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ON $\hat{\alpha}$g CONNECTED SPACES

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Abstract. In this paper, we define $\hat{\alpha}$ g connected spaces and derive some of its properties.

1.Introduction

Throughout this paper, a space means a topological space on which no separation axioms are assumed unless explicitly stated. In 1963, Levine [4] initiated semiopen sets and studied their properties. Mathematicians gave several papers containing interesting results about new types of sets. Njastad [5] defined α open sets. Senthilkumaran etal [6] defined a new type of sets called α g closed sets and studied their properties. In this paper we define α g connected spaces and derive some of its properties.

2. Preliminaries

Definition 2.1: A subset A of a topological space X is said to be
1)A preopen set if A $\subset$ int cl A and a preclosed set if cl int A $\subset$A
2)A regular open set if A = int cl A and a regular closed set if A = cl int A.
3)A semiopen set if A $\subset$ cl int A and a semi closed set if int cl A $\subset$ A
4)A α- open set if A $\subset$ int cl int A and a α- closed set if cl int cl A $\subset$A.
Definition 2.2: A subset A of a topological space X is called $\hat{\alpha}$ g closed set if int cl int A $\subset$ U whenever A $\subset$U and U is open in X.

The complement of $\hat{\alpha}$ g closed set in X is called $\hat{\alpha}$ g open set in X.

The intersection of all $\hat{\alpha}$ g closed sets containing A is called $\hat{\alpha}$ g closure of A and shall be denoted by $\hat{\alpha}$ g cl A. In general $\hat{\alpha}$ g cl A is not $\hat{\alpha}$ g closed in X.

The union of all $\hat{\alpha}$ g open sets that are contained in A is called $\hat{\alpha}$ g interior of A and shall be denoted by $\hat{\alpha}$ g int A. In general, $\hat{\alpha}$ g int A is not $\hat{\alpha}$ g open in X.

In what follows, let us assume arbitrary intersection of $\hat{\alpha}$ g closed sets in X is $\hat{\alpha}$ g closed in X. Then $\hat{\alpha}$ g cl A will be $\hat{\alpha}$ g closed in X and $\hat{\alpha}$ g int A is $\hat{\alpha}$ g open in X.

3. $\hat{\alpha}$g Separated sets

Definition 3.1 : Let X be a topological space . Two nonempty subsets A and B of X are said to be $\hat{\alpha}$ g separated ifand only if A$\cap$ $\hat{\alpha}$ g cl B = ϕ and $\hat{\alpha}$ g cl A$\cap$B = ϕ
These two conditions are equivalent to a single condition (A$\cap$ $\hat{\alpha}$ g cl B) $\cup$ ($\hat{\alpha}$ g cl A $\cap$ B) = ϕ
Remark 3.2 : A and B are $\hat{\alpha}$ g separated if and only if A and B are disjoint and neither of them contains the $\hat{\alpha}$ g limit point of the other.
Theorem 3.3: If A and B are $\hat{\alpha}$ g separated subsets of X and C$\subset$A and D$\subset$B, then C and D are $\hat{\alpha}$ g separated.
Proof : A$\cap$ $\hat{\alpha}$ g cl B = ϕ and $\hat{\alpha}$ g cl A$\cap$B=ϕ.
C$\subset$A$\Rightarrow$ $\hat{\alpha}$ g cl C $\subset$ $\hat{\alpha}$ g cl A and D$\subset$B$\Rightarrow$ $\hat{\alpha}$ g cl D$\subset$ $\hat{\alpha}$ g cl B
C $\cap$ $\hat{\alpha}$ g cl D $\subset$ A $\cap$ $\hat{\alpha}$ g cl B = ϕ

Similarly, $\hat{\alpha}$ g cl A $\cap$B = ϕ . Hence C and D are $\hat{\alpha}$ g separated.

Theorem 3.4 : Two $\hat{\alpha}$ g closed ($\hat{\alpha}$ g open) subset of the topological space X are $\hat{\alpha}$ g separated if and only if they are disjoint.

Proof : Since any two $\hat{\alpha}$ g separated sets are disjoint, we need only to show that two disjoint $\hat{\alpha}$ g closed ($\hat{\alpha}$ g open) sets are $\hat{\alpha}$ g separated. Let A and B be disjoint and $\hat{\alpha}$ g closed. Then A$\cap$ B = ϕ, $\hat{\alpha}$ g cl A = A, $\hat{\alpha}$ g cl B = B. So, $\hat{\alpha}$ g cl A$\cap$ B = ϕ and A$\cap$ $\hat{\alpha}$ g cl B = ϕ .

Hence A and B are $\hat{\alpha}$ g separated. Let A and B be disjoint and $\hat{\alpha}$ g open. Then $\hat{\alpha}$ g cl A' = A' and $\hat{\alpha}$ g cl B' = B'. A $\cap$B = ϕ $\Rightarrow$A$\subset$ B' and B $\subset$A'

$\Rightarrow$ $\hat{\alpha}$ g cl A $\subset$ $\hat{\alpha}$ g cl B' = B' and

$\hat{\alpha}$ g cl B $\subset$ $\hat{\alpha}$ g cl A' =A'

$\Rightarrow$ $\hat{\alpha}$ g cl A $\cap$B = ϕ and A$\cap$ $\hat{\alpha}$ g cl B = ϕ

Hence A and B are $\hat{\alpha}$ g separated.

Theorem 3.5 : If A and B are $\hat{\alpha}$ g separated sets of a topological space X, then

i)A$\cup$B is $\hat{\alpha}$ g closed $\Rightarrow$ A and B are $\hat{\alpha}$ g closed

ii)A$\cup$B is $\hat{\alpha}$ g open $\Rightarrow$A and B are $\hat{\alpha}$ g open

Proof : Let A and B be $\hat{\alpha}$ g separated sets of a topological space X so that A$\neq$ ϕ and B $\neq$ ϕ

$\hat{\alpha}$ g cl A $\cap$ B = ϕ and A $\cap$ $\hat{\alpha}$ g cl B = ϕ

Let A$\cup$B be $\hat{\alpha}$ g closed

$\hat{\alpha}$ g cl (A$\cup$B) = A$\cup$B. That is $\hat{\alpha}$ g cl A$\cup$ $\hat{\alpha}$ g cl B = A$\cup$B

$\hat{\alpha}$ g cl A = $\hat{\alpha}$ g cl A $\cap$ ($\hat{\alpha}$ g cl A$\cup$ $\hat{\alpha}$ g cl B)

= $\hat{\alpha}$ g cl A$\cap$ (A $\cup$B)

= ($\hat{\alpha}$ g cl A $\cap$A) $\cup$ ($\hat{\alpha}$ g cl A$\cap$ B)

= A$\cup$$\phi$ = A.

Hence A is $\hat{\alpha}$ g closed. Similarly, B is $\hat{\alpha}$ g closed.

Let A$\cup$B be $\hat{\alpha}$ g open.

$\hat{\alpha}$ g cl B is $\hat{\alpha}$ g closed. Hence X- $\hat{\alpha}$ g cl B is $\hat{\alpha}$ g open

(A$\cup$B) $\cap$ (X - $\hat{\alpha}$ g cl B) = (A$\cap$ (X - $\hat{\alpha}$ g cl B)) $\cup$ (B$\cap$ (X - $\hat{\alpha}$ g cl B))

=A $\cup$ ϕ = A.

Hence A is $\hat{\alpha}$ g open. Similarly B is $\hat{\alpha}$ g open.

Theorem 3.6 : Two disjoint sets A and B are $\hat{\alpha}$ g separated in a topological space X = A$\cup$B if and only if they are both $\hat{\alpha}$ g open and α g closed in X.

Proof : Let disjoint sets A and B be $\hat{\alpha}$ g separated in X. A $\cap$ $\hat{\alpha}$ g cl B = ϕ and $\hat{\alpha}$ g cl A$\cap$ B = ϕ

X = A $\cup$B. $\hat{\alpha}$ g cl A = $\hat{\alpha}$ g cl A $\cap$X = $\hat{\alpha}$ g cl A $\cap$ (A$\cup$B)

=($\hat{\alpha}$ g cl A $\cap$A) $\cup$ ($\hat{\alpha}$ g cl A$\cap$ B) = A.

Hence A is $\hat{\alpha}$ g closed. Similarly B is $\hat{\alpha}$ g closed.

Since A and B are disjoint and A $\cup$B = X, B is $\hat{\alpha}$ g open. Similarly A is $\hat{\alpha}$ g open.

Conversely, let disjoint sets A and B be both $\hat{\alpha}$ g open and $\hat{\alpha}$ g closed in X =A$\cup$B.

A is $\hat{\alpha}$ g closed in X. So, A = $\hat{\alpha}$ g cl A = $\hat{\alpha}$ g cl A $\cap$X

= $\hat{\alpha}$ g cl A $\cap$ (A$\cup$B) = ($\hat{\alpha}$ g cl A $\cap$A) $\cup$ ($\hat{\alpha}$ g cl A $\cap$B)

=A$\cup$($\hat{\alpha}$ g cl A$\cap$ B)

A$\cup$B = ϕ $\Rightarrow$ A$\cup$($\hat{\alpha}$ g cl A $\cap$B) = ϕ $\Rightarrow$ $\hat{\alpha}$ g cl A$\cap$ B = ϕ

Similarly, A $\cap$ $\hat{\alpha}$ g cl B = ϕ

So, A and B are $\hat{\alpha}$ g separated.

Theorem 3.7 : If A and B are proper subsets of a space X and both A and B are $\hat{\alpha}$ g closed and $\hat{\alpha}$ g open, then A – B is $\hat{\alpha}$ g separated from B – A.

Proof : (A – B) $\cap$ (B – A) = ϕ and A – B and B – A are non empty.

(A – B) $\cap$ $\hat{\alpha}$ g cl (B – A) = (A$\cap$ B') $\cap$ $\hat{\alpha}$ cl (B$\cap$ A') $\subset$ (A $\cap$B') $\cap$ ($\hat{\alpha}$ g cl B $\cap$ $\hat{\alpha}$ g cl A')

= (A $\cap$B' $\cap$ $\hat{\alpha}$ g cl B) $\cap$ (A$\cap$ B'$\cap$ $\hat{\alpha}$ g cl A')

$=(A\cap B'\cap B)\cap(A\cap B'\cap A')=\phi$
Similarly, $\hat{\alpha}$ g cl $(A-B)\cap(B-A)=\phi$
Hence A – B and B – A are $\hat{\alpha}$ g separated.

4. $\hat{\alpha}$g Connected and $\hat{\alpha}$g Disconnected sets

Definition 4.1 : Let X be a topological space. A subset A of X is said to be $\hat{\alpha}$ g disconnected if and only if it is the union of two non empty $\hat{\alpha}$ g separated sets. That is, if and only if there exists non empty sets C and D such that $C\cap$ $\hat{\alpha}$ g cl $D=\phi$, $\hat{\alpha}$ g cl $C\cap D=\phi$ and $A=C\cup D$. A is said to be $\hat{\alpha}$ g connected if and only if it is not $\hat{\alpha}$ g disconnected.
Remark 4.2 : The empty set is trivially $\hat{\alpha}$ g connected.
Also, every singleton set is $\hat{\alpha}$ g connected.
Definition 4.3 : Two points a and b of a topological space X is $\hat{\alpha}$ g connected if and only if they are contained in a $\hat{\alpha}$ g connected subset of X.
Theorem 4.4 : A topological space X is $\hat{\alpha}$ g disconnected if and only if there exists a non empty proper subset of X which is both $\hat{\alpha}$ g open and $\hat{\alpha}$ g closed in X.
Proof : Let A be a non empty preoper subset of X which is both $\hat{\alpha}$ g open and $\hat{\alpha}$ g closed. Let us show that X is $\hat{\alpha}$ g disconnected. . Let B = A'. As A is a proper subset, B is nonempty. Moeover, $A\cup B=X$ and $A\cap B=\phi$. As A is both $\hat{\alpha}$ g open and $\hat{\alpha}$ g closed, B is also both $\hat{\alpha}$ g open and $\hat{\alpha}$ g closed. Therefore, by theorem 3.6, A and B are $\hat{\alpha}$ g separated. Hence $X=A\cup B$, is $\hat{\alpha}$ g disconnected.
Conversely, let X be $\hat{\alpha}$ g disconnected. Then, there exists nonempty sets A and B such that $A\cap\hat{\alpha}$ g cl $B=\phi$, $\hat{\alpha}$ g cl $A\cap B=\phi$ and $X=A\cup B$ Now $A\subset\hat{\alpha}$ g cl A. So, $A\cap B=\phi$
Hence A = B'
Since B is nonempty and $B\cup=X$, it follows B = A' is a proper subset of X,
Now $A\cup\hat{\alpha}$ g cl $B=X$ as $A\cup B=X\Rightarrow A\cup\hat{\alpha}$ g cl $B=X$.
Also $A\cap\hat{\alpha}$ g cl $B=\phi\Rightarrow A=(\hat{\alpha}$ g cl B)' and similarly, $B=(\hat{\alpha}$ g cl A)'. Since $\hat{\alpha}$ g cl A and $\hat{\alpha}$ g cl B are $\hat{\alpha}$ g closed sets, B and A are $\hat{\alpha}$ g open sets. Since A = B', A is also $\hat{\alpha}$ g closed.
This completes the proof.
Remark 4.5 : In the above theorem, we have also shown that B is also a proper subset of X which is both $\hat{\alpha}$ g open and $\hat{\alpha}$ g closed.
Corollary 4.6 : A topological space X is $\hat{\alpha}$ g connected if and only if the only nonempty subset of X which is both $\hat{\alpha}$ g open and $\hat{\alpha}$ g closed in X is X itself.
Theorem 4.7 : A topological space X is $\hat{\alpha}$ g disconnected if and only if any one of the following statement holds.
i)X is the union of two disjoint nonempty $\hat{\alpha}$ g open sets.
ii)X is the union of two disjoint non empty $\hat{\alpha}$ g closed sets.
Proof : Let X be $\hat{\alpha}$ g disconnected. Then, there exists a nonempty proper subset A of X which is both $\hat{\alpha}$ g open and $\hat{\alpha}$ g closed. Then A' is also both $\hat{\alpha}$ g open and $\hat{\alpha}$ g closed. Also $A\cup A'=X$.
Hence the sets A and A' satisfy the requirement of (i) and (ii).
Conversely, let $X=A\cup B$ and $A\cap B=\phi$ where A and B are nonempty $\hat{\alpha}$ g open sets.
It follows that A = B', so that A is $\hat{\alpha}$ g closed. Since B is nonempty, A is a proper subset of X. Then A is a nonempty proper subsetof X. Then A is a nonepmpty propen subset of X which is both $\hat{\alpha}$ g open and $\hat{\alpha}$ g closed. Hence by theorem 4.4, X is $\hat{\alpha}$ g disconnected. Similarly, assuming (ii) we can prove X is $\hat{\alpha}$ g disconnected.
Remark 4.8 : $A\cup B$ is called a $\hat{\alpha}$ g disconnection of X.
Definition 4.9 : Let A be a subset of a topological space X. A point x of X is said to be $\hat{\alpha}$ g exterior point of A if it is $\hat{\alpha}$ g interior point of the complement A' of A. The set of all $\hat{\alpha}$ g exterior points of A is called the $\hat{\alpha}$ g exterior of A and shall be denoted by $\hat{\alpha}$ g ext (A).

Definition 4.10 : A point x of a topological space X is said to be a $\hat{\alpha}$g frontier point or $\hat{\alpha}$g boundary point of a subset A of X if it is neither a $\hat{\alpha}$g interior point nor $\hat{\alpha}$g exterior point of A. The set of all $\hat{\alpha}$g frontierpoints of A is called the $\hat{\alpha}$g frontier of A and shall be denoted by $\hat{\alpha}$g Fr(A).

Theorem 4.11 : A topological space X is $\hat{\alpha}$g connected if and only if every nonempty proper subset of X has a nonempty $\hat{\alpha}$g frontier.

Proof : Let every nonempty proper subset of X have a nonempty $\hat{\alpha}$g frontier. Let us show X is $\hat{\alpha}$g connected. Let X be $\hat{\alpha}$g disconnected. Then, there exists nonempty disjoint sets G and H both $\hat{\alpha}$g open and $\hat{\alpha}$g closed in X such that X = G$\cup$H. Therefore G = $\hat{\alpha}$g int G = $\hat{\alpha}$g cl G. But $\hat{\alpha}$g Fr(G) =ϕ, which is a contradiction to our hypothesis. Hence X must be $\hat{\alpha}$g connected.

Conversely, let X be $\hat{\alpha}$g connected. Let, if possible, there exists a nonempty proper subset of A of X such that $\hat{\alpha}$g Fr(A) = ϕ Now $\hat{\alpha}$g cl A = $\hat{\alpha}$g int A$\cup\hat{\alpha}$g Fr(A) by theorem 6.2[7]

= A $\cup\hat{\alpha}$g Fr(A) by theorem 6.4[7]

Hence A = $\hat{\alpha}$g int A = $\hat{\alpha}$g cl A, showing that A is both $\hat{\alpha}$g open and $\hat{\alpha}$g closed. Therefore X is $\hat{\alpha}$g disconnected, which is a contradiction. This completes the proof.

Theorem 4.12 : i) Every indiscrete topological space X , where X is non empty is $\hat{\alpha}$g connected.
ii)Every discrete topological space X, where X contains more than one point is $\hat{\alpha}$g connected.

Proof i) : There is no proper subset of X, which is both $\hat{\alpha}$g open and $\hat{\alpha}$g closed. So, X is $\hat{\alpha}$g connected.

ii) X contains more than one point . Every singleton subset of X is a nonempty proper subset of X which is both $\hat{\alpha}$g open and $\hat{\alpha}$g closed. Hence X is $\hat{\alpha}$g disconnected.

Theorem 4.13 : Let X be a topological space and let E be $\hat{\alpha}$g connected subset of X such that E$\subset$A$\cup$B where A and B are $\hat{\alpha}$g separated sets. Then E$\subset$A or E$\subset$B, that is E cannot intersect both A and B.

Proof : Since A and B are $\hat{\alpha}$g separated, A $\cap\hat{\alpha}$g cl B = ϕ and $\hat{\alpha}$g cl A$\cap$ B = ϕ

Now, E$\subset$A$\cup$B$\Rightarrow$E = E $\cap$ (A$\cup$B) = (E $\cap$A) $\cup$ (E$\cap$ B)

Let us prove one of the sets E$\cap$ A or E$\cap$ B is empty. For, if possible, none of these sets is empty That is, let E $\cap$A $\neq$ ϕ and E $\cap$B $\neq$ ϕ

Then (E $\cap$A) $\cap$ $\hat{\alpha}$g cl (E $\cap$B) $\subset$ (E $\cap$A) $\cap$ ($\hat{\alpha}$g cl E$\cap$ $\hat{\alpha}$g cl B) = (E$\cap$ ($\hat{\alpha}$g cl E $\cap\hat{\alpha}$g cl B)) $\cap$ (A$\cap$($\hat{\alpha}$g cl E $\cap\hat{\alpha}$g cl B)) = (E $\cap\hat{\alpha}$g cl B) $\cap\phi$= ϕ

Similarly, $\hat{\alpha}$g cl (E $\cap$A) $\cap$ (E $\cap$B) =ϕ

Hence E $\cap$A and E $\cap$B are $\hat{\alpha}$g separated sets. So, E is $\hat{\alpha}$g disconnected, which is a contradiction. Hence one of the sets E$\cap$ A or E $\cap$B is empty.

If E $\cap$A = ϕ , then E = E$\cap$ B, which implies E$\subset$B.

Corollary 4.14 : If E is a $\hat{\alpha}$g connected subset of a space X such that E$\subset$A$\cup$B, where A and B are disjoint $\hat{\alpha}$g open and $\hat{\alpha}$g closed subsets of X, then A and B are $\hat{\alpha}$g separated.

Proof : A and B are $\hat{\alpha}$g open with A$\cap$B = ϕ

Then A $\subset$ B' $\Rightarrow$ $\hat{\alpha}$g cl A $\subset\hat{\alpha}$g cl B' = B'

$\Rightarrow$ $\hat{\alpha}$g cl A$\cap$ B = ϕ

Similarly A$\cap$ $\hat{\alpha}$g cl B = ϕ

Hence A and B are $\hat{\alpha}$g separated.

Theorem 4.15 : Let E be a $\hat{\alpha}$g connected subset of a space X . If F is a subset of X such that E$\subset$F$\subset$ $\hat{\alpha}$g cl E, then F is $\hat{\alpha}$g connected. In particular $\hat{\alpha}$g cl E is $\hat{\alpha}$g connected.

Proof : Let F be $\hat{\alpha}$g disconnected. Then there exist nonempty sets A and B such that
A$\cap$ $\hat{\alpha}$g cl B = ϕ, $\hat{\alpha}$g cl A$\cap$ B = ϕ and A$\cup$B = F.

Since E$\subset$F = A$\cup$B, it follows from theorem 4.13 that E$\subset$A or E$\subset$B.

Let E$\subset$A. This implies $\hat{\alpha}$g cl E$\subset\hat{\alpha}$g cl A.

This implies $\hat{\alpha}$g cl E $\cap$B$\subset\hat{\alpha}$g cl A$\cap$ B = ϕ

That is $\hat{\alpha}$g cl E $\cap$B = ϕ . Also A$\cup$ = F$\subset\hat{\alpha}$g cl E

$\Rightarrow$B$\subset$F$\subset\hat{\alpha}$g cl E $\Rightarrow$ $\hat{\alpha}$g cl E $\cap$B = B

Hence B =ϕ , a contradiction. So, F must be $\hat{\alpha}$g connected. Again E $\subset\hat{\alpha}$g cl E$\subset\hat{\alpha}$g cl E, $\hat{\alpha}$g cl E is $\hat{\alpha}$g connected.

Theorem 4.16 : If every two points of a subset E of a topological space X are contained in some $\hat{\alpha}$g connected subset of E, then E is $\hat{\alpha}$g connected subset of X.

Proof : Let E be not $\hat{\alpha}$g connected. Then there exist non empty subsets A and B of X such that A$\cap\hat{\alpha}$g cl B = ϕ, $\hat{\alpha}$g cl A $\cap$B = ϕ and E = A $\cup$B.

Since A and B are non empty, there exists a point a$\in$A and a point b$\in$B. By hypothesis a and b must be contained in some $\hat{\alpha}$g connected subset F of E. Since F $\subset$A$\cup$B, by theorem 4.13, F$\subset$A or F$\subset$B. It follows a and b are both in A or B. Let a,b$\in$A. As b$\in$B, we have A$\cap$ B $\neq\phi$, a contradiction. Hence E must be $\hat{\alpha}$g connected.

Theorem 4.17 : A is a $\hat{\alpha}$g connected subset of a $\hat{\alpha}$g connected topological space X such that A' is the union of two $\hat{\alpha}$g separated sets B and C. Then A$\cup$B and A$\cup$C are $\hat{\alpha}$g connected.

Proof : Let A$\cup$B be $\hat{\alpha}$g disconnected. Then, there exist two nonempty $\hat{\alpha}$g separated sets G and H, whose union is A$\cup$B. A is a $\hat{\alpha}$g connected subset of $\hat{\alpha}$g disconnected set A$\cup$B having the $\hat{\alpha}$g separation

A$\cup$ = G$\cup$H. Then by theorem 4.13, either A$\subset$G or A$\subset$H.

Suppose that A$\subset$G.

Now, since A$\cup$B = G$\cup$H

A$\subset$G $\Rightarrow$ A $\cup$B $\subset$G $\cup$B$\Rightarrow$ G $\cup$H$\subset$ G$\cup$ B$\Rightarrow$ H$\subset$B

Since B and C are $\hat{\alpha}$g separated sets and H$\subset$B, C$\subset$C, it follows from theorem 3.3, that H and C are also $\hat{\alpha}$g separated. Thus the set H is $\hat{\alpha}$g separated from G as well as C and therefore H is $\hat{\alpha}$g separated from G $\cup$C

But A' = B $\cup$C$\Rightarrow$X = A$\cup$A' = A$\cup$(B$\cup$C)

$\Rightarrow$X = (A$\cup$B) $\cup$C$\Rightarrow$X = (G$\cup$H) $\cup$C = (G $\cup$C) $\cup$H

Consequently, X has been expressed as the union of nonempty $\hat{\alpha}$g separated sets. This implies X is $\hat{\alpha}$g disconnected, a contradiction.

Similar contradiction will arise if A$\subset$H.

Hence A$\cup$ is $\hat{\alpha}$g connected. Similarly A$\cup$C is $\hat{\alpha}$g connected.

$\hat{\alpha}$g continuity and $\hat{\alpha}$g connectedness

Definition 5.1 : Let X and Y be topological spaces. A mapping f ; X$\rightarrow$Y is said to be $\hat{\alpha}$g continuous if and only if the inverse image of every open set of Y is $\hat{\alpha}$g open in X.

Theorem 5.2 : If f is a $\hat{\alpha}$g continuous mapping of a $\hat{\alpha}$g connected space X onto an arbitrary topological space Y, then Y is connected.

Proof : Let Y be disconnected. Then there exists a nonempty proper subset G of Y which is both open and closed in Y. Since f is $\hat{\alpha}$g continuous and onto Y, f^{-1}(G) is a nonempty proper subset of X which is both $\hat{\alpha}$g open and $\hat{\alpha}$g closed in X. Therefore X is $\hat{\alpha}$g disconnected, a contradiction. Hence Y must be connected.

Theorem 5.3 : A topological space X is $\hat{\alpha}$g disconnected if and only if there exists a $\hat{\alpha}$g continuous mapping of X onto the discrete two point space {0,1}

Proof : Let X be $\hat{\alpha}$g disconnected. Then, there exist two nonempty disjoint $\hat{\alpha}$g open subsets G_1 and G_2 of of X such that X = $G_1\cup G_2$. Define a mapping f of X onto {0,1} by setting f(x) = 0 if x$\in G_1$ and f(x) = 1 if x$\in G_2$. Since {0,1} is discrete, its open sets are ϕ , {0},{1} and {0,1}

$f^{-1}(\{0\}) = G_1$, $f^{-1}(\{1\}) = G_2$ $f^{-1}(\phi) = \phi$, $f^{-1}(\{0,1\}) = X$

Hence f is $\hat{\alpha}$g continuous.

Conversely, let there exists a $\hat{\alpha}$g continuous mapping of X onto the discrete space {0.1}.

If X is $\hat{\alpha}$g connected, then {0,1} is connected by the above theorem . But this is impossible, since every discrete space is disconnected.

Theorem 5.4 : A space X is $\hat{\alpha}$g connected if and only if every $\hat{\alpha}$g continuous function f from X into the discrete two points space {0,1} is constant.

Proof : Let X be $\hat{\alpha}$g connected and let $y \in f(x) \subset \{0,1\}$, where $f : X \to \{0,1\}$ is any $\hat{\alpha}$g continuous function.Then {y} is both open and closed in the discrete space {0,1}. f is $\hat{\alpha}$g continuous. Therefore $f^{-1}(\{y\})$ is $\hat{\alpha}$g open and $\hat{\alpha}$g closed and nonempty. Since X is $\hat{\alpha}$g connected, $f^{-1}(\{y\}) = X$. Therefore f is a constant function.

Conversely, let every $\hat{\alpha}$g continuous function f ; $X \to \{0,1\}$ be constant.

Suppose, if possible, X be $\hat{\alpha}$g disconnected. Then, there exists a nonempty proper subset A of X which is both $\hat{\alpha}$g open and $\hat{\alpha}$g closed in X. Consider the characterstic function KA of A defined by

$$K_A(x) = \begin{cases} 1, & if\ x \in A \\ 0, & if\ x \in A' \end{cases}$$

Then K_A-1(ϕ) = ϕ, K_A-1 ({1}) = A, K_A-1 ({0}) = A' ,

K_A-1({0,1}) =X. Therefore K_A is $\hat{\alpha}$g continuous.

But K_A is non constant, a contradiction.

Hence X must be $\hat{\alpha}$g connected.

Definition 5.5 : A map $f : X \to Y$ is called $\hat{\alpha}$g irresolute if f^{-1} (V) is $\hat{\alpha}$g closed in X for every $\hat{\alpha}$g closed set V of Y. It is called irresolute if f^{-1} (V) is open in X for every open set V of Y.

Theorem 5.6 : A map $f : X \to Y$ is $\hat{\alpha}$g irresolute if for every $\hat{\alpha}$g open set A of Y, $f^{-1}(A)$ is $\hat{\alpha}$g open in X.

Proof : obvious

Theorem 5.7 : Let $f : X \to Y$ be a map. Then, if X is $\hat{\alpha}$g connected and f is $\hat{\alpha}$g is irresolute surjective, then Y is $\hat{\alpha}$g connected.

Proof : Let Y be $\hat{\alpha}$g disconnected. Then $Y = A \cup B$, where A and B are disjoint nonempty $\hat{\alpha}$g open subsets of Y, Since f is $\hat{\alpha}$g irresolute surjective, $X = f^{-1}(A) \cup$ f-1(B). $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint nonempty $\hat{\alpha}$g open subset of X. So, X is $\hat{\alpha}$g disconnected, a contradiction. This completes the proof.

References

[1] D.Andrijevic, Semipre open sets, Mat.Vesnik 38 (1986) 24- 32.

[2] C.G.Crossley, S.K.Hildebrand, Semi – closure, Texas J. Sci. 22 (1971) 99 – 112.

[3] S.Jafari, T.Noiri, Properties of β connected spaces, Acta. Math. Hungar.101 (2003) 227 – 236.

[4] N.Levine, Semi- open sets and Semi – continuity in topological spaces, Amer.Math. Monthly 70 (1963) 36 – 41.

[5] O.Njastad, On some classes of nearly open sets, Pacific. J. Math 15 (1965) 961 – 970.

[6] V.Senthilkumaran, R.Krishnakumar, Y.Palaniappan, On α generalized closed sets, Int.Natl.J. of Math. Archive, Accepted.

[7] V.Senthilkumaran, R.Krishnakumar, Y.Palaniappan, α g Exterior and α g Frontier in topological spaces , Int.Natl.J. of Math. Archive, Accepted.