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N POSITIVE INTEGERS WHOSE PRODUCT IS EQUAL TO M-TIMES OF THEIR SUM

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Abstract: In this article, the problem proposed by **Hitesh Jain** (2014) has been solved and generalized.

Introduction:

Recently,**Hitesh Jain** (2014), presented an open problem that for each positive integer $N > 1$, find N positive integers (not necessarily distinct) whose sum is equal to their roduct.
In this article, the above problem has been generalized. It is presented and solved that the product of N positive integers (not necessarily distinct) is M-times of their sum.

Problem presented by **Hitesh Jain** (2014):For each positive integer $N > 1$, find N positive integers (not necessarily distinct) whose sum is equal to their product.
Solution: For N=2, the required numbers are 2 and 2 as

$$2 + 2 = 2 \times 2.$$

For N=3, the required numbers are 1, 2 and 3 as

$$1 + 2 + 3 = 1 \times 2 \times 3.$$

For N=4, the required numbers are 1, 1, 2 and 4 as

$$1 + 1 + 2 + 4 = 1 \times 1 \times 2 \times 4.$$

For general N, the required numbers are 1, 1,….(N-2) times, 2 and N as

$$(1 + 1 + \cdots 1) + 2 + N = (1 \times 1 \times \ldots 1) \times 2 \times N.$$

Generalization of the Above Problem: For each positive integer $N > 1$, find N positive integers (not necessarily distinct) whose product is equal to M-times of their sum.
Solution: For M=1, the solution is given by:
For N=2, the required numbers are 2 and 2 as

$$2 + 2 = 2 \times 2.$$

For N=3, the required numbers are 1, 2 and 3 as

$$1 + 2 + 3 = 1 \times 2 \times 3.$$

For N=4, the required numbers are 1, 1, 2 and 4 as

$$1 + 1 + 2 + 4 = 1 \times 1 \times 2 \times 4.$$

For general N, the required numbers are 1, 1,….(N-2) times, 2 and N as

$$(1 + 1 + \cdots 1) + 2 + N = (1 \times 1 \times \ldots 1) \times 2 \times N.$$

For M=2, the solution is given by :
For N=2, the required numbers are 4 and 4 as

$$2(4 + 4) = 4 \times 4.$$

For N=3, the required numbers are 1, 4 and 5 as

$$2(1 + 4 + 5) = 1 \times 4 \times 5.$$

For N=4, the required numbers are 1, 1, 4 and 6 as

$$2(1 + 1 + 4 + 6) = 1 \times 1 \times 4 \times 6.$$

For general N, the required numbers are 1, 1,….(N-2) times, 4 and N+2 as

$$2\{(1 + 1 + \cdots 1) + 4 + (N + 2)\} = (1 \times 1 \times \ldots 1) \times 4 \times (N + 2).$$

For M=3, the solution is given by:
For N=2, the required numbers are 6 and 6 as

$$3(6 + 6) = 6 \times 6.$$

For N=3, the required numbers are 1, 6 and 7 as

$$3(1 + 6 + 7) = 1 \times 6 \times 7.$$

For N=4, the required numbers are 1, 1, 6 and 8 as

$$3(1+1+6+8) = 1 \times 1 \times 6 \times 8.$$

For general N, the required numbers are 1, 1,….(N-2) times, 6 and $N+4$ as

$$3\{(1+1+\cdots 1)+6+(N+4)\} = (1 \times 1 \times \ldots 1) \times 6 \times (N+4).$$

For general M, the solution is given by:

For N=2, the required numbers are $2M$ and $2M$ as

$$M(2M+2M) = 2M \times 2M.$$

For N=3, the required numbers are 1, $2M$ and $2M+1$ as

$$M\big(1+2M+(2M+1)\big) = 1 \times 2M \times (2M+1).$$

For N=4, the required numbers are 1, 1, $2M$ and $(2M+2)$ as

$$M\big(1+1+2M+(2M+2)\big) = 1 \times 1 \times 2M \times (2M+2).$$

For general N, the required numbers are 1, 1,….(N-2) times, , $2M$ and $(2M+N-2)$ as

$$M\{(1+1+\cdots 1)+2M+(2M+N-2)\} = (1 \times 1 \times \ldots 1) \times 2M \times (2M+N-2).$$

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