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Strongly Connectedness in Closure Space

[1]U.D.TAPI and [2]BHAGYASHRI A. DEOLE

[1]Department of Applied Mathematics and Computational Science

Shri G. S. Institute of Technology and Sciences (M.P.), India

[2]Department of Applied Mathematics and Computational Science

Shri G. S. Institute of Technology and Sciences (M.P.), India

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Abstract: A Čech closure space (X, u) is a set X with Čech closure operator u: P(X) → P(X) where P(X) is a power set of X, which satisfies u $\phi=\phi$, A ⊆uA for every A⊆X, u (A⋃B) = uA⋃uB, for all A, B ⊆ X. Many properties which hold in topological space hold in closure space as well. A topological space X is strongly connected if and only if it is not a disjoint union of countably many but more than one closed set. If X is strongly connected, and Ei"s are nonempty disjoint closed subsets of X, then X≠ E1⋃E2⋃. We further extend the concept of strongly connectedness in closure space.
The aim of this paper is to introduce and study the concept of **strongly connectedness in closure space.**

1. Introduction:

Čech closure space was introduced by Čech E. [1] in 1963. The modern notion of connectedness was proposed by Jorden (1893) and Schoenfliesz, and put on firm footing by Riesz [7] with the use of subspace topology. Many mathematicians such as EissaD.Habil, Khalid A. Elzenati[4], EissaD. Habil [5], Stadler B.M.R. and Stadler P.F. [6] have extended various concepts of strongly connectedness in topological space.
In this paper, we introduce **strongly connectedness in closure space** and study some of their properties.

2. Preliminaries:

Definition 2.1[2]**:-** An operator u: P(X) →P(X) defined on the power set P(X) of a set X satisfying the axioms:

1. u $\phi=\phi$,
2. A ⊆uA , for every A⊆X,
3. u(A⋃B)=uA⋃uB , for all A, B⊆X.

is called a Čech closure operator and the pair (X, u) is a Čech closure space.

Definition 2.2 [8]:- A closure space (X, u) is connected if and only if there exists a continuous function from X to the discrete space {0, 1} is constant. A subset A in a closure space (X, u) is said to be connected if A with the subspace topology is a connected space.

Definition 2.3[3]:A topological space X is strongly connected if and only if it is not a disjoint union of countably many but more than one closed set.
If X is strongly connected, and E_i"s are nonempty disjoint closed subsets of X, then

$X \neq E_1 \cup E_2 \cup$.............

3. Strongly connectedness in closure space:

Definition 3.1: A connected closure space (X, u) is said to be strongly connected if and only if it cannot be expressed as a disjoint union of count ably many but more than one closed sets.
In connected closure space (X, u), let E_1 and E_2 are two nonempty disjoint closed subsets of X then $X \neq E_1 \cup E_2$.
In strongly connected closure space (X, u), E_i,,s are nonempty disjoint closed subsets of X then $X \neq E_1 \cup E_2 \cup$.............

Example3.2: Let X= {a, b, c},
Define a closure function u: P(X) → P(X) such that
u{a}={a, b},u{b}=u{c}=u{b, c}={b, c},u{a, b}=u{a, c}=u(X)=X, u($\emptyset$)=$\emptyset$.

Hence (X, u) is a closure space.
Define a continuous function f: X→ {0, 1} such that f{a}=f{b}=f{c}=f{a, b}=f{a, c}=f{b, c}=1, f{ϕ}=ϕ.
Hence (X, u) is a connected closure space.
Here (X, u) is strongly connected closure space because we cannot find nonempty disjoint closed subsets E_i"s of X such that $X = E_1 \cup E_2 \cup$.............

Definition 3.3:- A connected closure space (X, u) is said to be strongly disconnected if and only if it can be expressed as a disjoint union of count ably many but more than one closed sets.

Theorem 3.4:- A continuous image of a strongly connected closure space is strongly connected closure space.

Proof: Let (X, u) is a strongly connected closure space. Suppose f (X) is not strongly connected closure space then by definition it can be expressed as a disjoint union of count ably many but more than one closed sets. Since f is continuous and the inverse image of closed set is still closed, so that X can be expressed as a disjoint union of count ably many but more than one closed sets. Hence (X, u) is a strongly disconnected closure space which is a contradiction. Therefore f(X) is strongly connected closure space.

Definition 3.5[9]:- Let (Z, u) be a closure space with more than one point .A closure space (X, u) is called z- connected closure space if and only if any continuous map from X to Z is constant.

Theorem3.6:-A connected closure space (X, u) is strongly connected if and only if it is z-connected closure space.

Proof: - Necessary condition: Suppose (X, u) is strongly connected closure space.
Let f: X→ {0, 1} = Z be any continuous map. By proposition 3.4, f(X) is strongly connected. The only strongly connected subsets of Z are the one point spaces. Hence f is constant; i.e. X is z-connected closure space.
Sufficient condition: Suppose connected closure space (X, u) is z-connected. If possible let (X, u) is not strongly connected closure space so that X can be expressed as a disjoint union of count ably many but more than one closed sets i. e. $X = \cup E_i$ .Then define a function f: X→Z= {0, 1} by taking f(x) =i whenever $x \in E_i$. This f is continuous but not constant. Hence X is not z-connected closure space which is a contradiction. Therefore, X is strongly connected closure space.
From the above theorem, strongly connected closure space is a special case of z-connectedness in closure space. Thus all the properties proved for z-connected closure space are applicable to strongly connected closure space.

Theorem 3.7:-A strongly connected closure space is connected. But converse is not true as the following example shows.

Example3.8:- Let X= {a, b, c} and define u: P(X) →P(X) as follows

u(∅)=∅,u{a}={a},u{b}={b},u{c}={c}, u{a, b}=u{a, c}=u{b, c}=u{X}=X. Hence (X, u) is a closure space.
We define a function f: X → {0, 1} such that f{a}=f{b}=f{c}=f{a, b}=f{a, c}=f{c, a}=f{X}=X, f{ϕ}=0.

Hence f is constant so that (X, u) is a connected closure space. However, X is not strongly connected because there exists pair wise semi separated sets {a}, {b}, {c} such that
X= {a} ∪{b}∪{c}.

Theorem 3.9:- The union of any family of strongly connected subsets of strongly connected closure space with a common point is strongly connected closure space.

Proof:-Let (X, u) is a strongly connected closure space. Let each {Ei: i∈Λ} is strongly connected subset of strongly connected closure space (X, u) and common point is y0.Let C= {⋃Ei: Ei⊆ X}, y0 ∈⋂Ei. For any continuous function f: C→ {0, 1}. Let ia: Ei→ C be the inclusion function. Each Ei is strongly connected, so that foia: Ei → {0, 1} is continuous and constant and ⋂ Ei ≠∅, so there exists a y0 such that y0 ∈ ⋂ Ei , i. e. foia is constant and equal to f(y0). Therefore f is constant and ⋃Ei is strongly trongly connected
Theorem 3.10:- Let A and B are subsets of a strongly connected closure space (X, u) such that A⊆B⊆A, where A is the closure of A. If A is strongly connected, then is strongly connected in closure space

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