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PARTICLE KNOTS IN TORIC MODULAR SPACE

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Abstract
The goal of this contribution is to relate quarks to knots or loops in a 6-space CP^3 that then ollapses into a torus in real 3-space P^3 instantaneously after the Big Bang, and massive inflation, when 3 quarks unite to form nucleons.

Introduction

Kedia et. al. in recent paper [10] investigate knotted structures in hydrodynamic fields such as current-guiding magnetic field lines in a plasma, or vortex lines of classical or quantum fluids which arise naturally as excitations that carry helicity that is a measure of the knottedness of the field. In particular their Fig.2g is a trefoil which is our Fig.3 without the quadrupole (that will be seen in Section 2 to collapse into a point) and the color-coding.

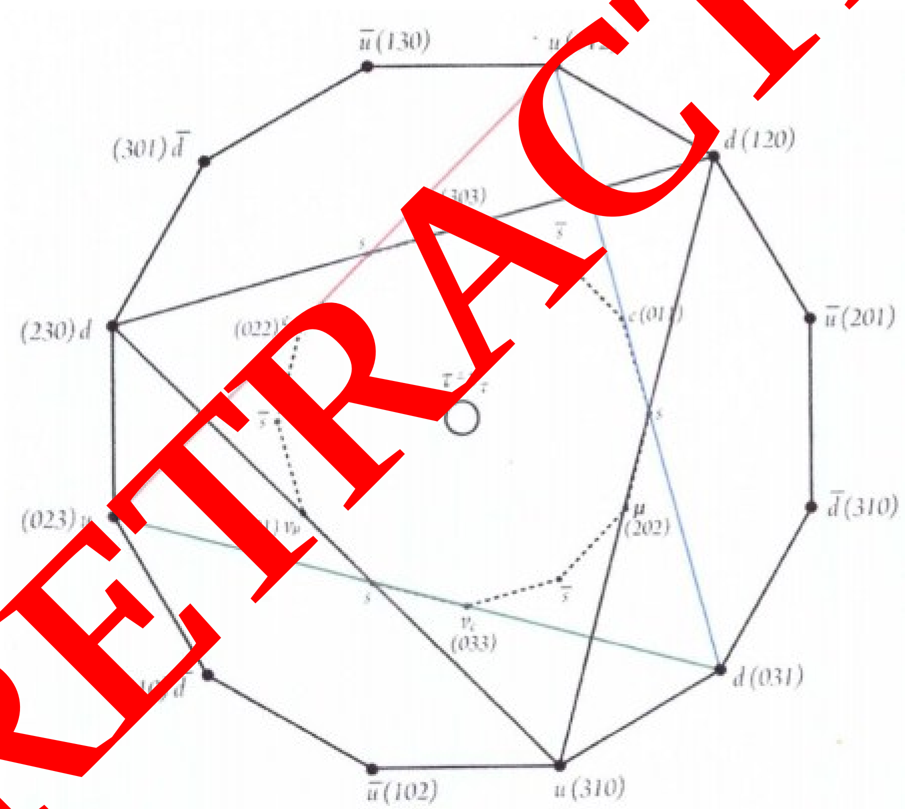

Fig. 1

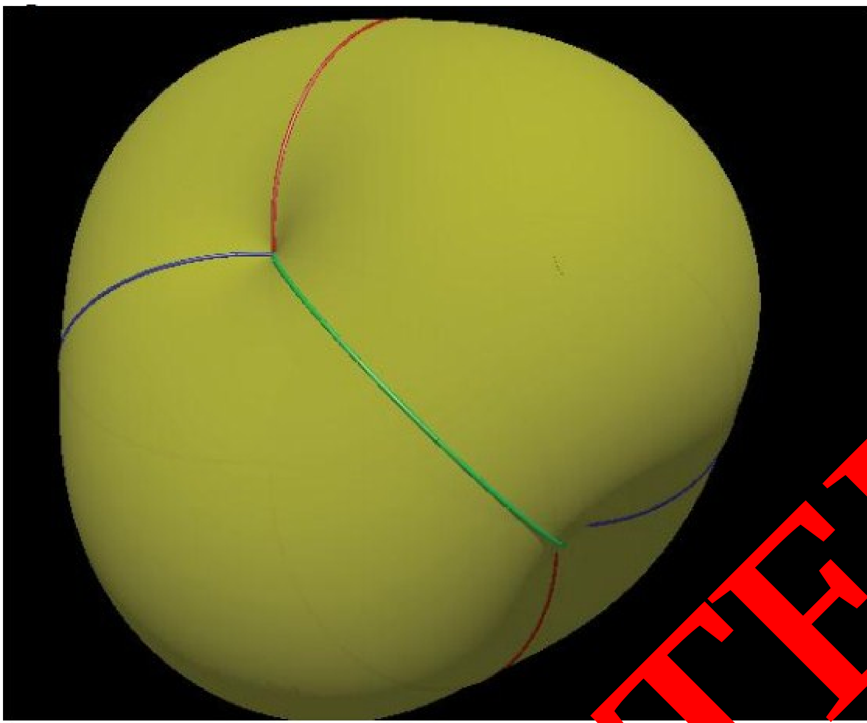

Fig. 2

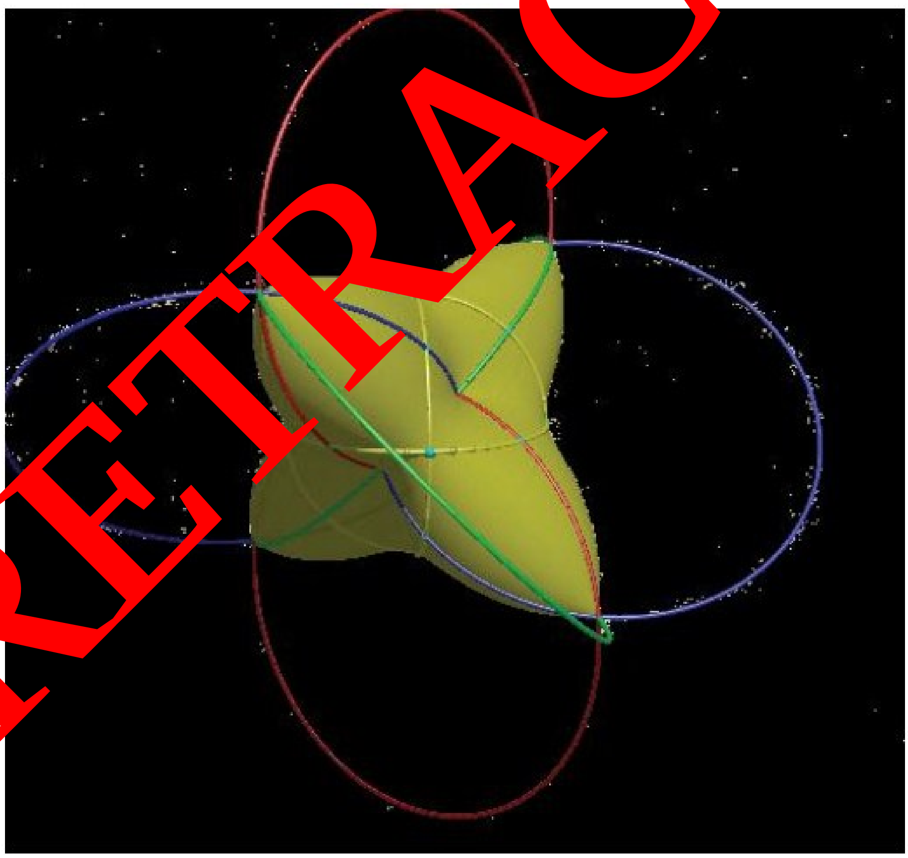

Fig. 3

The torus shown in Fig.2 is due to Marcelis [13] whose calculations in a projective space with 24 vertices appear to be unpublished, but are supported to some extent by Westy [18] (from the same school) who provides a color-coded complex map of the Riemann surface z that incorporates the phases of ω=120 degrees.

Murasagi [14] Ch.7 shows that Fig.2 is a trefoil (3,2) on a torus when we choose 3 points on the ends of a cylinder that can be joined to form the torus. This brings us to the goal of this contribution which is to relate the elementary particles to knots or loops in a 6-space. Here we will be guided by the work of Coxeter [5,6] who specifically labels the vertices of the torus appearing in Fig.1 by $0,\pm1,\omega,\omega^2$ where $\omega = \exp(2i\pi/3)$ so that a knot crosses the longitude of a torus at ω=120 degrees. Essentially this is a Galois Field GF(4) with permutations of ω raised to the powers 0,1,2,3 that will be considered in more detail in the next Section where we will show how 3 quarks in 6-space unite to become a nucleon in the projective space CP^3 which collapses to P^3 immediately after the Big Bang .Section 3 will employ the color-coding of Fig.1 as a model for Quantum Chromodynamics or QCD. Finally according to Rovelli [15] knots or loops in the 6-space described by E_6 employed by Coxeter may also describe Loop Quantum Gravity although details are beyond the scope of this contribution. Also Arvin [2] has considered knots on a torus as a model for elementary particles but excluding quarks. Again, Sundance O Billson Thompson, Smolin et.al. [17] also use knots as a model for Quantum Gravity and the Standard Model but utilize trinions instead of trefoils.

Coxeter Algebra

Fig.1 is a torus taken from Coxeter [5] which is an alternative to the graphs of su(3)c[illegible] × $su(3)_{spin}$ × su(3)isospin which is a triality sub-algebra of E_6.This was published later [illegible] as Fig.12.3B. Both graphs are orbifolds with 27 vertices that according to Slansky [1[illegible]] may be labeled by particles in the Standard Model or SM. However the actual labeling of the tritangent planes on a cubic surface (discussed by Hunt [10] Ch. 4) is new, but in line with Coxeter's labels. For example the up-quark u in Fig.1 is labeled by (012) indicated by $0,\omega,\omega^2$. Thus (023) on the same tritangent is simply a rotation through ω=120 degrees and so on. In this way we find an equilateral triangle labeled by the 3 quarks uud comprising a proton and another ddu for the neutron beginning with (120). There are 2 more tritangents (not labeled) for the anti-particles which complete the outer ring of Fig.1. But quarks also belong to a GF(4) ring and thus to a trefoil on a torus which is precisely the model adopted by Green, Schwarz and Witten [8] Section [illegible].2.

The quarks at the vertices of Fig.1 are trefoils illustrated by Fig.2, but the torus in Fig.1 only becomes a trefoil after the collapse of the inner ring just after the Big Bang when quarks in the 6-space CP^3 unite to build nucleons in the projective space P^3.

This is supported by Barth and Nieto [3] where only the 12 outer vertices and the center of Fig.1 are in P^3.Specifically these authors find 15 synthemes ,where a syntheme has 6 'fix-lines' that are the edges of an invariant tetrahedron such as u u d 0 representing a proton. However because there are only 3 vertices on the face of a syntheme the outer ring of Fig.1 carries the 4 stable particles proton, neutron and their anti-particles. Also since the tritangents are invariant under rotations there are actually 3x4=12 possible synthemes on the outer ring. Specifically each syntheme consists of 3 commuting operators. Thus 3 synthemes can be chosen for spin rotations about the 3 axes of 3-space. Thus introducing triality which is a characteristic of Toric-Calabi-Yau modular space that carry the Hessian Polyhedra in E_6 as discussed by Lie-Yang [12] and analysed by Coxeter [5].

Specifically each rotation in 3-space is also accompanied by a corresponding rotation in a parity 4-space when we permute 1,2,3. Charge conjugation belongs to a second set of 3 synthemes with the same rotations in a 3-space but a parity 5-space in another charge space [7].

In this way the 12 unstable particles *sss*, *overline(sss)*,$\mu^{\pm},\nu_\mu;\tau^{\pm},\nu_\tau$ do not appear in the compactification of CP^3 to P^3. This may be visualised as a collapse of the inner vertices to a point which carries the remaining 3 synthemes $e^{\pm},\nu_e$. labeled by {011,022,033 },{110,220,330} and {101,202,303} for the muon. In this process the masses m_τ,m_ν of the τ,μ reappear as stable deuterium 3
according to the relationship

$$m_\tau + m_\mu = m_p + m_n + m_e \qquad (1)$$

There is no heavy-ion decay and the same relation holds for the anti-particles. This equation is accurate if we assume that m_τ = 1777 MeV and m_μ = 101.4 MeV instead of the Fermi decomposition of muon decay in the weak interaction yielding 106 MeV. However in a recent publication Benjamin Brau et.al.[4] find a value of approximately 100 MeV for the mass of cosmic-ray muons so there is as yet some experimental uncertainty.

Quantum Chromodynamics,QCD

Returning again to Fig.1,when the inner vertices are contracted to a point at the origin the red, green and blue lines could serve as gluons on a new torus where a red upper path passes through the center before emerging at the circumference and giving way to a green gluon that in turn passes under the torus and then over to connect with a down quark and so on.The 3 color complex dimensions vanish when $CP^3 \to P^3$ but a torus knot remains in 3-space.

However Marcelis [13] calculates the dual set of 3 paths for the anti-gluons overlining (r,g,b) which appear in Fig.3 (without the quadrupole) so the gluon,antigluon linked trefoil give us the $SU(3)_c$ color symmetry underlining QCD as described by Griffiths [9],Section 9.1.For example when another quark is added after a rotation ω a red gluon may unite with an anti-blue to build r anti-b, then a following rotation would bring r to anti-r ,and so on before flow down to P3.In this way we can find 9 gluon pairs r anti-r,r anti- b ,r anti- g;b anti-r,b anti-b,b anti- g;g anti-r,g anti-b,g anti- g that are a basis for $SU(3)_c$ symmetry.

Finally Adams [1] p 273 also envisages the 3 colors r,b,g as three extra dimensions in a 6-space

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Fig.1 The Coxeter Polytope
Fig.2 Cayley Surface in Elliptic Space
Fig.3 Interior of Cayley Surface

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