

Magic Graphoidal on Class of Trees

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Abstract: B.D.Acharya and E. Sampathkumar [1] defined Graphoidal cover as partition of edge set of a graph G into internally disjoint paths (not necessarily open). The minimum cardinality of such cover is known as graphoidal covering number of G .

Let $G = \{V, E\}$ be a graph and let ψ be a graphoidal cover of G . Define f :

$V \cup E \rightarrow \{1, 2, \dots, p+q\}$ such that for every path $P = (v_0, v_1, v_2, \dots, v_n)$ in ψ with $f^*(P) = f(v_0) + f(v_n) + \sum_{i=1}^n f(v_{i-1}v_i) = k$, a constant, where f^* is the induced labeling on ψ . Then, we say that G admits ψ - magic graphoidal total labeling of G .

A graph G is called magic graphoidal if there exists a minimum graphoidal cover ψ of G such that G admits ψ - magic graphoidal total labeling.

In this paper, we proved that $[P_n; S_1]$, $[P_n; S_2]$, $T(n)$, $P_m \odot K_{1,3}$, $P_m \odot 2K_1$ and $K_{1,n} \odot 2K_1$ are magic graphoidal

1. INTRODUCTION

By a graph, we mean a finite simple and undirected graph. The vertex set and edge set of a graph G are denoted by $V(G)$ and $E(G)$ respectively. Terms and notations not used here are as in [3].

- 1.1. Definition :** Let $S_1 = (v_0, v_1)$ be a star and let $[P_n ; S_1]$ be the graph obtained from n copies of S_1 and the path $P_n = (u_1, u_2, \dots, u_n)$ by joining u_j with the vertex v_0 of the j^{th} copy of S_1 by means of an edge, for $1 \leq j \leq n$.
- 1.2. Definition :** Let $S_2 = (v_0, v_1, v_2)$ be a star and let $[P_n ; S_2]$ be the graph obtained from n copies of S_2 and the path $P_n = (u_1, u_2, \dots, u_n)$ by joining u_j with the vertex v_0 of the j^{th} copy of S_2 by means of an edge, for $1 \leq j \leq n$.
- 1.3. Defintion :** Let T be any Tree. Denote the tree, obtained from T by considering two copies of T by adding an edge between them, by $T_{(2)}$ and in general the graph obtained from $T_{(n-1)}$ and T by adding an edge between them is denoted by $T_{(n)}$.
- 1.4. Result [11] :** For a Tree T , $\gamma(T) = n-1$ where n is the number of pendent vertices of G .

2. PRELIMINARIES

Let $G = \{V, E\}$ be a graph with p vertices and q edges. A graphoidal cover ψ of G is a collection of (open) paths such that

- (i) every edge is in exactly one path of ψ
- (ii) every vertex is an interval vertex of atmost one path in ψ .

We define $\gamma(G) = \min_{\psi \in \zeta} |\psi|$,

where ζ is the collection of graphoidal covers ψ of G and γ is graphoidal covering number of G .

Let ψ be a graphoidal cover of G . Then we say that G admits ψ - magic graphoidal total labeling of G if there exists a bijection $f: V \cup E \rightarrow \{1, 2, \dots, p+q\}$ such that for every path $P = (v_0 v_1 v_2 \dots v_n)$ in ψ , then, $f^*(P) = f(v_0) + f(v_n) + \sum_{i=1}^n f(v_{i-1}v_i) = k$, a constant, where f^* is the induced labeling of ψ . A graph G is called magic graphoidal if there exists a minimum graphoidal cover ψ of G such that G admits ψ - magic graphoidal total labeling.

3. Magical Graphoidal on Trees

3.1. Theorem : $[P_n ; S_1]$, $(n - \text{even})$ is magic graphoidal.

Proof: Let $G = [P_n ; S_1]$

Let $V(G) = \{u_i, v_i, w_i : 1 \leq i \leq n\}$ and

$$E(G) = \{[(u_i v_i) \cup (v_i w_i) : 1 \leq i \leq n] \cup [(u_i u_{i+1}) : 1 \leq i \leq n-1]\}$$

Define $f: V \cup E \rightarrow \{1, 2, \dots, p+q\}$ by

$$f(w_1) = 1$$

$$f(w_1 v_1) = 2$$

$$f(v_1 u_1) = 3$$

$$f(u_{i+1}) = 6 + i \quad 1 \leq i \leq n-2$$

$$f(w_{i+1}) = 6n - i \quad 1 \leq i \leq n-1$$

$$f(v_{i+1} w_{i+1}) = 4n + 1 + i \quad 1 \leq i \leq n-1$$

$$f(u_{i+1} v_{i+1}) = 3n + 3 - 2i \quad 1 \leq i \leq (n/2)-1$$

$$f\left(u_{\frac{n}{2}+i} v_{\frac{n}{2}+i}\right) = 3n + 4 - 2i \quad 1 \leq i \leq \frac{n}{2}$$

$$f(u_i u_{i+1}) = \frac{3n}{2} + 4 + i \quad 1 \leq i \leq \frac{n}{2} - 1$$

$$f\left(u_{\frac{n}{2}-1+i} u_{\frac{n}{2}+i}\right) = n + 4 + i \quad 1 \leq i \leq \frac{n}{2}$$

Let $\psi = \{P_1 = (w_1, v_1, u_1, u_2, v_2, w_2), P_2 = (u_i, u_{i+1}, v_{i+1}, w_{i+1}) : 2 \leq i \leq n-1\}$

$$f^*[P_1] = f(w_1) + f(w_2) + f(w_1 v_1) + f(v_1 u_1) + f(u_1 u_2) + f(u_2 v_2) + f(v_2 w_2)$$

$$= 1 + 6n - 1 + 2 + 3 + \frac{3n}{2} + 4 + 1 + 3n + 3 - 2 + 4n + 2$$

$$= 13n + \frac{3n}{2} + 13 \text{ ----- (A)}$$

For $2 \leq i \leq (n/2)-1$,

$$f^*[P_2] = f(u_i) + f(w_{i+1}) + f(u_i u_{i+1}) + f(u_{i+1} v_{i+1}) + f(v_{i+1} w_{i+1})$$

$$= 6 + i - 1 + 6n - i + (3n/2) + 4 + i + 3n + 3 - 2i + 4n + 1 + i$$

$$= 13n + (3n/2) + 13 \text{ ----- (B)}$$

For $(n/2) \leq i \leq n-1$,

$$\begin{aligned}
 f^*[P_2] &= f(u_i) + f(w_{i+1}) + f(u_i u_{i+1}) + f(u_{i+1} v_{i+1}) + f(v_{i+1} w_{i+1}) \\
 &= 6+i-1+6n-i+n+4+i-(n/2)+1+3n+4-2(i+1-(n/2))+4n+1+i \\
 &= 13n+(3n/2)+13 \text{ ----- (C)}
 \end{aligned}$$

From (A), (B) and (C), we conclude that G admits ψ - magic graphoidal total labeling. Hence, $[P_n ; S_1]$, (n - even) is magic graphoidal.

For example, consider the graph $[P_6 ; S_1]$ shown in figure 1.

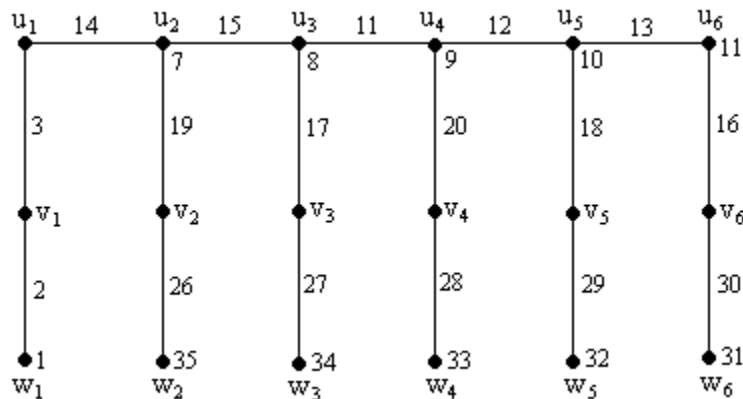


Figure 1 $[P_6 ; S_1]$

Clearly, $\psi = \{(w_1, v_1, u_1, u_2, v_2, w_2), (u_2, u_3, v_3, w_3), (u_3, u_4, v_4, w_4), (u_4, u_5, v_5, w_5), (u_5, u_6, v_6, w_6)\}$ is a minimum graphoidal cover and $[P_6 ; S_1]$ is magic graphoidal. Here the constant $K = 100$

3.2. Theorem : $[P_n ; S_1]$, (n - odd) is magic graphoidal.

Proof: Let $G = [P_n ; S_1]$

Let $V(G) = \{u_i, v_i, w_i : 1 \leq i \leq n\}$ and

$$E(G) = \{ [(u_i v_i) \cup (v_i w_i) : 1 \leq i \leq n] \cup [(u_i u_{i+1}) : 1 \leq i \leq n-1] \}$$

Define $f : V \cup E \rightarrow \{1, 2, \dots, p+q\}$ by

$$\begin{aligned}
 f(w_1) &= 1 \\
 f(w_1 v_1) &= 2 \\
 f(v_1 u_1) &= 3 \\
 f(u_{i+1}) &= 5 + 2i && 1 \leq i \leq (n-1)/2 \\
 f(w_{i+1}) &= 6n - i && 1 \leq i \leq n-1 \\
 f(v_{i+1} w_{i+1}) &= 4n + 1 + i && 1 \leq i \leq n-1 \\
 f(u_1 u_2) &= n + 5 \\
 f(u_{i+1} u_{i+2}) &= \frac{3n+11}{2} - i && 1 \leq i \leq \frac{n-1}{2} \\
 f(u_{n+1-i} u_{n-i}) &= \frac{3n+9}{2} + i && 1 \leq i \leq \frac{n-3}{2} \\
 f(u_{i+2} v_{i+2}) &= 3n + 2 - i && 1 \leq i \leq n-2 \\
 f(u_2 v_2) &= \frac{7n+3}{2}
 \end{aligned}$$

Let $\psi = \{P_1 = (w_1, v_1, u_1, u_2, v_2, w_2), P_2 = (u_i, u_{i+1}, v_{i+1}, w_{i+1}) : 2 \leq i \leq n - 1\}$

$$\begin{aligned}
 f^*[P_1] &= f(w_1) + f(w_2) + f(w_1v_1) + f(v_1u_1) + f(u_1u_2) + f(u_2v_2) + f(v_2w_2) \\
 &= 1 + 6n - 1 + 2 + 3 + n + 5 + \frac{7n + 3}{2} + 4n + 1 + 1 \\
 &= 14n + \frac{n + 1}{2} + 13 \text{ ----- (A)}
 \end{aligned}$$

For $2 \leq i < \frac{n+1}{2}$

$$\begin{aligned}
 f^*[P_2] &= f(u_i) + f(w_{i+1}) + f(u_iu_{i+1}) + f(u_{i+1}v_{i+1}) + f(v_{i+1}w_{i+1}) \\
 &= 5 + 2(i - 1) + 6n - i + \frac{3n + 11}{2} - (i - 1) + 3n + 2 - (i - 1) + 4n + 1 + i \\
 &= 14n + \frac{n + 1}{2} + 13 \text{ ----- (B)}
 \end{aligned}$$

For $\frac{n+1}{2} \leq i \leq n - 1$,

$$\begin{aligned}
 f^*[P_2] &= f(u_i) + f(w_{i+1}) + f(u_iu_{i+1}) + f(u_{i+1}v_{i+1}) + f(v_{i+1}w_{i+1}) \\
 &= 6 + 2\left(i - \frac{n + 1}{2}\right) + 6n - i + \frac{3n + 9}{2} + (n - i) + 3n + 2 - (i - 1) + 4n + 1 + i \\
 &= 14n + \frac{n + 1}{2} + 13 \text{ ----- (C)}
 \end{aligned}$$

From (A), (B) and (C), we conclude that ψ is minimum magic graphoidal cover

Hence, $[P_n ; S_1]$, (n - even) is magic graphoidal.

For example, consider the graph $[P_7 ; S_1]$ shown in figure 2.

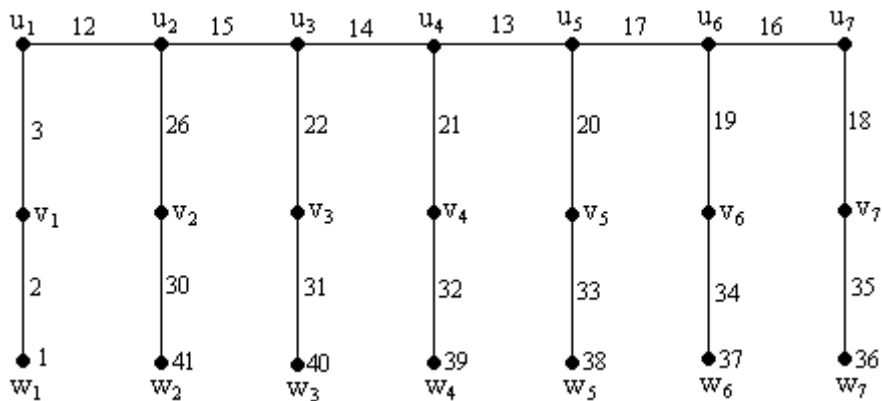


Figure 2 $[P_7 ; S_1]$

Clearly, $\psi = \{(w_1, v_1, u_1, u_2, v_2, w_2), (u_2, u_3, v_3, w_3), (u_3, u_4, v_4, w_4), (u_4, u_5, v_5, w_5), (u_5, u_6, v_6, w_6), (u_6, u_7, v_7, w_7)\}$ is a minimum graphoidal cover and $[P_7 ; S_1]$ is magic graphoidal. Here the constant $K = 115$.

3.3. Theorem : $[P_n ; S_2]$ is magic graphoidal.

Proof: Let $G = [P_n ; S_2]$

$$V(G) = \{(u_i, v_i) : 1 \leq i \leq n\}$$

$$E(G) = \{ [(u_i u_{i+1}) : 1 \leq i \leq n-1] \cup [(u_i v_i) : 1 \leq i \leq n] \\ \cup [(v_i v_{i1}) \cup (v_i v_{i2}) : 1 \leq i \leq n] \}$$

Define $f: V \cup E \rightarrow \{1, 2, \dots, p+q\}$ by

$$\begin{aligned} f(v_1) &= 1 \\ f(u_1 v_1) &= 2 \\ (u_1) &= 3 \\ f(u_{i+1}) &= 3 + i & 1 \leq i \leq n-2 \\ f(u_n) &= p + q \\ f(v_{i1}) &= n + 1 + i & 1 \leq i \leq n \\ f(v_{i+1}) &= 8n - 1 - i & 1 \leq i \leq n-1 \\ f(v_{i2}) &= 7n - i & 1 \leq i \leq n \\ f(v_i v_{i2}) &= 5n - 1 + i & 1 \leq i \leq n \\ f(u_{i+1} v_{i+1}) &= 4n + i & 1 \leq i \leq n-1 \\ f(u_i u_{i+1}) &= 4n + 1 - i & 1 \leq i \leq n-1 \\ f(v_i v_{i1}) &= 3n + 2 - i & 1 \leq i \leq n \end{aligned}$$

Let $\psi = \{P_1 = [(v_{i1}, v_i, v_{i2}) : 1 \leq i \leq n], P_2 = (v_1, u_1, u_2, v_2), P_3 = [(u_i, u_{i+1}, v_{i+1}) : 2 \leq i \leq n-1]\}$

For $1 \leq i \leq n$,

$$\begin{aligned} f^*[P_1] &= f(v_{i1}) + f(v_{i2}) + f(v_{i1} v_i) + f(v_i v_{i2}) \\ &= n + 1 + i + 7n - i + 3n + 2 - i + 5n - 1 + i \\ &= 16n + 2 \text{ ----- (A)} \end{aligned}$$

$$\begin{aligned} f^*[P_2] &= f(v_1) + f(v_2) + f(v_1 u_1) + f(u_1 u_2) + f(u_2 v_2) \\ &= 1 + 8n - 2 + 2 + 4n + 4n + 1 \\ &= 16n + 2 \text{ ----- (B)} \end{aligned}$$

For $2 \leq i \leq n-1$,

$$\begin{aligned} f^*[P_3] &= f(u_i) + f(v_{i+1}) + f(u_i u_{i+1}) + f(u_{i+1} v_{i+1}) \\ &= 3 + i - 1 + 8n - 1 - i + 4n + 1 - i + 4n + i \\ &= 16n + 2 \text{ ----- (C)} \end{aligned}$$

From (A), (B) and (C), we conclude that G admits ψ - magic graphoidal total labeling. Hence, $[P_n ; S_2]$ is magic graphoidal.

For example, consider the graphs $[P_3 ; S_2]$ and $[P_4 ; S_2]$ shown in figure 3.1 and 3.2.

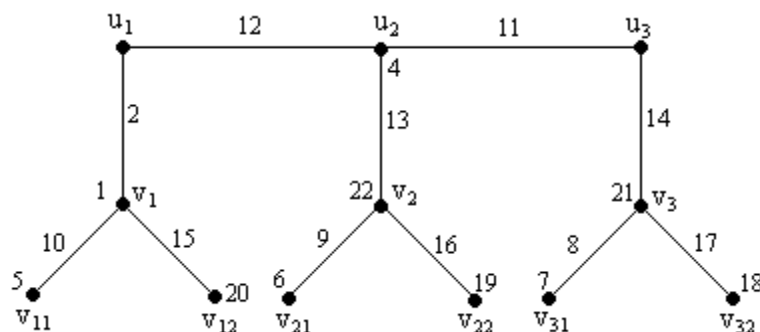


Figure 3.1 $[P_3 ; S_2]$

Clearly, $\psi = \{(v_{11}, v_1, v_{12}), (v_{21}, v_2, v_{22}), (v_{31}, v_3, v_{32}), (v_1, u_1, u_2, v_2), (u_2, u_3, v_3)\}$ is a minimum graphoidal cover and $[P_3 ; S_2]$ is magic graphoidal. Here the constant $K = 50$

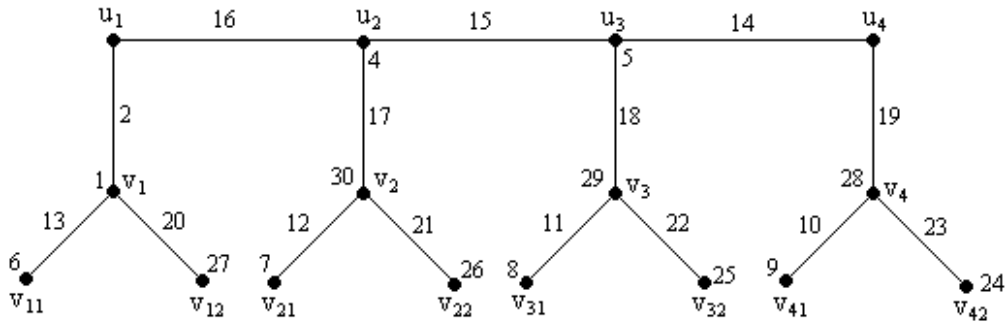


Figure 3.2 $[P_4 ; S_2]$

Clearly, $\psi = \{(v_{11},v_1,v_{12}), (v_{21},v_2,v_{22}), (v_{31},v_3,v_{32}), (v_{41},v_4,v_{42}), (v_1,u_1,u_2,v_2), (u_2,u_3,v_3), (u_3,u_4,v_4)\}$ is a minimum graphoidal cover and $[P_4 ; S_2]$ is magic graphoidal. Here the constant $K = 66$

3.4. Theorem : For a Tree, $T_{(n)}$ is magic graphoidal.

Proof :

Let $T_{(n)}$ be a graph such that

$$V[T_{(n)}] = \{u_{i1}, u_{i2}, u_{i3}, u_{i4}, u_{i5} : 1 \leq i \leq n\} \text{ and}$$

$$E[T_{(n)}] = \{[(u_{i1}u_{i2}), (u_{i2}u_{i3}), (u_{i3}u_{i4}), (u_{i4}u_{i5}) : 1 \leq i \leq n] \cup [(u_{in}u_{i+1n}) : 1 \leq i \leq n-1]\}$$

Define $f : V \cup E \rightarrow \{1, 2, \dots, p+q\}$ by

$f(u_{i1})$	$= i$	$1 \leq i \leq n$
$f(u_{i3})$	$= n + 1$	
$f(u_{i3}u_{i5})$	$= n + 2$	
$f(u_{i1}u_{i2})$	$= 2n + 3 - i$	$1 \leq i \leq n$
$f(u_{i+2,5}u_{i+3,5})$	$= 5n - 3 - 2i$	$1 \leq i \leq n - 3$
$f(u_{n+1-i,5}u_{n-i,5})$	$= 4n - 1 + i$	$n - 2 \leq i \leq n - 1$
$f(u_{n+1-i,2}u_{n+1-i,3})$	$= 5n - 2 + i$	$1 \leq i \leq n$
$f(u_{n+1-i,3}u_{n+1-i,4})$	$= 6n - 2 + i$	$1 \leq i \leq n$
$f(u_{n+1-i,3}u_{n+1-i,5})$	$= 7n - 2 + i$	$1 \leq i \leq n - 1$
$f(u_{i4})$	$= 8n - 2 + 2(i-1)$	$1 \leq i \leq n$
$f(u_{i+1,3})$	$= 8n - 1 + 2(i-1)$	$1 \leq i \leq n - 1$

$$\text{Let } \psi = \{P_1 = (u_{i1}, u_{i2}, u_{i3}, u_{i4}), P_2 = (u_{i3}, u_{i5}, u_{25}, u_{23}), P_3 = [(u_{i5}, u_{i+15}, u_{i+13}) : 2 \leq i \leq n - 1]\}$$

$$\begin{aligned} f^*[P_1] &= f(u_{i1}) + f(u_{i4}) + f(u_{i1}u_{i2}) + f(u_{i2}u_{i3}) + f(u_{i3}u_{i4}) \\ &= i + 8n - 2 + 2(i-1) + 2n + 3 - i + 5n - 2 + (n + 1 - i) + 6n - 2 + (n + 1 - i) \\ &= 23n - 3 \text{ ----- (A)} \end{aligned}$$

$$\begin{aligned} f^*[P_2] &= f(u_{i5}) + f(u_{i+1,3}) + f(u_{i5} u_{i+1,3}) + f(u_{i+1,5} u_{i+1,3}) \\ &= 2n + 2 + i - 1 + 8n - 1 + 2(i - 1) + 5n - 3 - 2(i - 2) + 7n - 2 + n - i \\ &= 23n - 3 \text{ ----- (B)} \end{aligned}$$

$$\begin{aligned} f^*[P_3] &= f(u_{i3}) + f(u_{23}) + f(u_{i3}u_{i5}) + f(u_{i5}u_{25}) + f(u_{25}u_{23}) \\ &= n + 1 + 8n - 1 + 2(i - 1) + 5n - 3 - 2(i - 2) + 7n - 2 + n - i \\ &= 23n - 3 \text{ ----- (C)} \end{aligned}$$

From (A), (B) and (C), we conclude that G admits ψ - magic graphoidal total labeling. Hence, $T_{(n)}$ is magic graphoidal.

For example, consider the graph $T_{(3)}$ shown in figure 4.

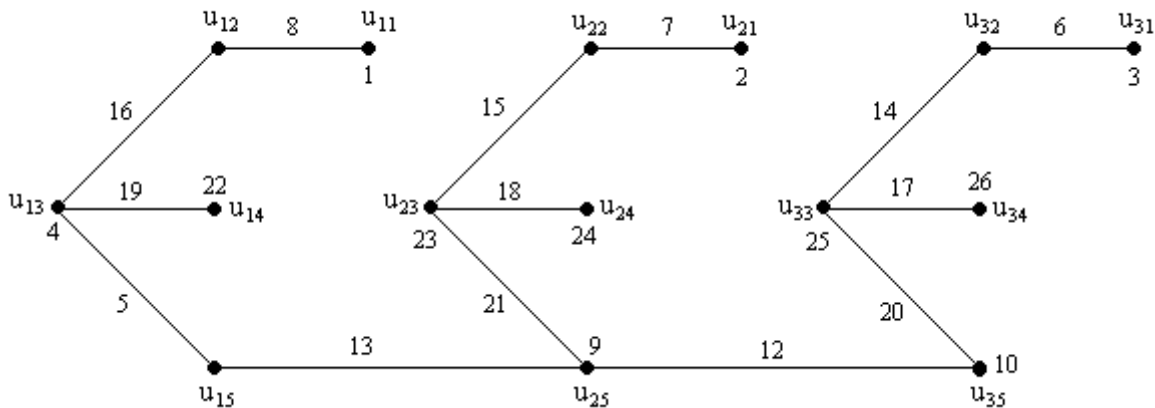


Figure 4 $T_{(3)}$

Clearly, $\psi = \{(u_{11}, u_{12}, u_{13}, u_{14}), (u_{21}, u_{22}, u_{23}, u_{24}), (u_{31}, u_{32}, u_{33}, u_{34}), (u_{13}, u_{15}, u_{25}, u_{23}), (u_{25}, u_{35}, u_{33})\}$ is a minimum graphoidal cover and $T_{(3)}$ is magic graphoidal. Here the constant $K = 66$

3.5. Theorem : The graph Double Crowned Star $K_{1,n} \odot K_1$ is magic graphoidal.

Proof : Let $G = K_{1,n} \odot 2K_1$

$$V(G) = \{u, [u_i : 1 \leq i \leq n], [(u_i, u_{i2}) : 1 \leq i \leq n]\} \text{ and}$$

$$E(G) = \{[(uu_i) : 1 \leq i \leq n] \cup [(u_i u_{i1}) \cup (u_i u_{i2}) : 1 \leq i \leq n]\}$$

Define $f : V \cup E \rightarrow \{1, 2, \dots, p+q\}$ by

$$\begin{aligned} f(u) &= 2n + 3 \\ f(u_{i1}) &= i & 1 \leq i \leq n \\ f(u_i) &= n + 1 \\ f(u_{i2}) &= n + 2 \\ f(u_{n+1-i,2}) &= n + 2 + i & 1 \leq i \leq n \\ f(u_i u_{i,1}) &= 2n + 3 + i & 1 \leq i \leq n \\ f(uu_1) &= 3n + 4 \\ f(u_{2+i}) &= 3n + 4 + i & 1 \leq i \leq n - 2 \\ f(uu_{n+1-i}) &= 4n + 2 + i & 1 \leq i \leq n - 1 \\ f(u_{n+1-i} u_{n+1-i, 2}) &= 5n + 1 + i & 1 \leq i \leq n \end{aligned}$$

$$\text{Let } \psi = \{P_1 = (u, u_1, u_2), P_2 = [(u, u_i) : 3 \leq i \leq n], P_3 = [(u_{i1}, u_i, u_{i2}) : 1 \leq i \leq n]\}$$

$$\begin{aligned} f^*[P_1] &= f(u) + f(u_1) + f(u_2) + f(u_1 u_2) \\ &= n + 1 + n + 2 + 3n + 4 + 5n + 1 \\ &= 10n + 8 \text{ ----- (A)} \end{aligned}$$

For $3 \leq i \leq n$,

$$\begin{aligned} f^*[P_2] &= f(u) + f(u_i) + f(uu_i) \\ &= 2n + 3 + 3n + 4 + i - 2 + 4n + 2 + n + 1 - i \\ &= 10n + 8 \text{ ----- (B)} \end{aligned}$$

For $1 \leq i \leq n$,

$$\begin{aligned}
 f^*[P_3] &= f(u_{i1}) + f(u_{i2}) + f(u_i u_i) + f(u_i u_{i2}) \\
 &= i + n + 2 + n + 1 - i + 2n + 3 + i + 5n + 1 + n + 1 - i \\
 &= 10n + 8 \text{ ----- (C)}
 \end{aligned}$$

From (A), (B) and (C), we conclude that G admits ψ - magic graphoidal total labeling. Hence, Double Crowned Star $K_{1,n} \odot 2K_1$ is magic graphoidal. For example, consider the graph $K_{1,5} \odot 2K_1$ shown in figure 5.

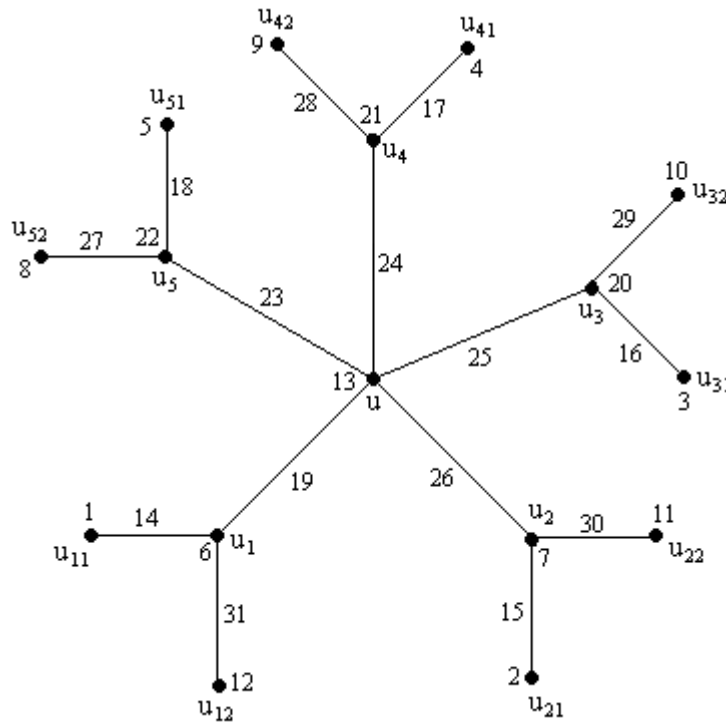


Figure 5. $K_{1,5} \odot 2K_1$

Clearly, $\psi = \{(u_{11}, u_1, u_{12}), (u_{21}, u_2, u_{22}), (u_{31}, u_3, u_{32}), (u_{41}, u_4, u_{42}), (u_{51}, u_5, u_{52}), (u_1, u, u_2), (u, u_3), (u, u_4), (u, u_5)\}$ is a minimum graphoidal cover and $K_{1,5} \odot 2K_1$ is magic graphoidal. Here the constant $K = 58$.

3.6. Theorem : $P_m \odot 2K_1$ is magical graphoidal.

Proof : Let $G = P_m \odot 2K_1$

$$\begin{aligned}
 V(G) &= \{ (u_i : 1 \leq i \leq m), (u_{ij} : 1 \leq i \leq m, 1 \leq j \leq 2) \} \text{ and} \\
 E(G) &= \{ [(u_i u_{i+1}) : 1 \leq i \leq m-1] \cup [(u_i u_{ij}) : 1 \leq i \leq m, 1 \leq j \leq 2] \}
 \end{aligned}$$

Let $\psi = \{P_1 = [(u_{i1}, u_i, u_{i2}) : 1 \leq i \leq m], P_2 = [(u_i, u_{i+1}) : 1 \leq i \leq m-1]\}$

Define $f : V \rightarrow \{0, 1, 2, \dots, 6m-1\}$ by

$$\begin{aligned}
 f(u_i u_{i1}) &= i & 1 \leq i \leq m \\
 f(u_i u_{i2}) &= 2m+1-i & 1 \leq i \leq m
 \end{aligned}$$

$$f(u_{2i}) = \begin{cases} 2m+i & 1 \leq i \leq \frac{m}{2} & \text{if } m \text{ is even} \\ 2m+i & 1 \leq i \leq \frac{m-1}{2} & \text{if } m \text{ is odd} \end{cases}$$

$$f(u_{i_1}) = \begin{cases} \frac{5m}{2} + i & 1 \leq i \leq m \text{ if } m \text{ is even} \\ \frac{5m-1}{2} + i & 1 \leq i \leq m \text{ if } m \text{ is odd} \end{cases}$$

$$f(u_{2i-1}) = \begin{cases} \frac{7m}{2} + i & 1 \leq i \leq \frac{m}{2} \text{ if } m \text{ is even} \\ \frac{7m-1}{2} + i & 1 \leq i \leq \frac{m+1}{2} \text{ if } m \text{ is odd} \end{cases}$$

$$f(u_{m+1-i} u_{m-i}) = 4m + i \quad 1 \leq i \leq m-1$$

$$f(u_{m+1-i}, 2) = 5m - 1 + i \quad 1 \leq i \leq m$$

Case (i) : when m is odd

For $1 \leq i \leq m$,

$$f^*[P_1] = f(u_{i_1}) + f(u_{i_2}) + f(u_{i_1}u_i) + f(u_iu_{i_2})$$

$$= \frac{5m-1}{2} + i + 5m - 1 + m + 1 - i + i + 2m + 1 - i$$

$$= \frac{21m+1}{2} \text{ ----- (A)}$$

For $1 \leq i \leq m-1, i \equiv 1 \pmod{2}$

$$f^*[P_2] = f(u_i) + f(u_{i+1}) + f(u_iu_{i+1})$$

$$= \frac{7m-1}{2} + \frac{i+1}{2} + 2m + \frac{i+1}{2} + 4m + m - i$$

$$= \frac{21m+1}{2} \text{ ----- (B)}$$

For $1 \leq i \leq m-1, i \equiv 0 \pmod{2}$

$$f^*[P_2] = f(u_i) + f(u_{i+1}) + f(u_iu_{i+1})$$

$$= 2m + \frac{i}{2} + \frac{7m-1}{2} + \frac{i+2}{2} + 4m + m - i$$

$$= \frac{21m+1}{2} \text{ ----- (C)}$$

From (A), (B) and (C), we conclude that G admits ψ - magic graphoidal total labeling. Hence, $P_m \odot 2K_1$ (m -odd) is magic graphoidal.

For example, consider the graph $P_5 \odot 2K_1$ shown in figure 6.1.

Case (ii) : when m is even

For $1 \leq i \leq m$,

$$f^*[P_1] = f(u_{i_1}) + f(u_{i_2}) + f(u_{i_1}u_i) + f(u_iu_{i_2})$$

$$\frac{5m}{2} = i + 5m - 1 + m + 1 - i + i + 2m + 1 - i$$

$$= \frac{21m + 2}{2} \text{ ----- (A)}$$

For $1 \leq i \leq m - 1, i \equiv 1 \pmod 2$

$$f^*[P_2] = f(u_i) + f(u_{i+1}) + f(u_i u_{i+1})$$

$$= \frac{7m}{2} + \frac{i+1}{2} + 2m + \frac{i+1}{2} + 4m + m - i$$

$$= \frac{21m + 2}{2} \text{ ----- (B)}$$

For $1 \leq i \leq m - 1, i \equiv 0 \pmod 2$

$$f^*[P_2] = f(u_i) + f(u_{i+1}) + f(u_i u_{i+1})$$

$$= 2m + \frac{i}{2} + \frac{7m}{2} + \frac{i+2}{2} + 4m + m - i$$

$$= \frac{21m + 2}{2} \text{ ----- (C)}$$

From (A), (B) and (C), we conclude that G admits ψ - magic graphoidal total labeling. Hence, $P_m \odot 2K_1$ (m -even) is magic graphoidal.

For example, consider the graph $P_4 \odot 2K_1$ shown in figure 6.2.

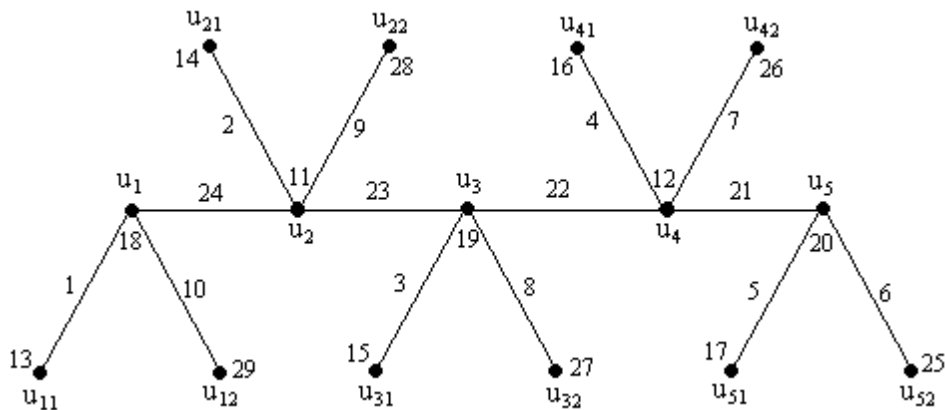


Figure 6.1 $P_5 \odot 2K_1$

Clearly, $\psi = \{(u_{11}, u_1, u_{12}), (u_{21}, u_2, u_{22}), (u_{31}, u_3, u_{32}), (u_{41}, u_4, u_{42}), (u_{51}, u_5, u_{52}), (u_1, u_2), (u_2, u_3), (u_3, u_4), (u_4, u_5)\}$ is a minimum graphoidal cover and $P_4 \odot 2K_1$ is magic graphoidal. Here the constant $K = 53$.

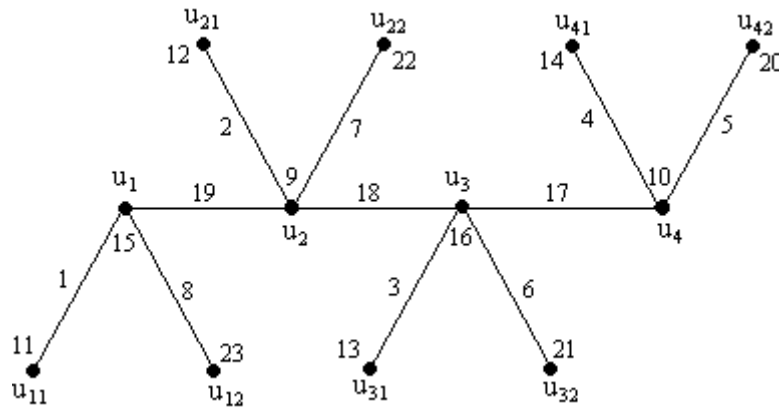


Figure 6.2 $P_4 \circledast 2K_1$

Clearly, $\psi = \{(u_{11}, u_1, u_{12}), (u_{21}, u_2, u_{22}), (u_{31}, u_3, u_{32}), (u_{41}, u_4, u_{42}), (u_1, u_2), (u_2, u_3), (u_3, u_4)\}$ is a minimum graphoidal cover and $P_4 \circledast 2K_1$ is magic graphoidal. Here the constant $K = 43$.

3.7. Theorem : $P_m \circledast K_{1,3}$ is magic graphoidal.

Proof : Let $G = P_m \circledast K_{1,3}$

$$V(G) = \{[(u_i v_i) : 1 \leq i \leq m], [(v_{ij}) : 1 \leq i \leq m, 1 \leq j \leq 3]\}$$

$$E(G) = \{[(u_i u_{i+1}) : 1 \leq i \leq m-1] \cup [(v_i v_{ij}) : 1 \leq i \leq m, 1 \leq j \leq 3]\}$$

$$\text{with } u_i = u_{i3}, 1 \leq i \leq m$$

Let $\psi = \{P_1 = [(v_{i1}, v_i, v_{i2}) : 1 \leq i \leq m], P_2 = [(v_i, u_i, u_{i+1}) : 1 \leq i \leq m-2],$

$$P_3 = (v_{m-1}, u_{m-1}, u_m, v_m)\}$$

Define $f : V \cup E \rightarrow \{1, 2, \dots, 8m-1\}$ by

$f(v_m)$	$= 1$	
$f(v_i v_{i1})$	$= i+1$	$1 \leq i \leq m$
$f(u_i v_i)$	$= m+1+i$	$1 \leq i \leq m-1$
$f(u_{m+1-i} u_{m-i})$	$= 2m+i$	$1 \leq i \leq m-1$
$f(v_{m+1-i} v_{m+1-i,2})$	$= 3m-1+i$	$1 \leq i \leq m$
$f(v_{i1})$	$= 4m-1+i$	$1 \leq i \leq m$
$f(v_i)$	$= 5m-1+i$	$1 \leq i \leq m-1$
$f(u_m v_m)$	$= 6m-1$	
$f(u_{m-i})$	$= 6m+i$	$1 \leq i \leq m-2$
$f(v_{m+1-i, 2})$	$= 7m-2+i$	$1 \leq i \leq m$

For $1 \leq i \leq m,$

$$\begin{aligned} f^*[P_1] &= f(v_{i1}) + f(v_{i2}) + f(v_{i1} v_i) + f(v_i v_{i2}) \\ &= 4m-1+i + 7m-2 + m+1-i + i+1 + 3m-1 + m+1-i \\ &= 16m-1 \text{ ----- (A)} \end{aligned}$$

For $1 \leq i \leq m-2,$

$$\begin{aligned} f^*[P_2] &= f(v_i) + f(u_{i+1}) + f(v_i u_i) + f(u_i u_{i+1}) \\ &= 5m-1+i + 6m + (m-i-1) + m+1+i + 2m+m-i \\ &= 16m-1 \text{ ----- (B)} \end{aligned}$$

$$\begin{aligned}
 f^*[P_3] &= f(v_{m-1}) + f(v_m) + f(v_{m-1}u_{m-1}) + f(u_{m-1}u_m) + f(u_mv_m) \\
 &= 6m - 2 + 1 + 2m + 2m + 1 + 6m - 1 \\
 &= 16m - 1 \text{ ----- (C)}
 \end{aligned}$$

From (A), (B) and (C), we conclude that G admits ψ - magic graphoidal total labeling. Hence, $P_m \odot K_{1,3}$ is magic graphoidal.

For example, consider the graph $P_4 \odot K_{1,3}$ shown in figure 7.

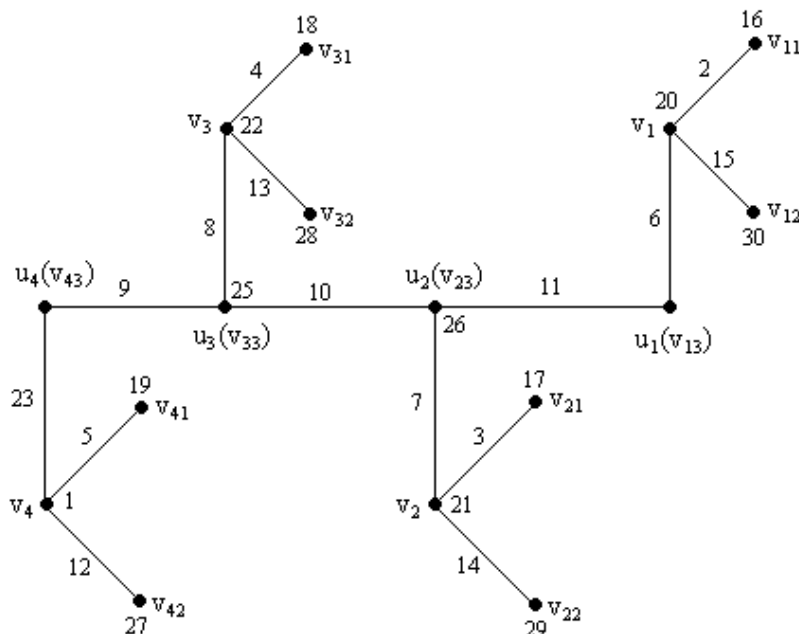


Figure 7. $P_4 \odot K_{1,3}$

Clearly, $\psi = \{(v_{11},v_1,v_{12}), (v_{21},v_2,v_{22}), (v_{31},v_3,v_{32}), (v_{41},v_4,v_{42}), (v_1,u_1,u_2), (v_1,u_2,u_3), (v_3,u_3,u_4,v_4)\}$ is magic graphoidal. Here the constant $K = 63$.

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