N POSITIVE INTEGERS WHOSE PRODUCT IS EQUAL TO M-TIMES OF THEIR SUM

Dr. Satish Kumar and Dr. HariKishan
Department of Mathematics
D.N. College, Meerut (U.P.), India.

Abstract: In this article, the problem proposed by Hitesh Jain (2014) has been solved and generalized.

Introduction:
Recently, Hitesh Jain (2014), presented an open problem that for each positive integer \( N > 1 \), find \( N \) positive integers (not necessarily distinct) whose sum is equal to their product.
In this article, the above problem has been generalized. It is presented and solved that the product of \( N \) positive integers (not necessarily distinct) is \( M \)-times of their sum.

Problem presented by Hitesh Jain (2014): For each positive integer \( N > 1 \), find \( N \) positive integers (not necessarily distinct) whose sum is equal to their product.

Solution: For \( N=2 \), the required numbers are 2 and 2 as
\[2 + 2 = 2 \times 2.\]

For \( N=3 \), the required numbers are 1, 2 and 3 as
\[1 + 2 + 3 = 1 \times 2 \times 3.\]

For \( N=4 \), the required numbers are 1, 1, 2 and 4 as
\[1 + 1 + 2 + 4 = 1 \times 1 \times 2 \times 4.\]

For general \( N \), the required numbers are \( 1, 1, \ldots, (N-2) \) times, 2 and \( N \) as
\[(1 + 1 + \cdots + 1) + 2 + N = (1 \times 1 \times \cdots \times 1) \times 2 \times N.\]

Generalization of the Above Problem: For each positive integer \( N > 1 \), find \( N \) positive integers (not necessarily distinct) whose product is equal to \( M \)-times of their sum.

Solution: For \( M=1 \), the solution is given by:
For \( N=2 \), the required numbers are 2 and 2 as
\[2 \times 2 = 2 \times 2.\]

For \( N=3 \), the required numbers are 1, 2 and 3 as
\[1 \times 2 \times 3.\]

For \( N=4 \), the required numbers are 1, 1, 2 and 4 as
\[1 \times 1 \times 2 \times 4.\]

For general \( N \), the required numbers are \( 1, 1, \ldots, (N-2) \) times, 2 and \( N \) as
\[(1 \times 1 \times \cdots \times 1) \times 2 \times N.\]

For \( M=2 \), the solution is given by:
For \( N=2 \), the required numbers are 4 and 4 as
\[2 \times (4 + 4) = 4 \times 4.\]

For \( N=3 \), the required numbers are 1, 4 and 5 as
\[2 \times (1 + 4 + 5) = 1 \times 4 \times 5.\]

For \( N=4 \), the required numbers are 1, 4 and 6 as
\[2 \times (1 + 1 + 4 + 6) = 1 \times 1 \times 4 \times 6.\]

For general \( N \), the required numbers are \( 1, 1, \ldots, (N-2) \) times, 4 and \( N+2 \) as
\[2 \times (1 \times 1 \times \cdots \times 1 + 4 + (N + 2)) = (1 \times 1 \times \cdots \times 1) \times 4 \times (N + 2).\]

For \( M=3 \), the solution is given by:
For \( N=2 \), the required numbers are 6 and 6 as
\[3 \times (6 + 6) = 6 \times 6.\]

For \( N=3 \), the required numbers are 1, 6 and 7 as
\[3 \times (1 + 6 + 7) = 1 \times 6 \times 7.\]
For $N=4$, the required numbers are 1, 1, 6 and 8 as
\[3(1 + 1 + 6 + 8) = 1 \times 1 \times 6 \times 8.\]
For general $N$, the required numbers are 1, 1, \ldots, $(N-2)$ times, 6 and $N+4$ as
\[3\left(\sum_{i=1}^{(N-2)} 1 + 6 + (N + 4)\right) = (1 \times 1 \times \ldots \times 1) \times 6 \times (N + 4).\]
For general $M$, the solution is given by:
For $N=2$, the required numbers are $2M$ and $2M$ as
\[M(2M + 2M) = 2M \times 2M.\]
For $N=3$, the required numbers are 1, $2M$ and $2M + 1$ as
\[M\left(1 + 2M + (2M + 1)\right) = 1 \times 2M \times (2M + 1).\]
For $N=4$, the required numbers are 1, 1, $2M$ and $(2M + 2)$ as
\[M\left(1 + 1 + 2M + (2M + 2)\right) = 1 \times 1 \times 2M \times (2M + 2).\]
For general $N$, the required numbers are 1, 1, \ldots, $(N-2)$ times, 2$M$ and $(2M + N - 2)$ as
\[M\left(\sum_{i=1}^{(N-2)} 1 + 2M + (2M + N - 2)\right) = (1 \times 1 \times \ldots \times 1) \times 2M \times (2M + N - 2).\]

References: