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An Exact Solution of modified KdV (mKdV) Equation as a reduction of Self-Dual Yang-Mills theory

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Abstract. It is known for quite a long time that Self-Dual Yang Mills (SDYM) theory reduce to Korteweg- DeVries equation, but recently Shehata and Alzaidy have proved that SDYM reduces to modified KdV equation. Therefore, this paper discusses an exact solution of modified Korteweg-DeVries equation with Mathematica. An implication of the proposed solution is that it is possible to consider hadrons as (a set of) KdV soliton.

Introduction

It is known for quite a long time that Self-Dual Yang Mills (SDYM) theory reduce to Korteweg- DeVries equation [1][2], but recently Shehata and Alzaidy have proved that SDYM reduces to modified KdV equation [3]. Therefore, this paper discusses an exact solution of modified Korteweg- DeVries equation with Mathematica. I use Mathematica rel. 9.0. An implication of the proposed solution is that it is possible to consider hadrons as (a set of) KdV soliton.

Self-Dual Yang Mills theory and its canonical reduction to Korteweg-DeVries equation

It has been shown since 1990s that many, and possibly all, integrable systems can be obtained by dimensional reduction of self-dual Yang Mills.[1] Moreover, according to Schiff [1] a remarkable piece of evidence for this was produced a few years ago by Mason and Sparling, who showed how to obtain the Korteweg-de Vries (KdV) and Nonlinear Schrodinger (NLS) equations from SDYM. Schiff also showed how the reduction method of Mason and Sparling could be extended to obtain certain three dimensional versions of the KdV and NLS equations from SDYM.[1] But it seems no one tries to reduce SDYM to mKDV, see also [2].
In this regard, it seems very interesting that A.R. Shehata and J.F. Alzaidy were able to reduce SDYM to mKdV equation in their 2011 paper.[3] The following is a summary of their canonical reduction of SDYM:

The SDYM equations can be written in compact form as follows [3, p.148]:

$$P_t + [P, R] = 0, \tag{1}$$

$$R_x - Q_t - [Q, R] = 0. \tag{2}$$

Let P take the canonical form

$$P = \begin{pmatrix} 0 & k \\ -k & 0 \end{pmatrix} \tag{3}$$

$$R = \begin{pmatrix} 0 & -u_{xx} - 2u^3 \\ u_{xx} + 2u^3 & 0 \end{pmatrix}, \tag{4}$$

$$Q = \begin{pmatrix} 0 & u \\ -u & 0 \end{pmatrix}, \tag{5}$$

From eq. (5) then they obtain the mKdV equation:

$$u_t + 6u_x u^2 + u_{xxx} = 0 \tag{6}$$

Solution of KdVequation with Mathematica

The KdV equation can be written as follows [6]:

$$u_t + 6u_x u + u_{xxx} = 0 \tag{7}$$

Meanwhile the non-dimensional KdV equation and its solution are given by:

$$\mathrm{DSolve}[D[u[t,x],t] + D[u[t,x],\{x,3\}] - 6u[t,x]D[u[t,x],x] == 0, u[t,x], \{t,x\}]$$

$$\{\{u[t,x] \to \frac{1}{6C[2]}(C[1] - 8C[2]^3 + 12C[2]^3\mathrm{Tanh}[tC[1] + xC[2] + C[3]]^2)\}\}$$

$$\{\{u[t,x] \to \frac{1}{6C[2]}(C[1] - 8C[2]^3 + 12C[2]^3\mathrm{Tanh}[tC[1] + xC[2] + C[3]]^2)\}\} [\![1,1,2]\!]$$

$$\frac{C[1] - 8C[2]^3 + 12C[2]^3\mathrm{Tanh}[tC[1] + xC[2] + C[3]]^2}{6C[2]}$$

Solution of modified KdV equation with Mathematica

Shehata and Alzaidy obtained mKdV equation [3]:

$$u_t + 6u_x u^2 + u_{xxx} = 0 \tag{8}$$

Its exact solution is given by:

$$\mathrm{DSolve}[D[u[t,x],t] + D[u[t,x],\{x,3\}] - D[u[t,x],x]6u[t,x]\text{^}2 == 0, u[t,x], \{t,x\}]$$

$$\{\{u[t,x] \to -C[2]\mathrm{Tanh}[xC[2] + 2tC[2]^3 + C[3]]\}, \{u[t,x] \to C[2]\mathrm{Tanh}[xC[2] + 2tC[2]^3 + C[3]]\}\}$$

An implication of the proposed solution is that it is possible to consider hadrons as (a set of) KdV soliton, see for example [8]. This proposition apparently deserves further investigations.
It seems worth mentioning here that there are also other approaches to find solutions of KdV/mKdV equations for example using Backlund transformation [3][4], and also using numerical programming [9].

Concluding remarks

It is known for quite a long time that Self-Dual Yang Mills (SDYM) theory reduce to Korteweg-DeVries equation, but recently Shehata and Alzaidy have proved that SDYM reduces to modified KdV equation. Therefore, this paper discusses an exact solution of modified Korteweg-DeVries equation with Mathematica. An implication of the proposed solution is that it is possible to consider hadrons as (a set of) KdV soliton. This proposition apparently deserves further investigations.

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