

## Magic Graphoidal on Class of Trees

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**Keywords:** Graphoidal Cover, Magic Graphoidal, Graphoidal Constant.

**Abstract:** B.D.Acharya and E. Sampathkumar [1] defined Graphoidal cover as partition of edge set of a graph  $G$  into internally disjoint paths (not necessarily open). The minimum cardinality of such cover is known as graphoidal covering number of  $G$ .

Let  $G = \{V, E\}$  be a graph and let  $\psi$  be a graphoidal cover of  $G$ . Define  $f$ :

$V \cup E \rightarrow \{1, 2, \dots, p+q\}$  such that for every path  $P = (v_0, v_1, v_2, \dots, v_n)$  in  $\psi$  with  $f^*(P) = f(v_0) + f(v_n) + \sum_{i=1}^n f(v_{i-1}v_i) = k$ , a constant, where  $f^*$  is the induced labeling on  $\psi$ . Then, we say that  $G$  admits  $\psi$  - magic graphoidal total labeling of  $G$ .

A graph  $G$  is called magic graphoidal if there exists a minimum graphoidal cover  $\psi$  of  $G$  such that  $G$  admits  $\psi$  - magic graphoidal total labeling.

In this paper, we proved that  $[P_n; S_1]$ ,  $[P_n; S_2]$ ,  $T(n)$ ,  $P_m \odot K_{1,3}$ ,  $P_m \odot 2K_1$  and  $K_{1,n} \odot 2$  are magic graphoidal

### 1. INTRODUCTION

By a graph, we mean a finite simple and undirected graph. The vertex set and edge set of a graph  $G$  are denoted by  $V(G)$  and  $E(G)$  respectively. Terms and notations not used here are as in [3].

- 1.1. Definition :** Let  $S_1 = (v_0, v_1)$  be a star and let  $[P_n ; S_1]$  be the graph obtained from  $n$  copies of  $S_1$  and the path  $P_n = (u_1, u_2, \dots, u_n)$  by joining  $u_j$  with the vertex  $v_0$  of the  $j^{\text{th}}$  copy of  $S_1$  by means of an edge, for  $1 \leq j \leq n$ .
- 1.2. Definition :** Let  $S_2 = (v_0, v_1, v_2)$  be a star and let  $[P_n ; S_2]$  be the graph obtained from  $n$  copies of  $S_2$  and the path  $P_n = (u_1, u_2, \dots, u_n)$  by joining  $u_j$  with the vertex  $v_0$  of the  $j^{\text{th}}$  copy of  $S_2$  by means of an edge, for  $1 \leq j \leq n$ .
- 1.3. Defintion :** Let  $T$  be any Tree. Denote the tree, obtained from  $T$  by considering two copies of  $T$  by adding an edge between them, by  $T_{(2)}$  and in general the graph obtained from  $T_{(n-1)}$  and  $T$  by adding an edge between them is denoted by  $T_{(n)}$ .
- 1.4. Result [11] :** For a Tree  $T$ ,  $\gamma(T) = n-1$  where  $n$  is the number of pendent vertices of  $G$ .

### 2. PRELIMINARIES

Let  $G = \{V, E\}$  be a graph with  $p$  vertices and  $q$  edges. A graphoidal cover  $\psi$  of  $G$  is a collection of (open) paths such that

- (i) every edge is in exactly one path of  $\psi$
- (ii) every vertex is an interval vertex of atmost one path in  $\psi$ .

We define  $\gamma(G) = \min_{\psi \in \zeta} |\psi|$ ,

where  $\zeta$  is the collection of graphoidal covers  $\psi$  of  $G$  and  $\gamma$  is graphoidal covering number of  $G$ .

Let  $\psi$  be a graphoidal cover of  $G$ . Then we say that  $G$  admits  $\psi$  - magic graphoidal total labeling of  $G$  if there exists a bijection  $f: V \cup E \rightarrow \{1, 2, \dots, p+q\}$  such that for every path  $P = (v_0 v_1 v_2 \dots v_n)$  in  $\psi$ , then,  $f^*(P) = f(v_0) + f(v_n) + \sum_{i=1}^n f(v_{i-1}v_i) = k$ , a constant, where  $f^*$  is the induced labeling of  $\psi$ . A graph  $G$  is called magic graphoidal if there exists a minimum graphoidal cover  $\psi$  of  $G$  such that  $G$  admits  $\psi$  - magic graphoidal total labeling.

**3. Magical Graphoidal on Trees**

**3.1. Theorem :**  $[P_n ; S_1]$ ,  $(n - \text{even})$  is magic graphoidal.

**Proof:** Let  $G = [P_n ; S_1]$

Let  $V(G) = \{u_i, v_i, w_i: 1 \leq i \leq n\}$  and

$$E(G) = \{ [(u_i v_i) \cup (v_i w_i) : 1 \leq i \leq n] \cup [(u_i u_{i+1}) : 1 \leq i \leq n-1] \}$$

Define  $f: V \cup E \rightarrow \{1, 2, \dots, p+q\}$  by

$$\begin{aligned} f(w_1) &= 1 \\ f(w_1 v_1) &= 2 \\ f(v_1 u_1) &= 3 \\ f(u_{i+1}) &= 6 + i && 1 \leq i \leq n-2 \\ f(w_{i+1}) &= 6n - i && 1 \leq i \leq n-1 \\ f(v_{i+1} w_{i+1}) &= 4n + 1 + i && 1 \leq i \leq n-1 \\ f(u_{i+1} v_{i+1}) &= 3n + 3 - 2i && 1 \leq i \leq (n/2)-1 \end{aligned}$$

$$f\left(u_{\frac{n}{2}+i} v_{\frac{n}{2}+i}\right) = 3n + 4 - 2i \quad 1 \leq i \leq \frac{n}{2}$$

$$f(u_i u_{i+1}) = \frac{3n}{2} + 4 + i \quad 1 \leq i \leq \frac{n}{2} - 1$$

$$f\left(u_{\frac{n}{2}-1+i} u_{\frac{n}{2}+i}\right) = n + 4 + i \quad 1 \leq i \leq \frac{n}{2}$$

Let  $\psi = \{P_1 = (w_1, v_1, u_1, u_2, v_2, w_2), P_2 = (u_i, u_{i+1}, v_{i+1}, w_{i+1}) : 2 \leq i \leq n-1\}$

$$f^*[P_1] = f(w_1) + f(w_2) + f(w_1 v_1) + f(v_1 u_1) + f(u_1 u_2) + f(u_2 v_2) + f(v_2 w_2)$$

$$= 1 + 6n - 1 + 2 + 3 + \frac{3n}{2} + 4 + 1 + 3n + 3 - 2 + 4n + 2$$

$$= 13n + \frac{3n}{2} + 13 \text{ ----- (A)}$$

For  $2 \leq i \leq (n/2)-1$ ,

$$f^*[P_2] = f(u_i) + f(w_{i+1}) + f(u_i u_{i+1}) + f(u_{i+1} v_{i+1}) + f(v_{i+1} w_{i+1})$$

$$= 6 + i - 1 + 6n - i + (3n/2) + 4 + i + 3n + 3 - 2i + 4n + 1 + i$$

$$= 13n + (3n/2) + 13 \text{ ----- (B)}$$

For  $(n/2) \leq i \leq n-1$ ,

$$\begin{aligned}
 f^*[P_2] &= f(u_i) + f(w_{i+1}) + f(u_i u_{i+1}) + f(u_{i+1} v_{i+1}) + f(v_{i+1} w_{i+1}) \\
 &= 6+i-1+6n-i+n+4+i-(n/2)+1+3n+4-2(i+1-(n/2))+4n+1+i \\
 &= 13n+(3n/2)+13 \text{ ----- (C)}
 \end{aligned}$$

From (A), (B) and (C), we conclude that G admits  $\psi$  - magic graphoidal total labeling. Hence,  $[P_n ; S_1]$ , (n - even) is magic graphoidal.

For example, consider the graph  $[P_6 ; S_1]$  shown in figure 1.

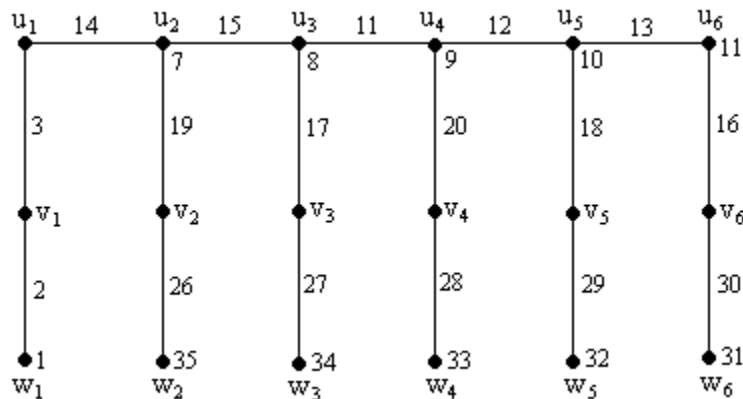


Figure 1  $[P_6 ; S_1]$

Clearly,  $\psi = \{(w_1, v_1, u_1, u_2, v_2, w_2), (u_2, u_3, v_3, w_3), (u_3, u_4, v_4, w_4), (u_4, u_5, v_5, w_5), (u_5, u_6, v_6, w_6)\}$  is a minimum graphoidal cover and  $[P_6 ; S_1]$  is magic graphoidal. Here the constant  $K = 100$

**3.2. Theorem :**  $[P_n ; S_1]$ , (n - odd) is magic graphoidal.

**Proof:** Let  $G = [P_n ; S_1]$

Let  $V(G) = \{u_i, v_i, w_i : 1 \leq i \leq n\}$  and

$$E(G) = \{ [(u_i v_i) \cup (v_i w_i) : 1 \leq i \leq n] \cup [(u_i u_{i+1}) : 1 \leq i \leq n-1] \}$$

Define  $f : V \cup E \rightarrow \{1, 2, \dots, p+q\}$  by

$$\begin{aligned}
 f(w_1) &= 1 \\
 f(w_1 v_1) &= 2 \\
 f(v_1 u_1) &= 3 \\
 f(u_{i+1}) &= 5 + 2i && 1 \leq i \leq (n-1)/2 \\
 f(w_{i+1}) &= 6n - i && 1 \leq i \leq n-1 \\
 f(v_{i+1} w_{i+1}) &= 4n + 1 + i && 1 \leq i \leq n-1 \\
 f(u_1 u_2) &= n + 5 \\
 f(u_{i+1} u_{i+2}) &= \frac{3n+11}{2} - i && 1 \leq i \leq \frac{n-1}{2} \\
 f(u_{n+1-i} u_{n-i}) &= \frac{3n+9}{2} + i && 1 \leq i \leq \frac{n-3}{2} \\
 f(u_{i+2} v_{i+2}) &= 3n + 2 - i && 1 \leq i \leq n-2 \\
 f(u_2 v_2) &= \frac{7n+3}{2}
 \end{aligned}$$

Let  $\psi = \{P_1 = (w_1, v_1, u_1, u_2, v_2, w_2), P_2 = (u_i, u_{i+1}, v_{i+1}, w_{i+1}) : 2 \leq i \leq n - 1\}$

$$\begin{aligned}
 f^*[P_1] &= f(w_1) + f(w_2) + f(w_1v_1) + f(v_1u_1) + f(u_1u_2) + f(u_2v_2) + f(v_2w_2) \\
 &= 1 + 6n - 1 + 2 + 3 + n + 5 + \frac{7n + 3}{2} + 4n + 1 + 1 \\
 &= 14n + \frac{n + 1}{2} + 13 \text{ ----- (A)}
 \end{aligned}$$

For  $2 \leq i < \frac{n+1}{2}$

$$\begin{aligned}
 f^*[P_2] &= f(u_i) + f(w_{i+1}) + f(u_iu_{i+1}) + f(u_{i+1}v_{i+1}) + f(v_{i+1}w_{i+1}) \\
 &= 5 + 2(i - 1) + 6n - i + \frac{3n + 11}{2} - (i - 1) + 3n + 2 - (i - 1) + 4n + 1 + i \\
 &= 14n + \frac{n + 1}{2} + 13 \text{ ----- (B)}
 \end{aligned}$$

For  $\frac{n+1}{2} \leq i \leq n - 1$ ,

$$\begin{aligned}
 f^*[P_2] &= f(u_i) + f(w_{i+1}) + f(u_iu_{i+1}) + f(u_{i+1}v_{i+1}) + f(v_{i+1}w_{i+1}) \\
 &= 6 + 2\left(i - \frac{n + 1}{2}\right) + 6n - i + \frac{3n + 9}{2} + (n - i) + 3n + 2 - (i - 1) + 4n + 1 + i \\
 &= 14n + \frac{n + 1}{2} + 13 \text{ ----- (C)}
 \end{aligned}$$

From (A), (B) and (C), we conclude that  $\psi$  is minimum magic graphoidal cover

Hence,  $[P_n ; S_1]$ , ( $n$  - even) is magic graphoidal.

For example, consider the graph  $[P_7 ; S_1]$  shown in figure 2.

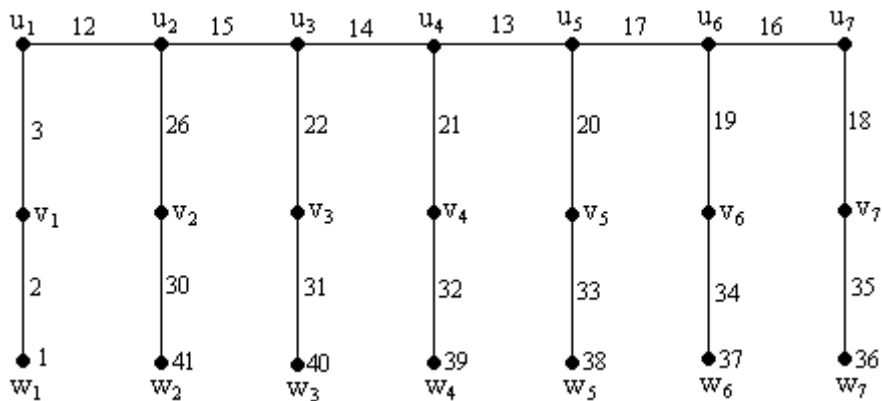


Figure 2  $[P_7 ; S_1]$

Clearly,  $\psi = \{(w_1, v_1, u_1, u_2, v_2, w_2), (u_2, u_3, v_3, w_3), (u_3, u_4, v_4, w_4), (u_4, u_5, v_5, w_5), (u_5, u_6, v_6, w_6), (u_6, u_7, v_7, w_7)\}$  is a minimum graphoidal cover and  $[P_7 ; S_1]$  is magic graphoidal. Here the constant  $K = 115$ .

**3.3. Theorem :**  $[P_n ; S_2]$  is magic graphoidal.

**Proof:** Let  $G = [P_n ; S_2]$

$$V(G) = \{(u_i, v_i) : 1 \leq i \leq n\}$$

$$E(G) = \{ [(u_i u_{i+1}) : 1 \leq i \leq n-1] \cup [(u_i v_i) : 1 \leq i \leq n] \\ \cup [(v_i v_{i1}) \cup (v_i v_{i2}) : 1 \leq i \leq n] \}$$

Define  $f: V \cup E \rightarrow \{1, 2, \dots, p+q\}$  by

$$\begin{aligned} f(v_1) &= 1 \\ f(u_1 v_1) &= 2 \\ f(u_1) &= 3 \\ f(u_{i+1}) &= 3 + i & 1 \leq i \leq n-2 \\ f(u_n) &= p + q \\ f(v_{i1}) &= n + 1 + i & 1 \leq i \leq n \\ f(v_{i+1}) &= 8n - 1 - i & 1 \leq i \leq n-1 \\ f(v_{i2}) &= 7n - i & 1 \leq i \leq n \\ f(v_i v_{i2}) &= 5n - 1 + i & 1 \leq i \leq n \\ f(u_{i+1} v_{i+1}) &= 4n + i & 1 \leq i \leq n-1 \\ f(u_i u_{i+1}) &= 4n + 1 - i & 1 \leq i \leq n-1 \\ f(v_i v_{i1}) &= 3n + 2 - i & 1 \leq i \leq n \end{aligned}$$

Let  $\psi = \{P_1 = [(v_{i1}, v_i, v_{i2}) : 1 \leq i \leq n], P_2 = (v_1, u_1, u_2, v_2), P_3 = [(u_i, u_{i+1}, v_{i+1}) : 2 \leq i \leq n-1]\}$

For  $1 \leq i \leq n$ ,

$$\begin{aligned} f^*[P_1] &= f(v_{i1}) + f(v_{i2}) + f(v_{i1} v_i) + f(v_i v_{i2}) \\ &= n + 1 + i + 7n - i + 3n + 2 - i + 5n - 1 + i \\ &= 16n + 2 \text{ ----- (A)} \end{aligned}$$

$$\begin{aligned} f^*[P_2] &= f(v_1) + f(v_2) + f(v_1 u_1) + f(u_1 u_2) + f(u_2 v_2) \\ &= 1 + 8n - 2 + 2 + 4n + 4n + 1 \\ &= 16n + 2 \text{ ----- (B)} \end{aligned}$$

For  $2 \leq i \leq n-1$ ,

$$\begin{aligned} f^*[P_3] &= f(u_i) + f(v_{i+1}) + f(u_i u_{i+1}) + f(u_{i+1} v_{i+1}) \\ &= 3 + i - 1 + 8n - 1 - i + 4n + 1 - i + 4n + i \\ &= 16n + 2 \text{ ----- (C)} \end{aligned}$$

From (A), (B) and (C), we conclude that  $G$  admits  $\psi$  - magic graphoidal total labeling. Hence,  $[P_n ; S_2]$  is magic graphoidal.

For example, consider the graphs  $[P_3 ; S_2]$  and  $[P_4 ; S_2]$  shown in figure 3.1 and 3.2.

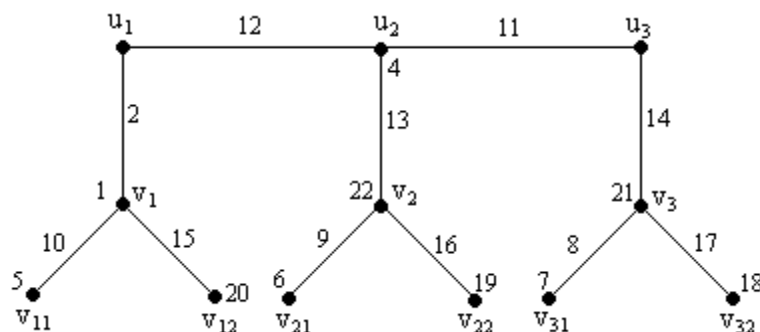


Figure 3.1  $[P_3 ; S_2]$

Clearly,  $\psi = \{(v_{11}, v_1, v_{12}), (v_{21}, v_2, v_{22}), (v_{31}, v_3, v_{32}), (v_1, u_1, u_2, v_2), (u_2, u_3, v_3)\}$  is a minimum graphoidal cover and  $[P_3 ; S_2]$  is magic graphoidal. Here the constant  $K = 50$

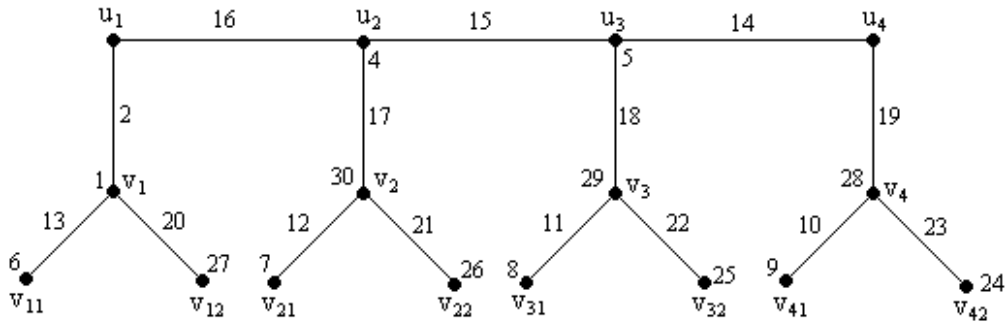


Figure 3.2  $[P_4 ; S_2]$

Clearly,  $\psi = \{(v_{11},v_1,v_{12}), (v_{21},v_2,v_{22}), (v_{31},v_3,v_{32}), (v_{41},v_4,v_{42}), (v_1,u_1,u_2,v_2), (u_2,u_3,v_3), (u_3,u_4,v_4)\}$  is a minimum graphoidal cover and  $[P_4 ; S_2]$  is magic graphoidal. Here the constant  $K = 66$

**3.4. Theorem :** For a Tree,  $T_{(n)}$  is magic graphoidal.

**Proof :**

Let  $T_{(n)}$  be a graph such that

$$V[T_{(n)}] = \{u_{i1}, u_{i2}, u_{i3}, u_{i4}, u_{i5} : 1 \leq i \leq n\} \text{ and}$$

$$E[T_{(n)}] = \{[(u_{i1}u_{i2}), (u_{i2}u_{i3}), (u_{i3}u_{i4}), (u_{i4}u_{i5}) : 1 \leq i \leq n] \cup [(u_{in}u_{i+1n}) : 1 \leq i \leq n-1]\}$$

Define  $f : V \cup E \rightarrow \{1, 2, \dots, p+q\}$  by

$f(u_{i1})$	$= i$	$1 \leq i \leq n$
$f(u_{i3})$	$= n + 1$	
$f(u_{i3}u_{i5})$	$= n + 2$	
$f(u_{i1}u_{i2})$	$= 2n + 3 - i$	$1 \leq i \leq n$
$f(u_{i+2,5}u_{i+3,5})$	$= 5n - 3 - 2i$	$1 \leq i \leq n - 3$
$f(u_{n+1-i,5}u_{n-i,5})$	$= 4n - 1 + i$	$n - 2 \leq i \leq n - 1$
$f(u_{n+1-i,2}u_{n+1-i,3})$	$= 5n - 2 + i$	$1 \leq i \leq n$
$f(u_{n+1-i,3}u_{n+1-i,4})$	$= 6n - 2 + i$	$1 \leq i \leq n$
$f(u_{n+1-i,3}u_{n+1-i,5})$	$= 7n - 2 + i$	$1 \leq i \leq n - 1$
$f(u_{i4})$	$= 8n - 2 + 2(i-1)$	$1 \leq i \leq n$
$f(u_{i+1,3})$	$= 8n - 1 + 2(i-1)$	$1 \leq i \leq n - 1$

$$\text{Let } \psi = \{P_1 = (u_{i1}, u_{i2}, u_{i3}, u_{i4}), P_2 = (u_{i3}, u_{i5}, u_{25}, u_{23}), P_3 = [(u_{i5}, u_{i+15}, u_{i+13}) : 2 \leq i \leq n - 1]\}$$

$$\begin{aligned} f^*[P_1] &= f(u_{i1}) + f(u_{i4}) + f(u_{i1}u_{i2}) + f(u_{i2}u_{i3}) + f(u_{i3}u_{i4}) \\ &= i + 8n - 2 + 2(i-1) + 2n + 3 - i + 5n - 2 + (n + 1 - i) + 6n - 2 + (n + 1 - i) \\ &= 23n - 3 \text{ ----- (A)} \end{aligned}$$

$$\begin{aligned} f^*[P_2] &= f(u_{i5}) + f(u_{i+1,3}) + f(u_{i5} u_{i+1,3}) + f(u_{i+1,5} u_{i+1,3}) \\ &= 2n + 2 + i - 1 + 8n - 1 + 2(i - 1) + 5n - 3 - 2(i - 2) + 7n - 2 + n - i \\ &= 23n - 3 \text{ ----- (B)} \end{aligned}$$

$$\begin{aligned} f^*[P_3] &= f(u_{i3}) + f(u_{23}) + f(u_{i3}u_{i5}) + f(u_{i5}u_{25}) + f(u_{25}u_{23}) \\ &= n + 1 + 8n - 1 + 2(i - 1) + 5n - 3 - 2(i - 2) + 7n - 2 + n - i \\ &= 23n - 3 \text{ ----- (C)} \end{aligned}$$

From (A), (B) and (C), we conclude that  $G$  admits  $\psi$  - magic graphoidal total labeling. Hence,  $T_{(n)}$  is magic graphoidal.

For example, consider the graph  $T_{(3)}$  shown in figure 4.

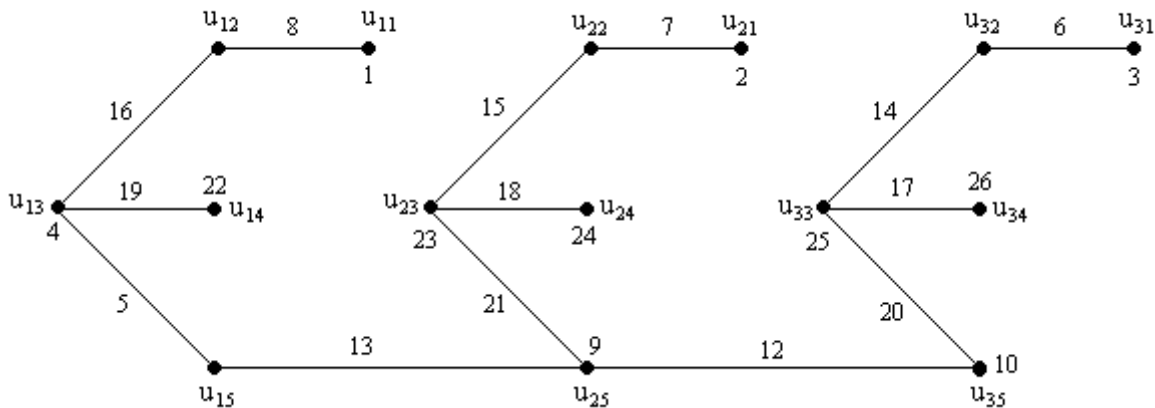


Figure 4  $T_{(3)}$

Clearly,  $\psi = \{(u_{11}, u_{12}, u_{13}, u_{14}), (u_{21}, u_{22}, u_{23}, u_{24}), (u_{31}, u_{32}, u_{33}, u_{34}), (u_{13}, u_{15}, u_{25}, u_{23}), (u_{25}, u_{35}, u_{33})\}$  is a minimum graphoidal cover and  $T_{(3)}$  is magic graphoidal. Here the constant  $K = 66$

**3.5. Theorem :** The graph Double Crowned Star  $K_{1,n} \odot K_1$  is magic graphoidal.

**Proof :** Let  $G = K_{1,n} \odot 2K_1$

$$V(G) = \{u, [u_i : 1 \leq i \leq n], [(u_i, u_{i2}) : 1 \leq i \leq n]\} \text{ and}$$

$$E(G) = \{[(uu_i) : 1 \leq i \leq n] \cup [(u_i u_{i1}) \cup (u_i u_{i2}) : 1 \leq i \leq n]\}$$

Define  $f : V \cup E \rightarrow \{1, 2, \dots, p+q\}$  by

$$\begin{aligned} f(u) &= 2n + 3 \\ f(u_{i1}) &= i & 1 \leq i \leq n \\ f(u_i) &= n + 1 \\ f(u_2) &= n + 2 \\ f(u_{n+1-i,2}) &= n + 2 + i & 1 \leq i \leq n \\ f(u_i u_{i,1}) &= 2n + 3 + i & 1 \leq i \leq n \\ f(uu_1) &= 3n + 4 \\ f(u_{2+i}) &= 3n + 4 + i & 1 \leq i \leq n - 2 \\ f(uu_{n+1-i}) &= 4n + 2 + i & 1 \leq i \leq n - 1 \\ f(u_{n+1-i} u_{n+1-i,2}) &= 5n + 1 + i & 1 \leq i \leq n \end{aligned}$$

$$\text{Let } \psi = \{P_1 = (u_1, u, u_2), P_2 = [(u, u_i) : 3 \leq i \leq n], P_3 = [(u_{i1}, u_i, u_{i2}) : 1 \leq i \leq n]\}$$

$$\begin{aligned} f^*[P_1] &= f(u_1) + f(u) + f(u_1 u) + f(uu_2) \\ &= n + 1 + n + 2 + 3n + 4 + 5n + 1 \\ &= 10n + 8 \text{ ----- (A)} \end{aligned}$$

For  $3 \leq i \leq n$ ,

$$\begin{aligned} f^*[P_2] &= f(u) + f(u_i) + f(uu_i) \\ &= 2n + 3 + 3n + 4 + i - 2 + 4n + 2 + n + 1 - i \\ &= 10n + 8 \text{ ----- (B)} \end{aligned}$$

For  $1 \leq i \leq n$ ,

$$\begin{aligned}
 f^*[P_3] &= f(u_{i1}) + f(u_{i2}) + f(u_{i1}u_i) + f(u_iu_{i2}) \\
 &= i + n + 2 + n + 1 - i + 2n + 3 + i + 5n + 1 + n + 1 - i \\
 &= 10n + 8 \text{ ----- (C)}
 \end{aligned}$$

From (A), (B) and (C), we conclude that  $G$  admits  $\psi$  - magic graphoidal total labeling. Hence, Double Crowned Star  $K_{1,n} \odot 2K_1$  is magic graphoidal. For example, consider the graph  $K_{1,5} \odot 2K_1$  shown in figure 5.

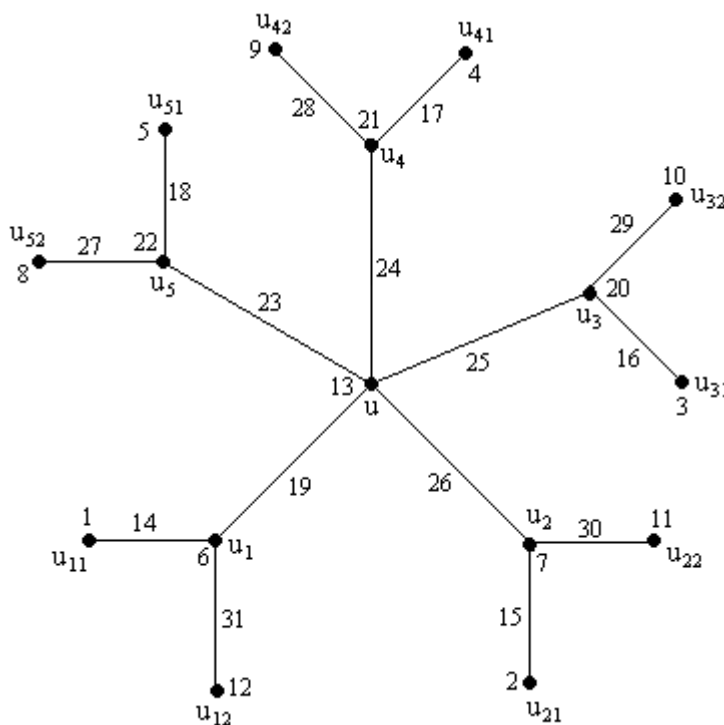


Figure 5.  $K_{1,5} \odot 2K_1$

Clearly,  $\psi = \{(u_{11}, u_1, u_{12}), (u_{21}, u_2, u_{22}), (u_{31}, u_3, u_{32}), (u_{41}, u_4, u_{42}), (u_{51}, u_5, u_{52}), (u_1, u, u_2), (u, u_3), (u, u_4), (u, u_5)\}$  is a minimum graphoidal cover and  $K_{1,5} \odot 2K_1$  is magic graphoidal. Here the constant  $K = 58$ .

**3.6. Theorem :**  $P_m \odot 2K_1$  is magical graphoidal.

**Proof :** Let  $G = P_m \odot 2K_1$

$$\begin{aligned}
 V(G) &= \{ (u_i : 1 \leq i \leq m), (u_{ij} : 1 \leq i \leq m, 1 \leq j \leq 2) \} \text{ and} \\
 E(G) &= \{ [(u_i u_{i+1}) : 1 \leq i \leq m-1] \cup [(u_i u_{ij}) : 1 \leq i \leq m, 1 \leq j \leq 2] \}
 \end{aligned}$$

Let  $\psi = \{P_1 = [(u_{i1}, u_i, u_{i2}) : 1 \leq i \leq m], P_2 = [(u_i, u_{i+1}) : 1 \leq i \leq m-1]\}$

Define  $f : V \rightarrow \{0, 1, 2, \dots, 6m-1\}$  by

$$\begin{aligned}
 f(u_i u_{i1}) &= i & 1 \leq i \leq m \\
 f(u_i u_{i2}) &= 2m+1-i & 1 \leq i \leq m
 \end{aligned}$$

$$f(u_{2i}) = \begin{cases} 2m+i & 1 \leq i \leq \frac{m}{2} & \text{if } m \text{ is even} \\ 2m+i & 1 \leq i \leq \frac{m-1}{2} & \text{if } m \text{ is odd} \end{cases}$$



$$f(u_{i_1}) = \begin{cases} \frac{5m}{2} + i & 1 \leq i \leq m \text{ if } m \text{ is even} \\ \frac{5m-1}{2} + i & 1 \leq i \leq m \text{ if } m \text{ is odd} \end{cases}$$

$$f(u_{2i-1}) = \begin{cases} \frac{7m}{2} + i & 1 \leq i \leq \frac{m}{2} \text{ if } m \text{ is even} \\ \frac{7m-1}{2} + i & 1 \leq i \leq \frac{m+1}{2} \text{ if } m \text{ is odd} \end{cases}$$

$$f(u_{m+1-i} u_{m-i}) = 4m + i \quad 1 \leq i \leq m - 1$$

$$f(u_{m+1-i}, 2) = 5m - 1 + i \quad 1 \leq i \leq m$$

**Case (i) :** when m is odd

For  $1 \leq i \leq m$ ,

$$f^*[P_1] = f(u_{i_1}) + f(u_{i_2}) + f(u_{i_1}u_i) + f(u_iu_{i_2})$$

$$= \frac{5m-1}{2} + i + 5m - 1 + m + 1 - i + i + 2m + 1 - i$$

$$= \frac{21m+1}{2} \text{ ----- (A)}$$

For  $1 \leq i \leq m - 1, i \equiv 1 \pmod{2}$

$$f^*[P_2] = f(u_i) + f(u_{i+1}) + f(u_iu_{i+1})$$

$$= \frac{7m-1}{2} + \frac{i+1}{2} + 2m + \frac{i+1}{2} + 4m + m - i$$

$$= \frac{21m+1}{2} \text{ ----- (B)}$$

For  $1 \leq i \leq m - 1, i \equiv 0 \pmod{2}$

$$f^*[P_2] = f(u_i) + f(u_{i+1}) + f(u_iu_{i+1})$$

$$= 2m + \frac{i}{2} + \frac{7m-1}{2} + \frac{i+2}{2} + 4m + m - i$$

$$= \frac{21m+1}{2} \text{ ----- (C)}$$

From (A), (B) and (C), we conclude that G admits  $\psi$  - magic graphoidal total labeling. Hence,  $P_m \odot 2K_1$  (m-odd) is magic graphoidal.

For example, consider the graph  $P_5 \odot 2K_1$  shown in figure 6.1.

**Case (ii) :** when m is even

For  $1 \leq i \leq m$ ,

$$f^*[P_1] = f(u_{i_1}) + f(u_{i_2}) + f(u_{i_1}u_i) + f(u_iu_{i_2})$$

$$\frac{5m}{2} = +i + 5m - 1 + m + 1 - i + i + 2m + 1 - i$$

$$= \frac{21m + 2}{2} \text{ ----- (A)}$$

For  $1 \leq i \leq m - 1, i \equiv 1 \pmod 2$

$$f^*[P_2] = f(u_i) + f(u_{i+1}) + f(u_i u_{i+1})$$

$$= \frac{7m}{2} + \frac{i+1}{2} + 2m + \frac{i+1}{2} + 4m + m - i$$

$$= \frac{21m + 2}{2} \text{ ----- (B)}$$

For  $1 \leq i \leq m - 1, i \equiv 0 \pmod 2$

$$f^*[P_2] = f(u_i) + f(u_{i+1}) + f(u_i u_{i+1})$$

$$= 2m + \frac{i}{2} + \frac{7m}{2} + \frac{i+2}{2} + 4m + m - i$$

$$= \frac{21m + 2}{2} \text{ ----- (C)}$$

From (A), (B) and (C), we conclude that  $G$  admits  $\psi$  - magic graphoidal total labeling. Hence,  $P_m \odot 2K_1$  ( $m$ -even) is magic graphoidal.

For example, consider the graph  $P_4 \odot 2K_1$  shown in figure 6.2.

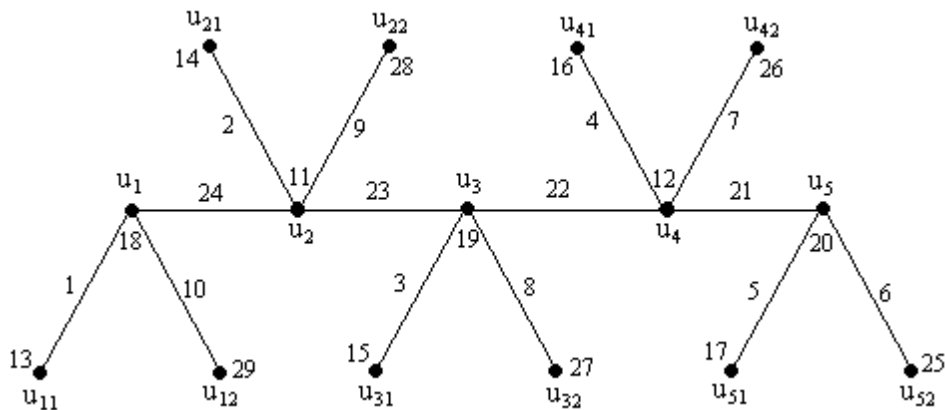


Figure 6.1  $P_5 \odot 2K_1$

Clearly,  $\psi = \{(u_{11}, u_1, u_{12}), (u_{21}, u_2, u_{22}), (u_{31}, u_3, u_{32}), (u_{41}, u_4, u_{42}), (u_{51}, u_5, u_{52}), (u_1, u_2), (u_2, u_3), (u_3, u_4), (u_4, u_5)\}$  is a minimum graphoidal cover and  $P_4 \odot 2K_1$  is magic graphoidal. Here the constant  $K = 53$ .

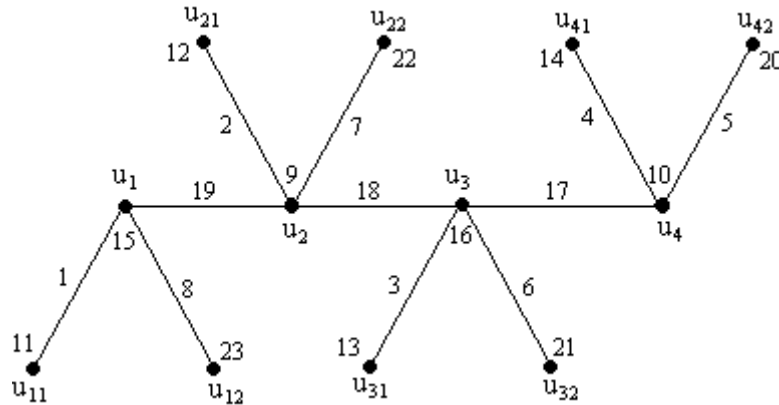


Figure 6.2  $P_4 \circledast 2K_1$

Clearly,  $\psi = \{(u_{11}, u_1, u_{12}), (u_{21}, u_2, u_{22}), (u_{31}, u_3, u_{32}), (u_{41}, u_4, u_{42}), (u_1, u_2), (u_2, u_3), (u_3, u_4)\}$  is a minimum graphoidal cover and  $P_4 \circledast 2K_1$  is magic graphoidal. Here the constant  $K = 43$ .

**3.7. Theorem :**  $P_m \circledast K_{1,3}$  is magic graphoidal.

**Proof :** Let  $G = P_m \circledast K_{1,3}$

$$V(G) = \{[(u_i v_i) : 1 \leq i \leq m], [(v_{ij}) : 1 \leq i \leq m, 1 \leq j \leq 3]\}$$

$$E(G) = \{[(u_i u_{i+1}) : 1 \leq i \leq m-1] \cup [(v_i v_{ij}) : 1 \leq i \leq m, 1 \leq j \leq 3]\}$$

$$\text{with } u_i = u_{i3}, 1 \leq i \leq m$$

Let  $\psi = \{P_1 = [(v_{i1}, v_i, v_{i2}) : 1 \leq i \leq m], P_2 = [(v_i, u_i, u_{i+1}) : 1 \leq i \leq m-2],$

$$P_3 = (v_{m-1}, u_{m-1}, u_m, v_m)\}$$

Define  $f : V \cup E \rightarrow \{1, 2, \dots, 8m-1\}$  by

$f(v_m)$	$= 1$	
$f(v_i v_{i1})$	$= i+1$	$1 \leq i \leq m$
$f(u_i v_i)$	$= m+1+i$	$1 \leq i \leq m-1$
$f(u_{m+1-i} u_{m-i})$	$= 2m+i$	$1 \leq i \leq m-1$
$f(v_{m+1-i} v_{m+1-i,2})$	$= 3m-1+i$	$1 \leq i \leq m$
$f(v_{i1})$	$= 4m-1+i$	$1 \leq i \leq m$
$f(v_i)$	$= 5m-1+i$	$1 \leq i \leq m-1$
$f(u_m v_m)$	$= 6m-1$	
$f(u_{m-i})$	$= 6m+i$	$1 \leq i \leq m-2$
$f(v_{m+1-i, 2})$	$= 7m-2+i$	$1 \leq i \leq m$

For  $1 \leq i \leq m,$

$$\begin{aligned} f^*[P_1] &= f(v_{i1}) + f(v_{i2}) + f(v_{i1} v_i) + f(v_i v_{i2}) \\ &= 4m-1+i + 7m-2 + m+1-i + i+1 + 3m-1 + m+1-i \\ &= 16m-1 \text{ ----- (A)} \end{aligned}$$

For  $1 \leq i \leq m-2,$

$$\begin{aligned} f^*[P_2] &= f(v_i) + f(u_{i+1}) + f(v_i u_i) + f(u_i u_{i+1}) \\ &= 5m-1+i + 6m + (m-i-1) + m+1+i + 2m+m-i \\ &= 16m-1 \text{ ----- (B)} \end{aligned}$$

$$\begin{aligned}
 f^*[P_3] &= f(v_{m-1}) + f(v_m) + f(v_{m-1}u_{m-1}) + f(u_{m-1}u_m) + f(u_mv_m) \\
 &= 6m - 2 + 1 + 2m + 2m + 1 + 6m - 1 \\
 &= 16m - 1 \text{ ----- (C)}
 \end{aligned}$$

From (A), (B) and (C), we conclude that  $G$  admits  $\psi$  - magic graphoidal total labeling. Hence,  $P_m \odot K_{1,3}$  is magic graphoidal.

For example, consider the graph  $P_4 \odot K_{1,3}$  shown in figure 7.

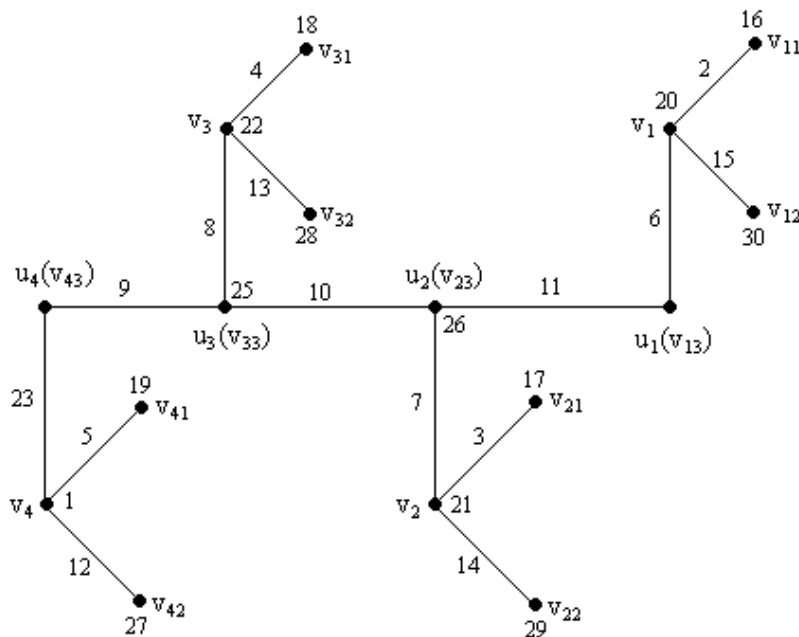


Figure 7.  $P_4 \odot K_{1,3}$

Clearly,  $\psi = \{(v_{11}, v_1, v_{12}), (v_{21}, v_2, v_{22}), (v_{31}, v_3, v_{32}), (v_{41}, v_4, v_{42}), (v_1, u_1, u_2), (v_1, u_2, u_3), (v_3, u_3, u_4, v_4)\}$  is magic graphoidal. Here the constant  $K = 63$ .

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