

## PARTICLE KNOTS IN TORIC MODULAR SPACE

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**Abstract:**The goal of this contribution is to relate quarks to knots or loops in a 6-space  $CP^3$  that then collapses into a torus in real 3-space  $P^3$  instantaneously after the Big Bang, and massive inflation, when 3 quarks unite to form nucleons.

### Introduction

Kedia et. al. in recent paper [9] investigate knotted structures in hydrodynamic fields such as current-guiding magnetic field lines in a plasma, or vortex lines of classical or quantum fields which arise naturally as excitations that carry helicity that is a measure of the knottedness of the field. In particular their Fig.2g is a trefoil which is our Fig.3 without the quadrupole (that will be seen in Section 2 to collapse into a point) and the color-coding.

The torus shown in Fig.2 is due to Marcelis [12] whose calculations in a projective space with 24 vertices appear to be unpublished, but are supported to some extent by Westy [16] (from the same school) who provides a color-coded complex map of the Riemann surface  $z$  that incorporates the phases of  $\omega=120$  degrees.

Murasagi [13] Ch.7 shows that Fig.2 is a trefoil (3,2) on a torus when we choose 3 points on the ends of a cylinder that can be joined to form the torus. This brings us to the goal of this contribution which is to relate the elementary particles to knots or loops in a 6-space. Here we will be guided by the work of Coxeter [5,6] who specifically labels the vertices of the torus appearing in Fig.1 by  $0, \pm 1, \omega, \omega^2$  where  $\omega = \exp(2i\pi/3)$  so that a knot crosses the longitude of a torus at  $\omega=120$  degrees. Essentially this is a Galois Field GF(4) with permutations of  $\omega$  raised to the powers 0,1,2,3 that will be considered in more detail in the next Section where we will show how 3 quarks in 6-space unite to become a nucleon in the projective space  $CP^3$  which collapses to 2  $P^3$  immediately after the Big Bang .Section 3 will employ the color-coding of Fig.1 as a model for Quantum Chromodynamics or QCD. Finally according to Rovelli [14] knots or loops in the 6-space described by  $E_6$  employed by Coxeter may also describe Loop Quantum Gravity although details are beyond the scope of this contribution. Also Arvin [2] has considered knots on a torus as a model for elementary particles but excluding quarks.

### Coxeter Algebra

Fig.1 is a torus taken from Coxeter [5] which is an alternative to the graph of  $su(3)_c \times su(3)_{spin} \times su(3)_{isospin}$  which is a sub-algebra of  $E_6$ .This was published later [6] as Fig.12.3B.Both graphs are orbifolds with 27 vertices that according to Slansky [15] may be labeled by particles in the Standard Model or SM. However the actual labeling of the tritangent planes on a cubic surface (discussed by Hunt [10] Ch. 4) is new but in line with Coxeter's labels. For example the up-quark  $u$  in Fig.1 is labeled by (012) indicated by  $0, \omega, \omega^2$ . Thus (023) on the same tritangent is simply a rotation through  $\omega=120$  degrees and so on. In this way we find an equilateral triangle labeled by the 3 quarks  $uud$  comprising a proton and another  $ddu$  for the neutron beginning with (120). There are 2 more tritangents (not labeled) for the anti-particles which complete the outer ring of Fig.1. But quarks also belong to a GF(4) ring and thus to a trefoil on a torus which is precisely the model adopted by Green, Schwarz and Witten [7] Section 9.5.2.

The quarks at the vertices of Fig.1 are trefoils illustrated by Fig.2, but the torus in Fig.1 only becomes a trefoil after the collapse of the inner ring just after the Big Bang when quarks in the 6-space  $CP^3$  unite to build nucleons in the projective space  $P^3$ .

This is supported by Barth and Nieto [3] where only the 12 outer vertices and the center of Fig.1 are in  $P^3$ . Specifically these authors find 15 synthemes, where a syntheme has 6 'fix-lines' that are the edges of an invariant tetrahedron such as  $u u d 0$  representing a proton. However because there are only 3 vertices on the face of a syntheme the outer ring of Fig.1 carries the 4 stable particles proton, neutron and their anti-particles. Also since the tritangents are invariant under rotations there are actually  $3 \times 4 = 12$  possible synthemes on the outer ring. Specifically each syntheme consists of 3 commuting operators. Thus 3 synthemes can be chosen for spin rotations about the 3 axes of 3 space. Thus introducing triality which is a characteristic of Toric-Calabi-Yau modular spaces that carry the Hessian Polyhedra in  $E_6$  as discussed by Lie-Yang [11] and analysed by Coxeter [5,6].

In this way the 12 unstable particles  $sss, \overline{(sss)}, \mu^\pm, \nu_\mu, \tau^\pm, \nu_\tau$  do not appear in the blow down of  $CP^3$  to  $P^3$ . This may be visualised as a collapse of the inner vertices to a point which carries the remaining 3 synthemes  $e^\pm, \nu_e$ . labeled by  $\{011,022,033\}, \{110,220,330\}$  and  $\{101,202,303\}$  for the muon. In this process the masses  $m_\tau, m_\nu$  of the  $\tau, \mu$  reappear as stable deuterium 3

according to the relationship

$$m_\tau + m_\mu = m_p + m_n + m_e \quad (1)$$

There is no heavy-ion decay and the same relation holds for the anti-particles, and the equation is accurate if we assume that  $m_\tau = 1777$  MeV and  $m_\mu = 101.4$  MeV instead of the Fermi decomposition of muon decay in the weak interaction yielding 106 MeV. However in a recent publication Benjamin Brau et.al.[4] find a value of approximately 100 MeV for the mass of cosmic-ray muons so there is as yet some experimental uncertainty.

### Quantum Chromodynamics, QCD

Returning again to Fig.1, when the inner vertices are contracted to a point at the origin the red, green and blue lines could serve as gluons on a new torus where a red upper path passes through the center before emerging at the circumference and giving way to a green gluon that in turn passes under the torus and then over to connect with a down quark and so on. The 3 color complex dimensions vanish when  $CP^3 \rightarrow P^3$  but a torus knot remains in 3-space.

However Marcelis [12] calculates the dual set of 3 paths for the anti-gluons  $\overline{(\#g,b)}$  which appear in Fig.3 (without the quadrupole) so the gluon, anti-gluon linked trefoil give us the  $SU(3)_c$  color symmetry underlining QCD as described by Griffiths [8], Section 9.1. For example when another quark is added after a rotation  $\omega$  a red gluon may unite with an anti-blue to build  $\overline{r} \overline{b}$ , then a following rotation would bring  $\overline{r}$  to  $\overline{r}$ , and so on before blow down to  $P^3$ . In this way we can find 9 gluon pairs  $\overline{r}, \overline{b}, \overline{g}; r, b, g$  that are a basis for  $SU(3)_c$  symmetry.

Finally Adams [1] p 273 also envisages the 3 colors  $r, b, g$  as three extra dimensions in a 6-space.

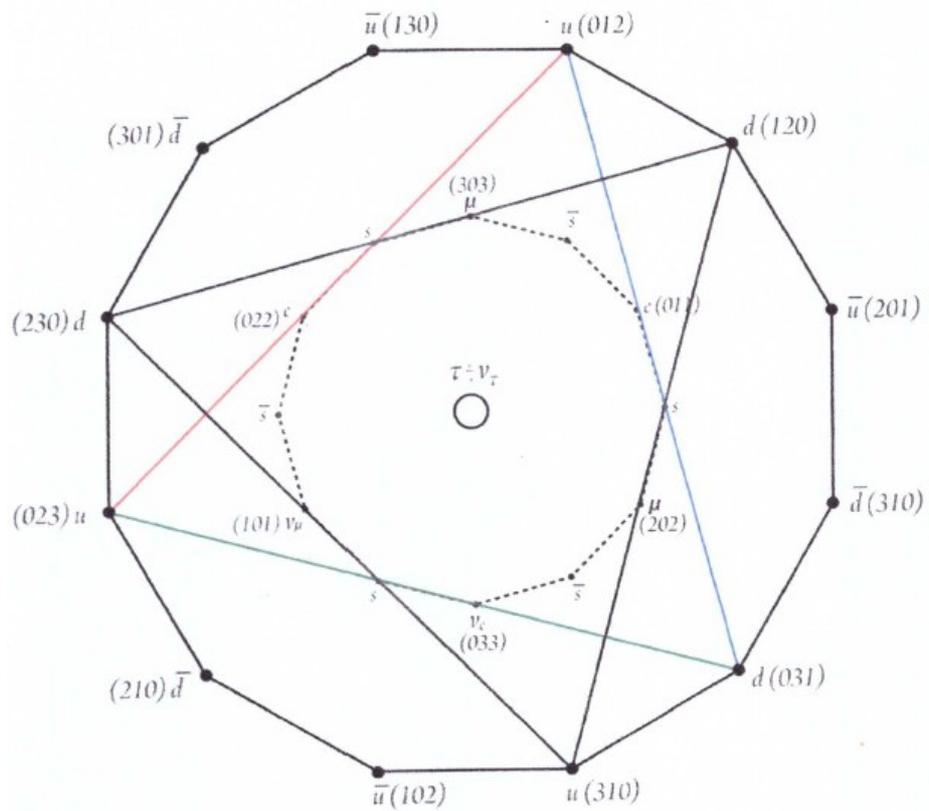


Fig.1 The Coxeter Polytope

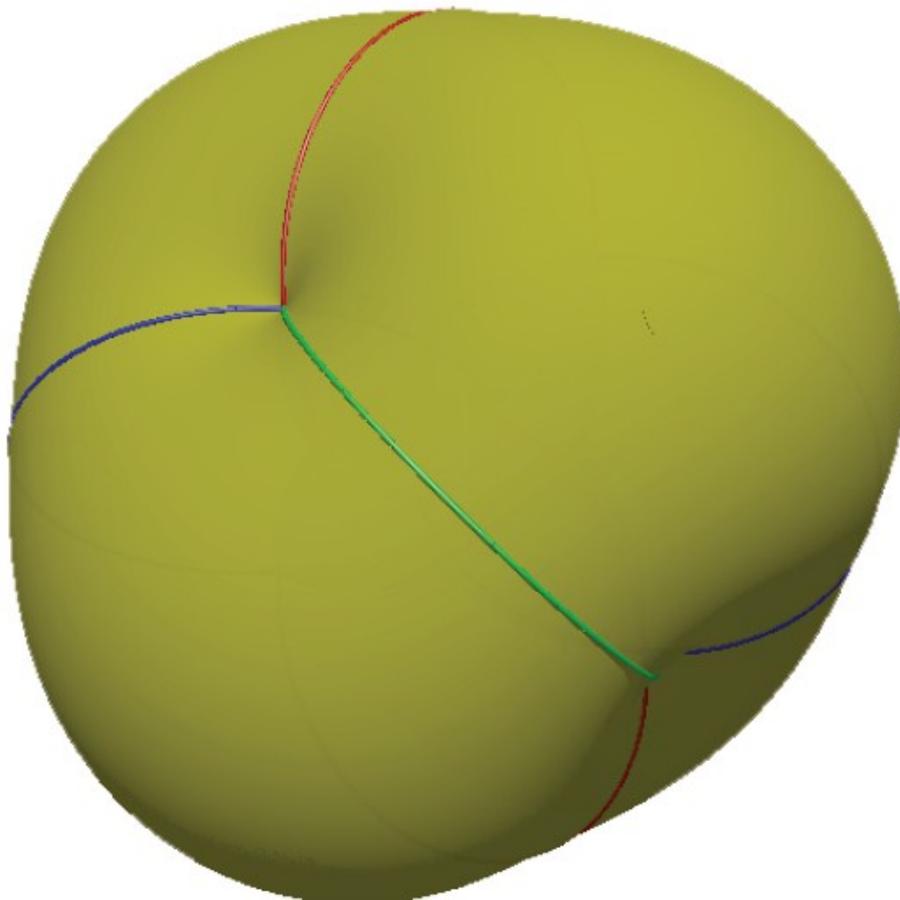


Fig.2 Cayley Surface in Elliptic Space

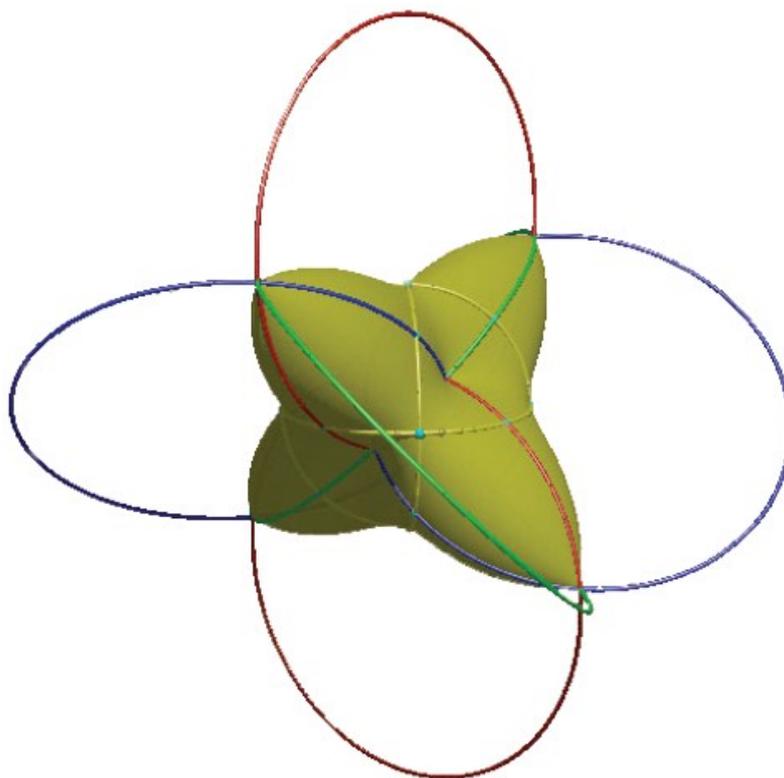


Fig.3 Interior of Cayley Surface

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