is the solution and proof of Beal's Conjecture.

ANALYSIS CASE 2

$$A^x + B^y = C^z$$

For $u \neq v$; $0 \leq u, v < n$ (u or v or both ≥ 2)

$$K^n N^n + P^{n+u} N^{n+u} = S^{n+v} N^n \quad /: N^n$$

$$K^n + P^{n+u} N^u = S^{n+v} N^v$$

$$K^n + P^{n+u} N^u - S^{n+v} N^v = 0$$

If x, y, and z are all different in case $0 \leq u, v < n$, (u or v or both ≥ 2 ; $u \neq v$) then it is not possible that common prime factor N and A, B, and C are all integers simultaneously, except for the particular case where:

$$\underline{v = 2} \quad K = 2$$

$$\underline{u = 0} \quad P = 1$$

$$\underline{n = 3} \quad S = 1$$

Then equation (1) takes the form

$$K^n N^n + N^n = N^{n+v}$$

$$K^3 N^3 + N^3 = N^{3+2}$$

$$N^2 = K^3 + 1$$

$$N^2 = 2^3 + 1 = 9$$

$$N = 3$$

$$2^3 N^n + N^n = N^{n+2}$$

$$2^3 N^3 + N^3 = N^5$$

$$2^3 3^3 + 3^3 = 3^5$$

$$6^3 + 3^3 = 3^5$$

If the equation $2^3 N^n + N^n = N^{n+2}$ is multiplied by N^{tn} , ($t = 1, 2, 3, 4, \dots$), we obtain different form of same equation .

E.g.1 $N = 3$
 $n = 3$
 $t = 1$
 $tn = 1 \times 3 = 3$

$$2^3 N^n + N^n = N^{n+2} / N^{tn}$$

$$2^3 3^3 + 3^3 = 3^5 / 3^3$$

$$2^3 9^3 + 3^6 = 3^{5+3}$$

$$18^3 + 3^6 = 3^8 \text{ or if:}$$

$$3^6 = 9^3 ; 3^8 = 9^4 \text{ then:}$$

$$18^3 + 9^3 = 9^4$$

E.g.2 $n = 3$
 $t = 2$
 $nt = 2 \times 3 = 6$
 $N = 3$

$$2^3 N^n + N^n = N^{n+2} / N^{tn}$$

$$2^3 3^3 + 3^3 = 3^5 / 3^6$$

$$2^3 3^9 + 3^9 = 3^{11} \text{ or if: } 3^9 = 27^3$$

$$54^3 + 27^3 = 3^{11}$$

ANALYSIS CASE

1. $v = m$

a.1 $K^{n+m} N^m + P^{n+m} N^{n+m} = S^n N^n$
 $N^n (K^{n+m} N^m + P^{n+m} N^m) = S^n N^n$
 $N^m (K^{n+m} + P^{n+m}) = S^n$

$$N^m = \frac{S^n}{K^{n+m} + P^{n+m}}$$

For the common factor N to be a positive integer, the following conditions must be satisfied: S^n must be divisible by $K^{n+m} + P^{n+m}$ without a remainder, in that case we can write:

$$K^{n+m} + P^{n+m} = S$$

$$N^m = \frac{S^n}{S} = S^{n-1}$$

$$N = S^{\frac{n-1}{m}} = S^q$$

The following condition must be fulfilled:

$$q = \frac{n-1}{m} \text{ - must be positive integers } \geq 1$$

$$K^{n+m} N^{n+m} + P^{n+m} N^{n+m} = S^n N^n$$

$$q = \frac{n-1}{m} = \frac{3-1}{2} = 1$$

$$S = K^{n+m} + P^{n+m} = 3^{3+2} + 2^{3+2}$$

$$S = 275$$

$$N = S^{\frac{n-1}{m}} = 275^{\frac{3-1}{2}} = 275$$

$$\begin{aligned} K^{3+2} N^{3+2} + P^{3+2} N^{3+2} &= S^3 N^3 \\ 3^{3+2} 275^{3+2} + 2^{3+2} 275^{3+2} &= 275^3 275^3 \\ 3^5 275^5 + 2^5 275^5 &= 275^6 \end{aligned}$$

As we can see equation:

$$3^5 275^5 + 2^5 275^5 = 275^6$$

becomes equation (1)

$$K^n N^n + P^n N^n = N^{n+1}$$

For $K = 3$ $n = 5$

$$P = 2 \quad v = 1$$

$$S = 1 \quad u = 0$$

$$N = K^n + P^n =$$

$$N = 3^5 + 2^5 = 275$$

$$3^5 275^5 + 2^5 275^5 = 275^6$$

$$\text{b.1 } K^n N^n + P^n N^n = S^{n+m} N^{n+m}$$

$$N^n (K^n + P^n) = S^{n+m} N^{n+m}$$

$$N^n = \frac{K^n + P^n}{S^{n+m}} \quad S > P$$

For the common factor N to be a positive integer, the following conditions must be satisfied:

K^n must be divisible by $S^{n+m} - P^{n+m}$ without a remainder, in that case we can write:

$$S^{n+m} - P^{n+m} = K \quad S > P$$

$$N = K^{\frac{n-1}{m}} = K^q$$

The following condition must be fulfilled:

$$q = \frac{n-1}{m} \text{ - must be positive integer } \geq 1$$

E.g 1 $n = 3$

$$m = 1$$

$$S = 3$$

$$P = 2$$

$$q = \frac{n-1}{m} = \frac{3-1}{1} = 2$$

$$K^n N^n + P^{n+m} N^{n+m} = S^{n+m} N^{n+m}$$

$$N^m = K^{n-1}$$

$$N = K^2$$

$$K = S^{n+m} - P^{n+m}$$

$$K = 3^4 - 2^4 = 65$$

$$N = K^2 = 65^2$$

$$65^3 (65^2)^3 + 2^{3+1} (65^2)^{3+1} = 3^{3+1} (65^2)^{3+1}$$

$$65^9 + 2^4 65^8 = 3^4 65^8$$

$$65^4 65^5 + 2^4 65^4 65^4 = 3^4 65^4 65^4 \quad /:65^4$$

$$65^5 + 2^4 65^4 = 3^4 65^4$$

$$65^5 + 130^4 = 195^4$$

As we can see equation

$$2^4 65^4 + 65^5 = 3^4 65^4$$

$$130^4 + 65^5 = 195^4$$

becomes equation (2)

$$K^n N^n + N^{n+1} = S^n N^n$$

For: $K = 2 \quad n = 4$

$$S = 3 \quad u = 1$$

$$P = 1 \quad v = 0$$

$$N = S^n - K^n$$

$$N = 3^4 - 2^4 = 65$$

$$2^4 65^4 + 65^5 = 3^4 65^4$$

E.g.2. $n = 3$

$$m = 2$$

$$S = 3$$

$$P = 2$$

$$q = \frac{n-1}{m} = \frac{3-1}{2} = 1$$

$$K^n N^n + P^{n+m} N^{n+m} = S^{n+m} N^{n+m}$$

$$N^m = K^{n-1}$$

$$N = K$$

$$K = S^{n+m} - P^{n+m}$$

$$K = 3^5 - 2^5 = 211$$

$$N = K = 211$$

$$211^3 211^3 + 2^{3+2} 211^{3+2} = 3^{3+2} 211^{3+2}$$

$$2^5 211^5 + 211^6 = 3^5 211^5$$

$$422^5 + 211^6 = 633^5$$

As we can see equation:

$$2^5 211^5 + 211^6 = 3^5 211^5$$

becomes equation (2)

$$K^n N^n + N^{n+1} = S^n N^n$$

$$\text{For } K = 2 \quad n = 5$$

$$P = 1 \quad u = 1$$

$$S = 3 \quad v = 0$$

$$N = S^5 - K^5$$

$$N = 3^5 - 2^5$$

$$N = 211$$

$$2^5 211^5 + 211^6 = 3^5 211^5$$

$$2) \quad u = v = m ; \quad m = 0$$

$$K^n N^n + P^{n+m} N^{n+m} = S^{n+m} N^{n+m}$$

$$K^n N^n + P^n N^n = S^n N^n$$

$$K^n + P^n = S^n \quad \text{or}$$

$$A^n + B^n = C^n \quad \text{for } n > 2$$

We get the equation for FERMAT'S CONJECTURE

Or Fermat's Last Theorem

Following the Theorem 1: x, y, and z can never be all equal or different simultaneously, for x, y, and z > 2

The equation $A^n + B^n = C^n$ for $n > 2$ has no integer solutions other than trivial solution in which at least one of the variables is zero.

For $n = 2$ we get Pythagorean theorem.

C. EXAMPLES AND DISCUSSION

EXAMPLE: 1

$$K^n N^n + P^{n+u} N^{n+u} = S^n N^n$$

$$n \geq 3 ; K, P, S \geq 1, u \neq v ; u, v = 0, 1$$

$$a) \quad u = v$$

$$v = 1$$

$$K^n N^n + P^n N^n = S^{n+1} N^{n+1}$$

$$N = K^n + P^n$$

$$\text{E.g.1. } K = 3 \quad \underline{n = 3}$$

$$P = 2 \quad u = 0$$

$$S = 1 \quad v = 1$$

$$K^n N^n + P^n N^n = S^{n+1} N^{n+1}$$

$$N = K^n + P^n$$

$$N = 3^3 + 2^3$$

$$N = 35$$

$$3^3 35^3 + 2^3 35^3 = 35^4$$

$$105^3 + 70^3 = 35^4$$

If N is common factor:

$$N = N_1 \times N_2 \times \dots$$

$$N_1 = \frac{K_1^n + P_1^n}{S_1^{n+1}}$$

$N_1 = f(K_1, P_1, S_1)$ – common prime factor

$$K_1, P_1, S_1 \geq 1$$

$N = 35$ is common factor

$$N = 5 \times 7 = N_1 \times N_2$$

$N_1 = 5$ and $N_2 = 7$ are common prime factors

And we get the same if we write:

$$3^3 N_1^3 N_2^3 + 2^3 N_1^3 N_2^3 = N_1^4 N_2^4$$

$$3^3 5^3 7^3 + 2^3 5^3 7^3 = 5^4 7^4$$

$$N_1 = 5 \quad ; \quad N_2 = 7$$

$$K_1 = 3N_2 = 2 \times 7 = 21$$

$$P_1 = 2N_2 = 14$$

$$S_1 = N_2 = 7$$

$$K_1^3 N_1^3 + P_1^3 N_1^3 = S_1^4 N_1^4$$

$$21^3 N_1^3 + 14^3 N_1^3 = 7^4 N_1^4$$

$$N_1 = \frac{K_1^n + P_1^n}{S_1^{n+1}}$$

$$N_1 = \frac{21^3 + 14^3}{7^4} = 5$$

$$21^3 5^3 + 14^3 5^3 = 7^4 5^4$$

$$105^3 + 70^3 = 35^7$$

If multiply

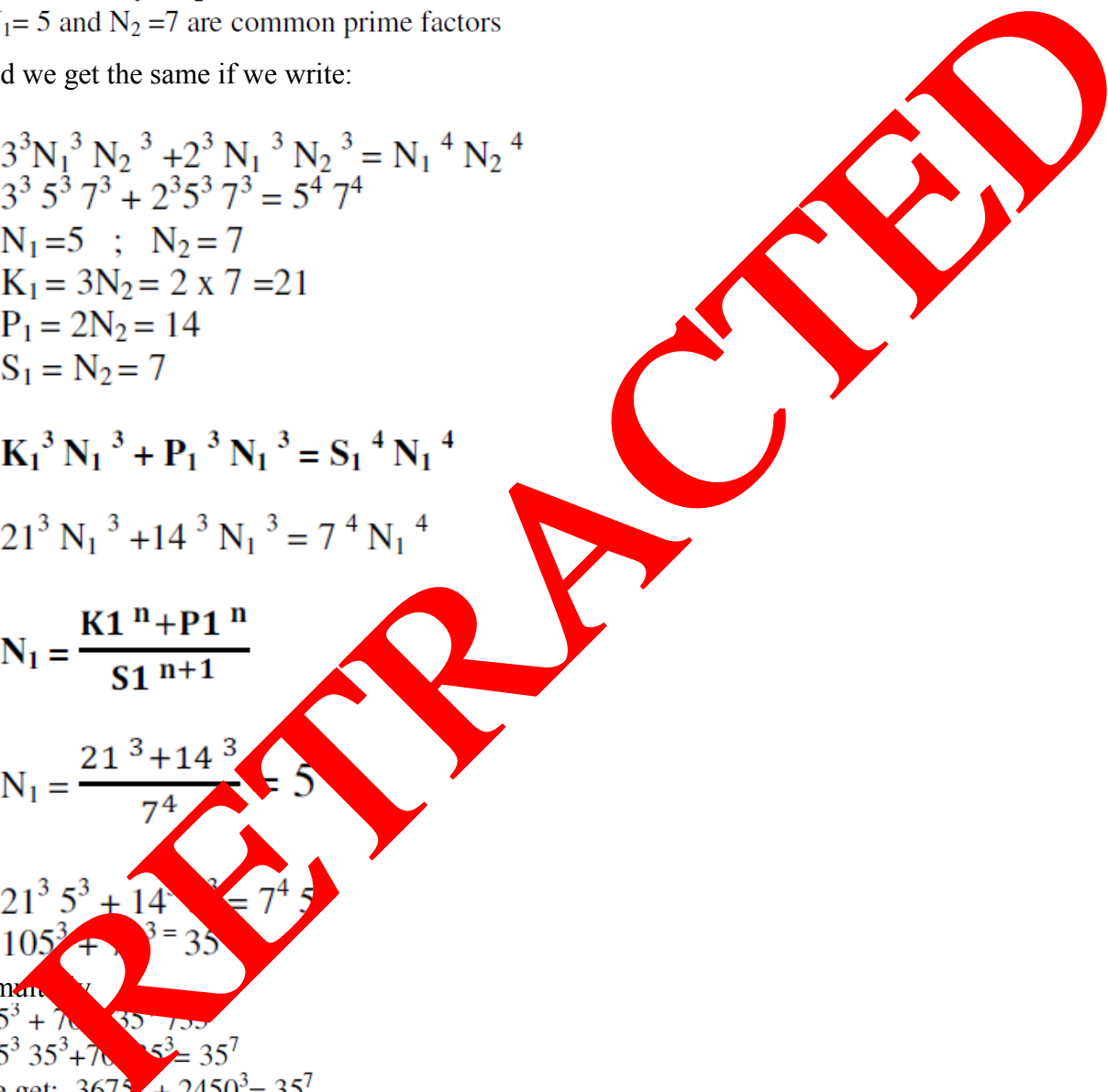
$$105^3 + 70^3 = 35^7$$

$$105^3 35^3 + 70^3 35^3 = 35^7$$

$$\text{We get: } 3675^3 + 2450^3 = 35^7$$

Where is:

$$N_1 = 5 \text{ common prime factor}$$



$$\begin{aligned} \text{E.g.2. } K &= 2 & n &= 3 \\ P &= 2 & u &= 0 \\ S &= 1 & v &= 1 \end{aligned}$$

$$\begin{aligned} K^n N^n + P^n N^n &= N^{n+1} \\ N &= K^n + P^n \end{aligned}$$

$$\begin{aligned} N &= 2^3 + 2^3 \\ N &= 16 \\ 2^3 16^3 + 2^3 16^3 &= 16^4 \\ 32^3 + 32^3 &= 16^4 \text{ or} \\ 2^{15} + 2^{15} &= 4^8 \end{aligned}$$

$N_i = 2$ is common prime factor

$$\begin{aligned} \text{For: } n &\geq 3 \\ u &= 0 ; v = 1 \\ K, P, S &\geq 1 \end{aligned}$$

Or: $N_i = f(K_i, P_i, S_i)$ -common prime factor $K_i, P_i, S_i, \geq 1$ - positive integers

Multiple equation with N^{nt} or N^{int}

$$t = \{1, 2, 3, \dots\},$$

we obtain infinite number of solutions, in different form.

EXAMPLE: 2

$$K^n N^n + P^{n+u} N^{n+u} = N^{n+v}$$

$$n \geq 3 ; K, P, S \geq 1 ; u \neq v ; u, v = \{0, 1\}$$

$$\begin{aligned} \text{b) } u &= 1 \\ v &= 0 \end{aligned}$$

$$\begin{aligned} K^n N^n + N^{n+1} &= S^n N^n \\ N &= S^n - K^n \quad S > K \end{aligned}$$

$$\begin{aligned} \text{E.g.1. } K &= 1 & n &= 4 \\ S &= 2 & u &= 1 \\ P &= 1 & v &= 0 \end{aligned}$$

$$\begin{aligned} K^n N^n + N^{n+1} &= S^n N^n \\ N &= S^n - K^n \end{aligned}$$

$$\begin{aligned} N &= 2^4 - 1 \\ N &= 15 \\ 15^4 + 15^5 &= 2 \cdot 15^4 \\ 15^4 + 15^5 &= 30^4 \end{aligned}$$

For N is common factor:

$$N = N_1 N_2 \dots$$

$$N_1 = \frac{S^1 n - K^1 n}{P^1 n+1}$$

$N = 15$ is common factor

$$N = 5 \times 3 = N_1 \times N_2$$

$N_1 = 5$ and $N_2 = 3$ are common prime factors. And we get the same if we write:

$$N_1^4 N_2^4 + N_1^5 N_2^5 = 2^4 N_1^4 N_2^4$$

$$3^4 5^4 + 3^5 5^5 = 2^4 3^4 5^4$$

$$N_1=5 ; N_2=3$$

$$K_1 = N_2 = 3$$

$$P_1 = N_2 = 3$$

$$S_1 = 2 N_2 = 2 \times 3 = 6$$

$$K_1^4 N_1^4 + P_1^5 N_1^5 = S_1^4 N_1^4$$

$$3^4 N_1^4 + 3^5 N_1^5 = 2^4 3^4 N_1^4$$

$$N_1 = \frac{S_1^n - K_1^n}{P_1^{n+1}}$$

$$N_1 = \frac{6^4 - 3^4}{3^5} = 5$$

$$5^4 3^4 + 5^5 3^5 = 2^4 5^4 3^4$$

$$15^4 + 15^5 = 30^4$$

E.g. 2. $K = 1$ $v = 5$
 $S = 2$ $v = 1$
 $P = 1$ $v =$

$$K^n N^n + 1^{n+1} = S^n 1^n$$

$$N = \frac{S^n - K^n}{P^{n+1}}$$

$$N = \frac{2^5 - 1}{1^{5+1}}$$

$$N = 31$$

$$31^5 + 31^6 = 2^5 31^5$$

$$31^5 + 31^6 = 62^5$$

N = 31 is a common prime factor

$$\begin{aligned} \text{E.g.3. } K &= 3 & n &= 4 \\ S &= 5 & u &= 1 \\ P &= 1 & v &= 0 \end{aligned}$$

$$\begin{aligned} K^n N^n + N^{n+1} &= S^n N^n \\ N &= S^n - K^n \end{aligned}$$

$$\begin{aligned} N &= 5^4 - 3^4 \\ N &= 544 \end{aligned}$$

$$3^4 544^4 + 544^5 = 5^4 544^4$$

$$\text{If: } 544 = 32 \times 17$$

$$\begin{aligned} 3^4 32^4 17^4 + 32^5 17^5 &= 5^4 32^4 17^4 \\ 96^4 17^4 + 32^5 17^5 &= 160^4 17^4 \end{aligned}$$

$$\begin{aligned} K_1^n N_1^n + P_1^{n+1} N_1^{n+1} &= S_1^n N_1^n \\ N_1 &= \frac{S_1^n - K_1^n}{P_1^{n+1}} = \frac{160^4 - 96^4}{32^5} \end{aligned}$$

$$\begin{aligned} N_1 &= 17 \\ 3^4 32^4 17^4 + 32^5 17^5 &= 5^4 32^4 17^4 \quad /: 32^4 \\ 3^4 17^4 + 32 \times 17^5 &= 5^4 17^4 \\ 32 &= 2^5 \\ 3^4 17^4 + 2^5 17^5 &= 5^4 17^4 \end{aligned}$$

$$\begin{aligned} K_2^n N_2^n + P_2^{n+1} N_2^{n+1} &= S_2^n N_2^n \\ K_2 &= 3; P_2 = 2; S_2 = 5; n = 4 \end{aligned}$$

$$N_2 = \frac{S_2^n - K_2^n}{P_2^{n+1}} = \frac{5^4 - 3^4}{2^5} = 17$$

$N^2 = 17$ is a common prime factor

For: $n \geq 3$; $u = 1$; $v = 0$; $K, P, S \geq 1$
 Or $N_i = f(K_i, P_i, S_i)$ is a common prime factor
 K_i, P_i, S_i are positive integers ≥ 1

Multiple or divide equations with Nnt or $Nint$ $t = \{1, 2, 3, \dots\}$, we get a different form of equation and obtain an infinite number of solutions.

From the analysis above we can conclude that every solution is a part off and belongs to the solution set of equations (1), (1.1), (2) and (2.1), which are solution of the Beal Conjecture.

D. MORE EXAMPLES

of applications of equations (1), (1.1), (2) and (2.1) as solutions of Beal Conjecture

$$1. \quad \begin{array}{l} K = 1 \quad \underline{n = 5} \\ P = 1 \quad u = 0 \\ S = 1 \quad v = 1 \end{array}$$

$$\begin{array}{l} K^n N^n + P^n N^n = N^{n+1} \\ N = K^n + P^n \end{array}$$

$$\begin{array}{l} N = 1 + 1 = 2 \\ 2^5 + 2^5 = 2^6 \quad \text{or} \\ 2^5 + 2^5 = 4^3 \end{array}$$

$$\underline{N = 2}$$

$$2. \quad \begin{array}{l} K = 3 \quad \underline{n = 3} \\ P = 3 \quad u = 0 \\ S = 1 \quad v = 1 \end{array}$$

$$\begin{array}{l} K^n N^n + P^n N^n = N^{n+1} \\ N = K^n + P^n \end{array}$$

$$N = 3^3 + 3^3$$

$$N = 54$$

$$3^3 54^3 + 3^3 54^3 = 54^4$$

$$3^3 2^3 27^3 + 3^3 2^3 27^3 = 2^4 27^4$$

$$162^3 + 162^3 = 54^4$$

$$\begin{array}{l} \underline{N_1 = 3} \\ N_2 = 2 \end{array}$$

$$3. \quad \begin{array}{l} K = 4 \quad \underline{n = 3} \\ P = 1 \quad u = 0 \\ S = 1 \quad v = 1 \end{array}$$

$$\begin{array}{l} K^n N^n + P^n N^n = N^{n+1} \\ N = K^n + P^n \end{array}$$

RETRACTED

$$N = 4^3 + 1^3$$

$$N = 65$$

$$4^3 65^3 + 65^3 = 65^4$$

$$4^3 13^3 5^3 + 13^3 5^3 = 13^4 5^4$$

$$52^3 5^3 + 13^3 5^3 = 13^4 5^4$$

$$260^3 + 65^3 = 65^4$$

$$\frac{N_1 = 5}{N_2 = 13}$$

4. $K = 1 \quad n = 3$
 $P = 1 \quad u = 1$
 $S = 2 \quad v = 0$

$$K^n N^n + N^{n+1} = S^n N^n$$

$$N = S^n - K^n$$

$$N = 2^3 - 1$$

$$N = 7$$

$$7^3 + 7^4 = 2^3 7^3$$

$$7^3 + 7^4 = 14^3$$

$$N = 7$$

5. $K = 2 \quad n = 5$
 $P = 1 \quad u = 0$
 $S = 1 \quad v = 1$

$$K^n N^n + P^n N^n = N^{n+1}$$

$$N = K^n + P^n$$

$$N = 2^5 + 1$$

$$N = 33$$

$$2^5 33^5 + 33^5 = 33^6$$

$$2^5 3^5 11^5 + 3^5 11^5 = 3^6 11^6$$

$$6^5 11^5 + 3^5 11^5 = 3^6 11^6$$

$$66^5 + 33^5 = 33^6$$

$$\frac{N_1 = 11}{N_2 = 3}$$

6. $K = 2 \quad n = 6$
 $P = 1 \quad u = 0$
 $S = 1 \quad v = 1$

$$K^n N^n + P^n N^n = N^{n+1}$$

$$N = K^n + P^n$$

$$N = 2^6 + 1$$

$$N = 65$$

$$2^6 65^6 + 65^6 = 65^7$$

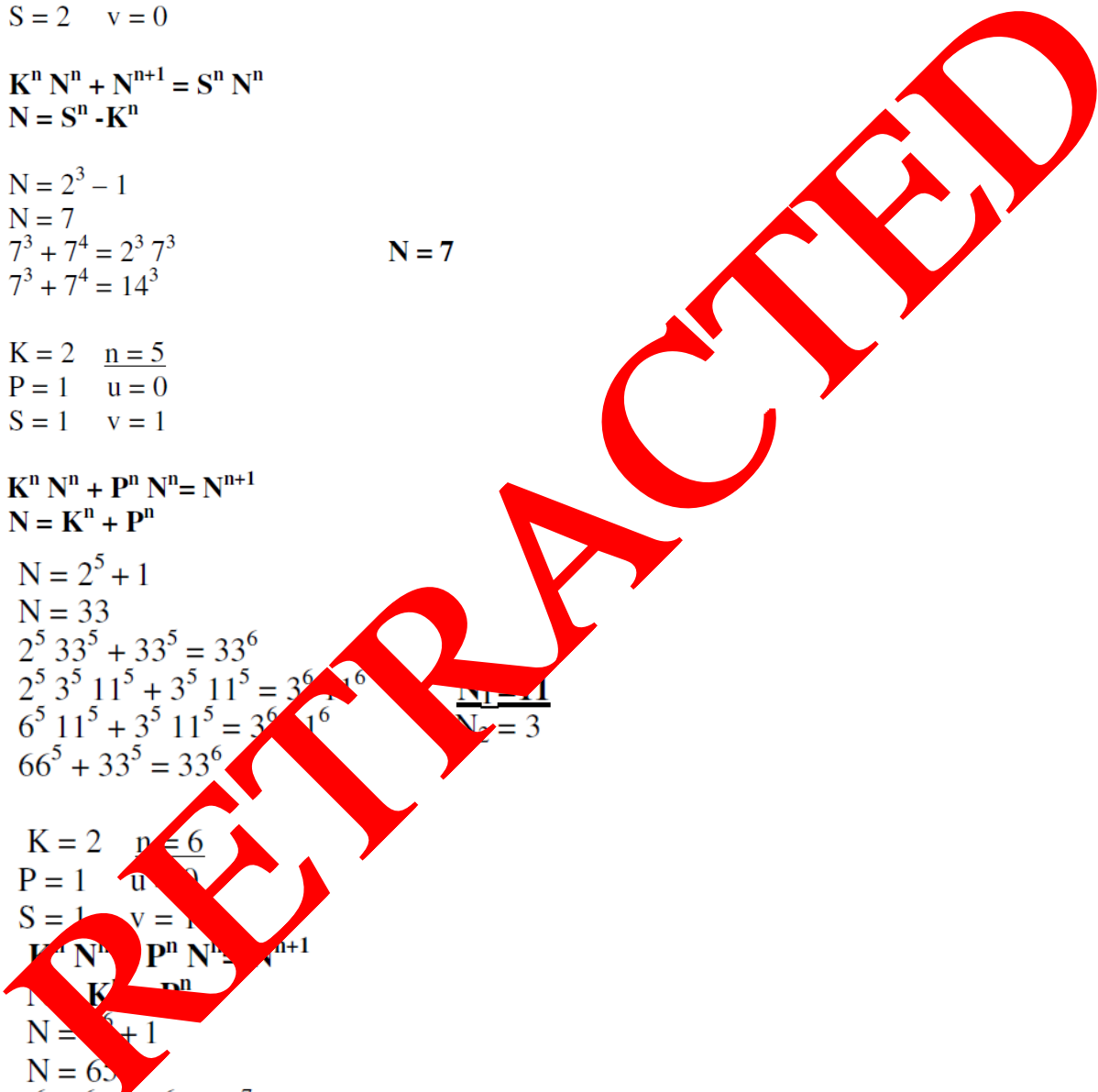
$$2^6 5^6 13^6 + 5^6 13^6 = 5^7 13^7$$

$$10^6 13^6 + 5^6 13^6 = 5^7 13^7$$

$$130^6 + 65^6 = 65^7$$

$$\frac{N_1 = 13}{N_2 = 5}$$

7. $K = 2 \quad n = 4$
 $P = 1 \quad u = 0$
 $S = 1 \quad v = 1$



$$K^n N^n + P^n N^n = N^{n+1}$$

$$N = K^n + P^n$$

$$N = 2^4 + 1$$

$$N = 17$$

$$2^4 17^4 + 17^4 = 17^5$$

$$34^4 + 17^4 = 17^5$$

N = 17

8. $K = 2 \quad n = 3$

$P = 1 \quad u = 1$

$S = 3 \quad v = 0$

$$K^n N^n + N^{n+1} = S^n N^n$$

$$N = S^n - K^n$$

$$N = 3^3 - 2^3$$

$$N = 19$$

$$2^3 19^3 + 19^4 = 3^3 19^3$$

$$38^3 + 19^4 = 57^3$$

N = 19

.....

CONCLUSION

Starting from the Beal Conjecture and

$A^x + B^y = C^z$ where

A, B, C, x, y, and z are positive integers, x, y, and z > 2 than A, B, and C must have a common prime factor.

Then equations (1), (1.1), and (2.1), are solutions of the Beal Conjecture for every value of K, P and $S \geq 1$; $n \geq 3$; and $(u,v) \in \{0, 1\}$ including particular case where $(u = 0 ; v = 2)$, as shown. This is the solution and proof of Beal Conjecture.

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