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# Special Dio 3 – tuples

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**Abstract.** We search for three distinct polynomials with integer coefficient such that the product of any two members of the set minus their sum and increased by a non- zero integer (or polynomial with integer coefficient) is a perfect square.

**Introduction:**

The problem of constructing the set with property that the product of any two its distinct elements is one less than a square has a very long history and such sets were studied by Diophantus. A set of m positive integers $\{a_1, a_2, \ldots a_m\}$ is called a Diophantine m-tuple

$$a_i * a_j + 1 \text{ is a perfect square,} \tag{1}$$

a perfect square for all $1 \le i \prec j \le m$. Many generalizations of this problem (1) were considered since antiquity, for example by adding a fixed integer n instead of 1, looking kth powers instead of squares or considering the powers over domains other than Z or Q. Many mathematicians consider the problem of the existence of Diophantine quadruples with the property *D(n)* for any arbitrary integer n and also for any linear polynomials in n. In this context one may refer [1-16]. The above results motivated us the following definition:

A set of three distinct polynomials with integer coefficient $(a_1, a_2, a_3)$ is said to be a special dio 3- tuple with property *D(n)* if $a_i * a_j - (a_i + a_j) + n$ is a perfect square for all $1 \le i \prec j \le 3$.

In the above definition n may be a non – zero integer or polynomial with integer coefficients. In this communication we consider a few special dio 3 tuples of polygonal numbers from $t_{6,n}$ to $t_{10,n}$ and centered polygonal numbers from $ct_{6,n}$ to $ct_{10,n}$ with their corresponding properties.

**Notations:**

$t_{m,n} = n\left(1 + \frac{(n-1)(m-2)}{2}\right) =$ Polygonal number of rank n with sides m

$ct_{m,n} = \frac{mn(n+1)}{2} + 1 =$ Centered Polygonal number of rank n with sides m

**Construction of Dio 3 - tuples for Hexagonal numbers**

Let $a = t_{6,n}$, $b = t_{6,n-2}$ be Hexagonal number of rank n and n-2 respectively such that $ab - (a+b) + 4n^2 - 10n + 11$ is a perfect square say $\gamma^2$

Let c be any non zero integer such that

$$ac-(a+c)+4n^2-10n+11=\alpha^2 \quad (2)$$
$$bc-(b+c)+4n^2-10n+11=\beta^2 \quad (3)$$

On solving equations (2) and (3), we get

$$(b-1)\alpha^2-(a-1)\beta^2=(a-b)+\left(4n^2-10n+11\right)(b-a) \quad (4)$$

Assuming $\alpha=x+(a-1)T$ and $\beta=x+(b-1)T$, in (4) it reduces to

$$x^2=(b-1)(a-1)T^2+\left(4n^2-10n+10\right) \quad (5)$$

The initial solution of equation (5) is given by

$$T_o=1 \text{ and } x_0=n^2-5n+1 \quad (6)$$

Therefore, $\alpha=4n^2-6n$ (7)

On substuting the values of α and $a$ in equation (2), we get

$$c=8n^2-20n+11$$
$$=t_{6,2n-2}-2n+1$$

Therefore, triple $\left(t_{6,n},t_{6,n-2},t_{6,2n-2}-2n+1\right)$ is Dio 3- tuple with property $D\left(4n^2-10n+11\right)$

For simplicity, we present in the table below a few Dio 3- tuples for polygonal numbers from $t_{6,n}$ to $t_{10,n}$ with suitable properties.

| a | b | c | D(n) |
|---|---|---|---|
| $t_{6,n}$ | $t_{6,n-2}$ | $t_{18,n}-13n+9$ | $D(10)$ |
| | $t_{6,n-1}$ | $t_{6,2n-2}+6n-4$ | $D\left(10n^2-15n+7\right)$ |
| $2t_{7,n}$ | $2t_{7,n-2}$ | $2t_{7,2n-2}-6n+5$ | $D\left(10n^2-26n+35\right)$ |
| | | $t_{42,n}-33n+29$ | $D(30)$ |
| | $2t_{7,n-1}$ | $2t_{7,2n-2}+14n+13$ | $D\left(25n^2-40n+17\right)$ |
| $t_{8,n}$ | $t_{8,n-2}$ | $t_{8,2n-2}-4n+7$ | $D\left(18n^2-48n+32\right)$ |
| | | $t_{26,n}-21n+7$ | $D(17)$ |
| | $t_{8,n-1}$ | $t_{8,2n-2}+8n-6$ | $D\left(18n^2-30n+14\right)$ |
| $2t_{9,n}$ | $2t_{9,n-2}$ | $2t_{9,2n-2}-10n+11$ | $D\left(28n^2-76n+74\right)$ |
| | | $t_{58,n}-49n+45$ | $D(54)$ |
| | $2t_{9,n-1}$ | $2t_{9,2n-2}+18n-19$ | $D\left(35n^2-60n+28\right)$ |
| $t_{10,n}$ | $t_{10,n-2}$ | $t_{10,2n-2}+6n-1$ | $D\left(24n^2-66n+47\right)$ |
| | | $t_{34,n}-29n+25$ | $D(26)$ |
| | $t_{10,n-1}$ | $t_{10,2n-2}+10n-10$ | $D\left(20n^2-35n+16\right)$ |

**Construction of Dio 3 - tuples for Centered Hexagonal numbers**

Let $a = ct_{6,n}$, $b = ct_{6,n-2}$ be Centered Hexagonal number of rank n and n-2 respectively such that $ab-(a+b)+24n^2-30n+2$ is a perfect square say $\gamma^2$

Let c be any non zero integer such that

$$ac-(a+c)+24n^2-30n+2=\alpha^2 \tag{8}$$

$$bc-(b+c)+24n^2-30n+4=\beta^2 \tag{9}$$

On solving equations (8) and (9), we get

$$(b-1)\alpha^2-(a-1)\beta^2=(a-b)+\left(24n^2-30n+2\right)(b-a) \tag{10}$$

Assuming $\alpha = x+(a-1)T$ and $\beta = x+(b-1)T$ in (10), it reduces to

$$x^2=(b-1)(a-1)T^2+\left(24n^2-30n+1\right) \tag{11}$$

The initial solution of equation (11) is given by

$$T_o=1 \text{ and } x_0=3n^2-3n+1 \tag{12}$$

Therefore, $\alpha = 6n^2+1$ (13)

On substuting the values of α and $a$ in equation (8), we get

$$c=12n^2-12n+9$$
$$= ct_{6,2n-2}+6n+8$$

Therefore, triple $\left(ct_{6,n}, ct_{6,n-2}, ct_{6,2n-2}+6n+8\right)$ is Dio 3- tuple with property $D\left(24n^2-30n+2\right)$

For simplicity, we present in the table below a few Dio 3- tuples for Centered polygonal numbers from $ct_{6,n}$ to $ct_{10,n}$ with suitable properties.

| a | b | C | D(n) |
|---|---|---|---|
| $ct_{6,n}$ | $ct_{6,n-2}$ | $ct_{24,n}-24n$ | $D(10)$ |
| | $ct_{6,n-1}$ | $ct_{6,2n-2}+14n-4$ | $D\left(-12n^3+19n^2-4n+2\right)$ |
| $2ct_{7,n}$ | $2ct_{7,n-2}$ | $2ct_{7,2n-2}+14n+9$ | $D\left(140n^2-140n+2\right)$ |
| | | $ct_{56,n}-56n+4$ | $D(22)$ |
| | $2ct_{7,n-1}$ | $2ct_{7,2n-2}+32n-9$ | $D\left(-70n^3+88n^2-20n+4\right)$ |
| $ct_{8,n}$ | $ct_{8,n-2}$ | $ct_{8,2n-2}+8n-8$ | $D\left(40n^2-40n+2\right)$ |
| | | $ct_{32,n}+32n$ | $D(17)$ |
| | $ct_{8,n-1}$ | $ct_{8,2n-2}+20n-4$ | $D\left(-16n^3+28n^2-4n+2\right)$ |
| $2ct_{9,n}$ | $2ct_{9,n-2}$ | $2ct_{9,2n-2}+18n+11$ | $D\left(234n^2-234n+7\right)$ |
| | | $ct_{72,n}-72n+4$ | $D(46)$ |
| | $2ct_{9,n-1}$ | $2ct_{9,2n-2}+24n-13$ | $D\left(-126n^3+148n^2-28n+4\right)$ |
| $ct_{10,n}$ | $ct_{10,n-2}$ | $ct_{10,2n-2}+10n+2$ | $D\left(60n^2-60n+2\right)$ |
| | | $ct_{40,n}-40n$ | $D(26)$ |
| | $ct_{10,n-1}$ | $ct_{10,2n-2}+26n-8$ | $D\left(-20n^3+39n^2-4n+2\right)$ |

**Conclusion:**

In this paper we have presented a few examples of constructing a special Dio 3 tuples for polygonal numbers and centered polygonal numbers with suitable properties. To conclude one may search for Dio 3 – tuples for higher order polygonal numbers and centered polygonal numbers with their corresponding suitable properties.

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