

N POSITIVE INTEGERS WHOSE PRODUCT IS EQUAL TO M-TIMES OF THEIR SUM

Dr. Satish Kumar and Dr. HariKishan

Department of Mathematics
D.N. College, Meerut (U.P.), India.

Abstract: In this article, the problem proposed by **Hitesh Jain** (2014) has been solved and generalized.

Introduction:

Recently, **Hitesh Jain** (2014), presented an open problem that for each positive integer $N > 1$, find N positive integers (not necessarily distinct) whose sum is equal to their product.

In this article, the above problem has been generalized. It is presented and solved that the product of N positive integers (not necessarily distinct) is M -times of their sum.

Problem presented by **Hitesh Jain** (2014): For each positive integer $N > 1$, find N positive integers (not necessarily distinct) whose sum is equal to their product.

Solution: For $N=2$, the required numbers are 2 and 2 as

$$2 + 2 = 2 \times 2.$$

For $N=3$, the required numbers are 1, 2 and 3 as

$$1 + 2 + 3 = 1 \times 2 \times 3.$$

For $N=4$, the required numbers are 1, 1, 2 and 4 as

$$1 + 1 + 2 + 4 = 1 \times 1 \times 2 \times 4.$$

For general N , the required numbers are 1, 1, ..., $(N-2)$ times, 2 and N as

$$(1 + 1 + \dots + 1) + 2 + N = (1 \times 1 \times \dots \times 1) \times 2 \times N.$$

Generalization of the Above Problem: For each positive integer $N > 1$, find N positive integers (not necessarily distinct) whose product is equal to M -times of their sum.

Solution: For $M=1$, the solution is given by:

For $N=2$, the required numbers are 2 and 2 as

$$2 + 2 = 2 \times 2.$$

For $N=3$, the required numbers are 1, 2 and 3 as

$$1 + 2 + 3 = 1 \times 2 \times 3.$$

For $N=4$, the required numbers are 1, 1, 2 and 4 as

$$1 + 1 + 2 + 4 = 1 \times 1 \times 2 \times 4.$$

For general N , the required numbers are 1, 1, ..., $(N-2)$ times, 2 and N as

$$(1 + 1 + \dots + 1) + 2 + N = (1 \times 1 \times \dots \times 1) \times 2 \times N.$$

For $M=2$, the solution is given by :

For $N=2$, the required numbers are 4 and 4 as

$$2(4 + 4) = 4 \times 4.$$

For $N=3$, the required numbers are 1, 4 and 5 as

$$2(1 + 4 + 5) = 1 \times 4 \times 5.$$

For $N=4$, the required numbers are 1, 1, 4 and 6 as

$$2(1 + 1 + 4 + 6) = 1 \times 1 \times 4 \times 6.$$

For general N , the required numbers are 1, 1, ..., $(N-2)$ times, 4 and $N+2$ as

$$2\{(1 + 1 + \dots + 1) + 4 + (N + 2)\} = (1 \times 1 \times \dots \times 1) \times 4 \times (N + 2).$$

For $M=3$, the solution is given by:

For $N=2$, the required numbers are 6 and 6 as

$$3(6 + 6) = 6 \times 6.$$

For $N=3$, the required numbers are 1, 6 and 7 as

$$3(1 + 6 + 7) = 1 \times 6 \times 7.$$

For $N=4$, the required numbers are 1, 1, 6 and 8 as

$$3(1 + 1 + 6 + 8) = 1 \times 1 \times 6 \times 8.$$

For general N , the required numbers are 1, 1, ..., $(N-2)$ times, 6 and $N+4$ as

$$3\{(1 + 1 + \dots + 1) + 6 + (N + 4)\} = (1 \times 1 \times \dots \times 1) \times 6 \times (N + 4).$$

For general M , the solution is given by:

For $N=2$, the required numbers are $2M$ and $2M$ as

$$M(2M + 2M) = 2M \times 2M.$$

For $N=3$, the required numbers are 1, $2M$ and $2M + 1$ as

$$M(1 + 2M + (2M + 1)) = 1 \times 2M \times (2M + 1).$$

For $N=4$, the required numbers are 1, 1, $2M$ and $(2M + 2)$ as

$$M(1 + 1 + 2M + (2M + 2)) = 1 \times 1 \times 2M \times (2M + 2).$$

For general N , the required numbers are 1, 1, ..., $(N-2)$ times, , $2M$ and $(2M + N - 2)$ as

$$M\{(1 + 1 + \dots + 1) + 2M + (2M + N - 2)\} = (1 \times 1 \times \dots \times 1) \times 2M \times (2M + N - 2).$$

References:

- [1] Andrew, G.E. (1992): Number Theory. Hindustan Publishing Corporation, New-Delhi, India.
- [2] Guy, R.K. (2004): Unsolved Problems in Number Theory. Springer.
- [3] Jain, H. (2014): Problem 1, Problems and Solutions Section, Mathematics News Letter 24(4), 111.
- [4] Telang, S.G. (1996): Number Theory. Tata McGraw. Hill, New-Delhi.