# AN INVERSE THERMOELASTIC PROBLEM OF CIRCULAR PLATE.

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**Abstract**: The aim of this work is to determine the unknown temperature, displacement and thermal stresses on the upper surface of the circular plate subjected to arbitrary known interior temperature under Steady-state field. The fixed circular edge is thermally insulated and temperature of a lower surface of plate is kept at zero.The governing heat conduction equation has been solved by using the Hankel transform technique. The results are obtained in series form in terms of Bessel's functions and results have been computed numerically and illustrated graphically.

## 1. Introduction

As known, thermal behaviors of structures must be considered in many situations. Study of thermal effect on deformations and stresses of a plate, especially a circular plate is increasingly important. Firstly, the problems of circular plates are more complicated and thus more attractive to many scientists. Secondly, there are practical requirements for thick plates in various modern projects, such as high building, raceway, high-way, container wharf, and so on.

Ashida et al. [1] discussed the inverse transient thermoelastic problem for a composite circular plate. Deshmukh et al. [2] are also discussed on an inverse transient problem of quasi-static thermal deflection of a thin clamped circular plate. Grysa et al [3] studied the one dimensional problem of temperature and the heat flux at the surface of a thermo elastic slab. Kulkarni et al. [5] studied an inverse transient problem of quasi-static thermal stresses in a thick circular plate. Noda [6] discussed an analytical method for an inverse problem of coupled thermal stress fields in a circular cylinder.

Also Roy Choudhary [7] studied a rate of quasi-static stress in a thin circular plate due to transient temperature applied along the circumference of a circle over the upper face. Recently Gaikwad et al. [4] studied an exact solution of unsteady-state thermoelastic problem of a circular plate.

The present paper deals with the determination of unknown temperature, displacement and thermal stresses on the upper surface of circular plate subjected to arbitrary known interior temperature under steady state field. The fixed edge is thermally insulated and temperature of a lower surface of plate is kept at zero. The governing heat conduction equation has been solved by using the Hankel transform technique. The results are obtained in series form in terms of Bessel's functions and results have been computed numerically and illustrated graphically.

This paper contains new and novel contribution of stresses in circular plate under steady state. The results presented here will be more useful in engineering problem particularly in the determination of the state of strain in circular plate constituting foundations of containers for hot gases or liquids, in the foundations for furnaces etc.

## 2. Statement of the problem

Consider a circular plate of radius $a$ and thickness $2h$ occupying space $D : 0 \leq r \leq a , -h \leq z \leq h$ .The thermoelastic displacement function as in [6] is governed by Poisson's equation

$$\nabla^2 U = (1+\nu)a_t T \tag{1}$$

with $U = 0$ at $r = 0$ and $r = a$. (2)

where
$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$$

$\nu$ and $a_t$ are the Poisson's ratio and the linear coefficient of thermal expansion of the material of the plate and T is the temperature of the plate satisfying the differential equation

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = 0 \tag{3}$$

Subject to the boundary conditions

$$\frac{\partial T}{\partial r} = 0 \qquad\qquad \text{at } r = a, -h \leq z \leq h \tag{4}$$

$$\frac{\partial T}{\partial z} + k_1 T = f(r) \qquad\qquad \text{at } z = \xi, -h \leq \xi \leq h, \ 0 \leq r \leq a \tag{5}$$

$$\frac{\partial T}{\partial z} - k_2 T = 0 \qquad\qquad \text{at } z = -h, 0 \leq r \leq a \tag{6}$$

and

$$\frac{\partial T}{\partial r} = g(r) \qquad\qquad \text{(unknown)} \qquad \text{at } z = h, 0 \leq r \leq a \tag{7}$$

where $k_1$ and $k_2$ are the radiation constants on the plane two surfaces.
The stress functions $\sigma_{rr}$ and $\sigma_{\theta\theta}$ are given by,

$$\sigma_{rr} = -2\mu\frac{1}{r}\frac{\partial U}{\partial r} \tag{8}$$

$$\sigma_{\theta\theta} = -2\mu\frac{\partial^2 U}{\partial r^2} \tag{9}$$

where $\mu$ is the Lame's constant, while each of the stress functions $\sigma_{rz}$, $\sigma_{zz}$ and $\sigma_{\theta z}$ are zero within the plate in the plane state of stress. The equations (1) to (9) constitute the mathematical formulation of the problem under consideration.

### 3. Solution of the Problem

To obtain the expressions for temperature $T(r, z)$ introduce the finite Hankel transform over the variable r and its inverse transform defined in [8] as

$$\overline{T}(\xi_n, z) = \int_0^a r\, J_0(\xi_n, r)T(r,z)dr \tag{10}$$

$$T(r,z) = \sum_{n=1}^{\infty} \left( \frac{2J_0(\xi_n, r)}{a^2 J_0^2(\xi_n, a)} \right) \overline{T}(\xi_n, z) \tag{11}$$

where $\xi_1, \xi_2, \cdots\cdots$ is the transcendental equation

$$J1\,(\xi\,a) = 0 \tag{12}$$

$J_n(x)$ is Bessel function of the first kind of order $r$

$$\frac{d^2\overline{T}}{dz^2} - \xi_n^2 \overline{T} = 0 \tag{13}$$

where $T$ is the Hankel transform of $T$ .
On solving Eq. (13) under the conditions given in Eq. (4) and Eq. (5),
One obtains

$$\overline{T} = \sum_{n=1}^{\infty} \overline{f}(\xi_n) \left[ \frac{\xi_n \cosh[\xi_n(z+h)] + k_2 \sinh[\xi_n(z+h)]}{\left(\xi_n^2 + k_1 k_2\right)\sinh(2\xi_n h) + \xi_n(k_1 + k_2)\cosh(2\xi_n h)} \right] \tag{14}$$

On applying the inverse Hankel transform defined in Eq. (11) to Eq. (14), one obtain the expression for the temperature as

$$T(r,z) = \sum_{n=1}^{\infty} \overline{f}(\xi_n) \left( \frac{2J_0(\xi_n r)}{a^2 J_0^2(\xi_n a)} \right) \left[ \frac{\xi_n \cosh[\xi_n(z+h)] + k_2 \sinh[\xi_n(z+h)]}{\left(\xi_n^2 + k_1 k_2\right)\sinh(2\xi_n h) + \xi_n(k_1 + k_2)\cosh(2\xi_n h)} \right] \tag{15}$$

Unknown temperature as,

$$g(r) = -\sum_{n=1}^{\infty} \xi_n \overline{f}(\xi_n) \left( \frac{2J_1(\xi_n r)}{a^2 J_0^2(\xi_n a)} \right)$$

$$\times \left[ \frac{\xi_n \cosh(2h\xi_n) + k_2 \sinh(2h\xi_n)}{\left(\xi_n^2 + k_1 k_2\right)\sinh(2\xi_n h) + \xi_n(k_1 + k_2)\cosh(2\xi_n h)} \right] \tag{16}$$

where $\quad \overline{f}(\xi_n) = \int_0^a r\,J_0(\xi_n, r) f(r)\,dr \tag{17}$

**Determination of Thermoelastic Displacement**

On putting the values of temperature $T\,(r,\,z,\,t\,)$ from Eq. (16) in Eq. (1), one obtain the thermoelastic displacement function $U\,(r,\,z,\,t)$ as,

$$U(r,z) = -(1+v)\frac{2a_t}{a^2} \sum_{n=1}^{\infty} \overline{f}(\xi_n) \left( \frac{J_0(\xi_n r)}{\xi_n^2 J_0^2(\xi_n a)} \right)$$

$$\times \left[ \frac{\xi_n \cosh[\xi_n(z+h)] + k_2 \sinh[\xi_n(z+h)]}{\left(\xi_n^2 + k_1 k_2\right)\sinh(2\xi_n h) + \xi_n(k_1 + k_2)\cosh(2\xi_n h)} \right] \tag{18}$$

## Determination of Stress Functions

Using Eq. (17) in Eq. (8) and (9), one obtains the stress functions $\sigma_{rr}$ and $\sigma_{\theta\theta}$ as,

$$\sigma_{rr} = -\frac{4(1+v)\mu a_t}{ra^2} \sum_{n=1}^{\infty} \overline{f}(\xi_n) \left( \frac{J_1(\xi_n r)}{\xi_n J_0^2(\xi_n a)} \right)$$

$$\times \left[ \frac{\xi_n \cosh\left[\xi_n(z+h)\right] + k_2 \sinh\left[\xi_n(z+h)\right]}{\left(\xi_n^2 + k_1 k_2\right)\sinh(2\xi_n h) + \xi_n(k_1+k_2)\cosh(2\xi_n h)} \right] \tag{19}$$

$$\sigma_{\theta\theta} = -\frac{4(1+v)\mu a_t}{a^2} \sum_{n=1}^{\infty} \overline{f}(\xi_n) \left( \frac{J_1'(\xi_n r)}{J_0^2(\xi_n a)} \right)$$

$$\times \left[ \frac{\xi_n \cosh\left[\xi_n(z+h)\right] + k_2 \sinh\left[\xi_n(z+h)\right]}{\left(\xi_n^2 + k_1 k_2\right)\sinh(2\xi_n h) + \xi_n(k_1+k_2)\cosh(2\xi_n h)} \right] \tag{20}$$

## Special Case and Numerical Results

Set

$$f(r) = (r^2 - a^2)^2 \tag{21}$$

Applying finite Hankel transform as defined in Eq. (17) to (21), one obtains

$$\overline{f}(\xi_n) = \int_0^a r(r^2 - a^2)^2 J_0(\xi_n, r)dr$$

$$\overline{f}(\xi_n) = \frac{8a\left\{\left(8 - a^2\xi_n^2\right)J_1(\xi_n a) - 4aJ_0(\xi_n a)\right\}}{\xi_n^5} \tag{22}$$

The numerical calculation have been carried out for steel (SN 50C) plate with parameters

Set $a = 1m$, $h = 0.2m$. Thermal diffusivity $k = 15.9 \times 10^{-6}$ ($m^2 \, s^{-1}$) and Poisson ratio $v = 0.281$ With $\xi_1 = 3.8317$, $\xi_2 = 7.0156$, $\xi_3 = 10.1735$, $\xi_4 = 13.3237$, $\xi_5 = 16.470$, $\xi_6 = 19.46159$, $\xi_7 = 22.7601$. are the roots of transcendental equations $J_1(\xi a) = 0$ as in [9].

## Concluding Remarks:

In this paper, we have discussed the steady-state thermoelastic problem for a circular plate on outer curved surface of the circular plate. The fixed circular edge is thermally insulated and zero temperature is maintained on lower surface.

The finite Hankel transform technique is used to obtain the numerical results. The thermoelastic behavior is examined such as unknown temperature, displacement and stresses that are obtained can be applied to the design of useful structures or in engineering applications. Also any particular case of special interest can be derived by assigning suitable values to the parameters and functions in the expressions (15), (18)- (20).
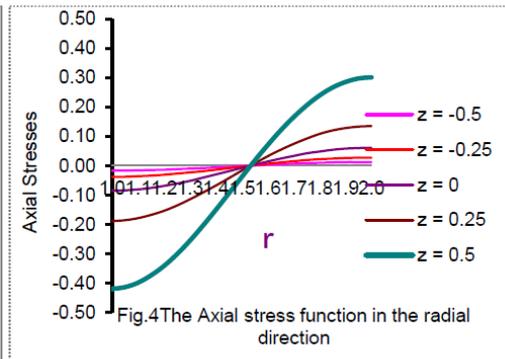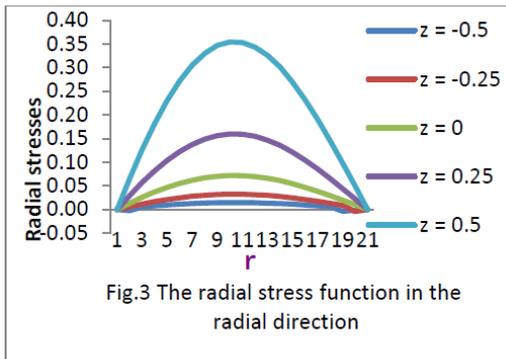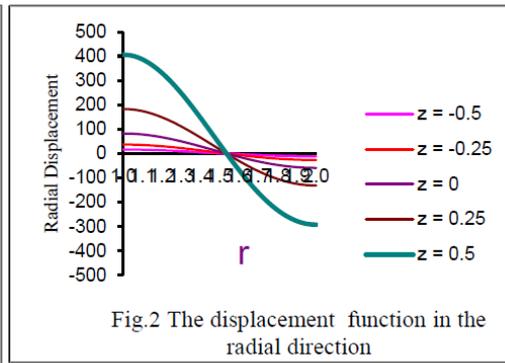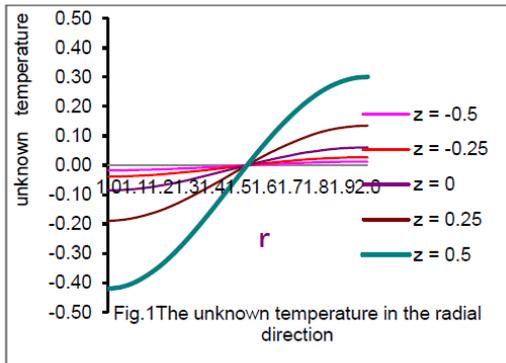
Fig.1 The unknown temperature in the radial direction

Fig.2 The displacement function in the radial direction

Fig.3 The radial stress function in the radial direction

Fig.4 The Axial stress function in the radial direction

**Fig.1.** The unknown temperature decreases with the thickness of the circular region $0 \leq r \leq 0.8$ where as it is an increase within the annular region $0.8 \leq r \leq 1$.

**Fig.2.** The radial displacement increases with the thickness of the circular plate.

**Fig.3.** The radial stress function $\sigma rr$ is increases with the circular plate and it shows the normal curve. Also it develops the tensile stresses in radial direction.

**Fig.4.** The stress function $\sigma_{\theta\theta}$ is decreases with the thickness of the circular plate and it is negligible for small thickness. Also it develops the tensile stresses in radial direction.

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