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Special Dio 3 – tuples

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Abstract. We search for three distinct polynomials with integer coefficient such that the product of any two members of the set minus their sum and increased by a non- zero integer (or polynomial with integer coefficient) is a perfect square.

Introduction:

The problem of constructing the set with property that the product of any two its distinct elements is one less than a square has a very long history and such sets were studied by Diophantus. A set of m positive integers $\{a_1, a_2, \ldots a_m\}$ is called a Diophantine m-tuple

$$a_i * a_j + 1 \text{ is a perfect square,} \tag{1}$$

a perfect square for all $1 \le i \prec j \le m$. Many generalizations of this problem (1) were considered since antiquity, for example by adding a fixed integer n instead of 1, looking kth powers instead of squares or considering the powers over domains other than Z or Q. Many mathematicians consider the problem of the existence of Diophantine quadruples with the property *D(n)* for any arbitrary integer n and also for any linear polynomials in n. In this context one may refer [1-16]. The above results motivated us the following definition:

A set of three distinct polynomials with integer coefficient (a_1, a_2, a_3) is said to be a special dio 3- tuple with property *D(n)* if $a_i * a_j - (a_i + a_j) + n$ is a perfect square for all $1 \le i \prec j \le 3$.

In the above definition n may be a non – zero integer or polynomial with integer coefficients. In this communication we consider a few special dio 3 tuples of polygonal numbers from $t_{6,n}$ to $t_{10,n}$ and centered polygonal numbers from $ct_{6,n}$ to $ct_{10,n}$ with their corresponding properties.

Notations:

$t_{m,n} = n\left(1 + \frac{(n-1)(m-2)}{2}\right) =$ Polygonal number of rank n with sides m

$ct_{m,n} = \frac{mn(n+1)}{2} + 1 =$ Centered Polygonal number of rank n with sides m

Construction of Dio 3 - tuples for Hexagonal numbers

Let $a = t_{6,n}$, $b = t_{6,n-2}$ be Hexagonal number of rank n and n-2 respectively such that $ab - (a+b) + 4n^2 - 10n + 11$ is a perfect square say γ^2

Let c be any non zero integer such that

$$ac-(a+c)+4n^2-10n+11=\alpha^2 \tag{2}$$

$$bc-(b+c)+4n^2-10n+11=\beta^2 \tag{3}$$

On solving equations (2) and (3), we get

$$(b-1)\alpha^2-(a-1)\beta^2=(a-b)+\left(4n^2-10n+11\right)(b-a) \tag{4}$$

Assuming $\alpha=x+(a-1)T$ and $\beta=x+(b-1)T$, in (4) it reduces to

$$x^2=(b-1)(a-1)T^2+\left(4n^2-10n+10\right) \tag{5}$$

The initial solution of equation (5) is given by

$$T_o=1 \text{ and } x_0=n^2-5n+1 \tag{6}$$

Therefore,

$$\alpha=4n^2-6n \tag{7}$$

On substuting the values of α and a in equation (2), we get

$$c=8n^2-20n+11$$

$$=t_{6,2n-2}-2n+1$$

Therefore, triple $\left(t_{6,n},t_{6,n-2},t_{6,2n-2}-2n+1\right)$ is Dio 3- tuple with property $D\left(4n^2-10n+11\right)$

For simplicity, we present in the table below a few Dio 3- tuples for polygonal numbers from $t_{6,n}$ to $t_{10,n}$ with suitable properties.

a	b	c	D(n)
$t_{6,n}$	$t_{6,n-2}$	$t_{18,n}-13n+9$	$D(10)$
	$t_{6,n-1}$	$t_{6,2n-2}+6n-4$	$D\left(10n^2-15n+7\right)$
$2t_{7,n}$	$2t_{7,n-2}$	$2t_{7,2n-2}-6n+5$	$D\left(10n^2-26n+35\right)$
		$t_{42,n}-33n+29$	$D(30)$
	$2t_{7,n-1}$	$2t_{7,2n-2}+14n+13$	$D\left(25n^2-40n+17\right)$
$t_{8,n}$	$t_{8,n-2}$	$t_{8,2n-2}-4n+7$	$D\left(18n^2-48n+32\right)$
		$t_{26,n}-21n+7$	$D(17)$
	$t_{8,n-1}$	$t_{8,2n-2}+8n-6$	$D\left(18n^2-30n+14\right)$
$2t_{9,n}$	$2t_{9,n-2}$	$2t_{9,2n-2}-10n+11$	$D\left(28n^2-76n+74\right)$
		$t_{58,n}-49n+45$	$D(54)$
	$2t_{9,n-1}$	$2t_{9,2n-2}+18n-19$	$D\left(35n^2-60n+28\right)$
$t_{10,n}$	$t_{10,n-2}$	$t_{10,2n-2}+6n-1$	$D\left(24n^2-66n+47\right)$
		$t_{34,n}-29n+25$	$D(26)$
	$t_{10,n-1}$	$t_{10,2n-2}+10n-10$	$D\left(20n^2-35n+16\right)$

Construction of Dio 3 - tuples for Centered Hexagonal numbers

Let $a = ct_{6,n}$, $b = ct_{6,n-2}$ be Centered Hexagonal number of rank n and n-2 respectively such that $ab-(a+b)+24n^2-30n+2$ is a perfect square say γ^2

Let c be any non zero integer such that

$$ac-(a+c)+24n^2-30n+2=\alpha^2 \tag{8}$$

$$bc-(b+c)+24n^2-30n+4=\beta^2 \tag{9}$$

On solving equations (8) and (9), we get

$$(b-1)\alpha^2-(a-1)\beta^2=(a-b)+\left(24n^2-30n+2\right)(b-a) \tag{10}$$

Assuming $\alpha = x+(a-1)T$ and $\beta = x+(b-1)T$ in (10), it reduces to

$$x^2=(b-1)(a-1)T^2+\left(24n^2-30n+1\right) \tag{11}$$

The initial solution of equation (11) is given by

$$T_o=1 \text{ and } x_0=3n^2-3n+1 \tag{12}$$

Therefore,

$$\alpha=6n^2+1 \tag{13}$$

On substuting the values of α and a in equation (8), we get

$$c=12n^2-12n+9$$

$$=ct_{6,2n-2}+6n+8$$

Therefore, triple $\left(ct_{6,n}, ct_{6,n-2}, ct_{6,2n-2}+6n+8\right)$ is Dio 3- tuple with property $D\left(24n^2-30n+2\right)$

For simplicity, we present in the table below a few Dio 3- tuples for Centered polygonal numbers from $ct_{6,n}$ to $ct_{10,n}$ with suitable properties.

a	b	C	D(n)
$ct_{6,n}$	$ct_{6,n-2}$	$ct_{24,n}-24n$	$D(10)$
	$ct_{6,n-1}$	$ct_{6,2n-2}+14n-4$	$D\left(-12n^3+19n^2-4n+2\right)$
$2ct_{7,n}$	$2ct_{7,n-2}$	$2ct_{7,2n-2}+14n+9$	$D\left(140n^2-140n+2\right)$
		$ct_{56,n}-56n+4$	$D(22)$
	$2ct_{7,n-1}$	$2ct_{7,2n-2}+32n-9$	$D\left(-70n^3+88n^2-20n+4\right)$
$ct_{8,n}$	$ct_{8,n-2}$	$ct_{8,2n-2}+8n-8$	$D\left(40n^2-40n+2\right)$
		$ct_{32,n}+32n$	$D(17)$
	$ct_{8,n-1}$	$ct_{8,2n-2}+20n-4$	$D\left(-16n^3+28n^2-4n+2\right)$
$2ct_{9,n}$	$2ct_{9,n-2}$	$2ct_{9,2n-2}+18n+11$	$D\left(234n^2-234n+7\right)$
		$ct_{72,n}-72n+4$	$D(46)$
	$2ct_{9,n-1}$	$2ct_{9,2n-2}+24n-13$	$D\left(-126n^3+148n^2-28n+4\right)$
$ct_{10,n}$	$ct_{10,n-2}$	$ct_{10,2n-2}+10n+2$	$D\left(60n^2-60n+2\right)$
		$ct_{40,n}-40n$	$D(26)$
	$ct_{10,n-1}$	$ct_{10,2n-2}+26n-8$	$D\left(-20n^3+39n^2-4n+2\right)$

Conclusion:

In this paper we have presented a few examples of constructing a special Dio 3 tuples for polygonal numbers and centered polygonal numbers with suitable properties. To conclude one may search for Dio 3 – tuples for higher order polygonal numbers and centered polygonal numbers with their corresponding suitable properties.

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