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Magic Graphoidal on Class of Trees

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Abstract: B.D.Acharya and E. Sampathkumar [1] defined Graphoidal cover as partition of edge set of a graph G into internally disjoint paths (not necessarily open). The minimum cardinality of such cover is known as graphoidal covering number of G.

Let G = {V, E} be a graph and let ψ be a graphoidal cover of G. Define f:

$V \cup E \rightarrow \{1, 2, \ldots, p+q\}$ such that for every path $P = (v_0, v_1, v_2, \ldots v_n)$ in ψ with $f^*(P) = f(v_0) + f(v_n) + \sum_1^n f(v_{i-1}v_i) = k$, a constant, where f^* is the induced labeling on ψ. Then, we say that G admits ψ - magic graphoidal total labeling of G.

A graph G is called magic graphoidal if there exists a minimum graphoidal cover ψ of G such that G admits ψ - magic graphoidal total labeling.

In this paper, we proved that $[P_n;S_1]$, $[P_n;S_2]$, $T_{(n)}$, $P_m \odot K_{1,3}$, $P_m \odot 2K_1$ and $K_{1,n}$ $2\odot_1$ are magic graphoidal

1. INTRODUCTION

By a graph, we mean a finite simple and undirected graph. The vertex set and edge set of a graph G are denoted by V(G) and E(G) respectively. Terms and notations not used here are as in [3].

1.1. Definition : Let $S_1 = (v_0, v_1)$ be a star and let $[P_n ; S_1]$ be the graph obtained from n copies of S_1 and the path $P_n = (u_1, u_2, \ldots, u_n)$ by joining u_j with the vertex v_0 of the j^{th} copy of S_1 by means of an edge, for $1 \leq j \leq n$.

1.2. Definition : Let $S_2 = (v_0, v_1, v_2)$ be a star and let $[P_n ; S_2]$ be the graph obtained from n copies of S_2 and the path $P_n = (u_1, u_2, \ldots, u_n)$ by joining u_j with the vertex v_0 of the j^{th} copy of S_2 by means of an edge, for $1 \leq j \leq n$.

1.3. Defintion : Let T be any Tree. Denote the tree, obtained from T by considering two copies of T by adding an edge between them, by $T_{(2)}$ and in general the graph obtained from $T_{(n-1)}$ and T by adding an edge between them is denoted by $T_{(n)}$.

1.4. Result [11] : For a Tree T, $\gamma(T)$ = n-1 where n is the number of pendent vertices of G.

2. PRELIMINARIES

Let G = {V, E} be a graph with p vertices and q edges. A graphoidal cover ψ of G is a collection of (open) paths such that

(i) every edge is in exactly one path of ψ

(ii) every vertex is an interval vertex of atmost one path in ψ.

We define $\gamma(G) = \min_{\psi \in \zeta} |\psi|$,

where ζ is the collec tion of graphoidal covers ψ of G and

γ is graphoidal covering number of G.

Let ψ be a graphoidal cover of G. Then we say that G admits ψ - magic graphoidal total labeling of G if there exists a bijection f: $V \cup E \rightarrow \{1, 2, \ldots, p+q\}$ such that for every path P $=(v_0 v_1 v_2 \ldots v_n)$ in ψ, then, $f^*(P) = f(v_0) + f(v_n) + \sum_1^n f(v_{i-1}v_i) = k$, a constant, where f^* is the induced labeling of ψ. A graph G is called magic graphoidal if there exists a minimum graphoidal cover ψ of G such that G admits ψ - magic graphoidal total labeling.

3. Magical Graphoidal on Trees

3.1. Theorem : $[P_n ; S_1]$, (n - even) is magic graphoidal.

Proof: Let $G = [P_n ; S_1]$

Let $V(G) = \{u_i, v_i, w_i: \ 1 \leq i \leq n\}$ and
$E(G) = \{ [(u_iv_i) \cup (v_iw_i) : 1 \leq i \leq n] \cup [(u_iu_{i+1}) : 1 \leq i \leq n-1]\}$

Define $f : V \cup E \rightarrow \{1, 2, \ldots, p+q\}$ by

$f(w_1) = 1$

$f(w_1v_1) = 2$

$f(v_1u_1) = 3$

$f(u_{i+1}) = 6 + i \qquad 1 \leq i \leq n-2$

$f(w_{i+1}) = 6n - i \qquad 1 \leq i \leq n-1$

$f(v_{i+1}w_{i+1}) = 4n + 1 + i \qquad 1 \leq i \leq n-1$

$f(u_{i+1}v_{i+1}) = 3n + 3 - 2i \qquad 1 \leq i \leq (n\backslash 2)-1$

$$f\left(u_{\frac{n}{2}+i}v_{\frac{n}{2}+i}\right) = 3n + 4 - 2i \quad 1 \leq i \leq \frac{n}{2}$$

$$f(u_iu_{i+1}) = \frac{3n}{2} + 4 + i \qquad 1 \leq i \leq \frac{n}{2} - 1$$

$$f\left(u_{\frac{n}{2}-1+i}u_{\frac{n}{2}+i}\right) = n + 4 + i \quad 1 \leq i \leq \frac{n}{2}$$

Let $\psi = \{P_1 = (w_1, v_1, u_1, u_2, v_2, w_2),\ P_2 = (u_i, u_{i+1}, v_{i+1}, w_{i+1}) : \ 2 \leq i \leq n-1\}$

$f^*[P_1] = f(w_1) + f(w_2) + f(w_1v_1) + f(v_1u_1) + f(u_1u_2) + f(u_2v_2) + f(v_2w_2)$

$$= 1 + 6n - 1 + 2 + 3 + \frac{3n}{2} + 4 + 1 + 3n + 3 - 2 + 4n + 2$$

$$= 13n + \frac{3n}{2} + 13 \text{ ---------- } \textbf{(A)}$$

For $2 \leq i \leq (n\backslash 2) - 1$,

$f^*[P_2] = f(u_i) + f(w_{i+1}) + f(u_iu_{i+1}) + f(u_{i+1}v_{i+1}) + f(v_{i+1}w_{i+1})$

$= 6 + i - 1 + 6n - i + (3n\backslash 2) + 4 + i + 3n + 3 - 2i + 4n + 1 + i$

$= 13n + (3n\backslash 2) + 13$ **---------- (B)**

For $(n\backslash 2) \leq i \leq n - 1$,

$f^*[P_2] = f(u_i) + f(w_{i+1}) + f(u_iu_{i+1}) + f(u_{i+1}v_{i+1}) + f(v_{i+1}w_{i+1})$

$= 6+i-1+6n-i+n+4+i-(n\backslash 2)+1+3n+4-2(i+1-(n\backslash 2))+4n+1+i$

$= 13n+(3n\backslash 2)+13$ **---------- (C)**

From (A), (B) and (C), we conclude that G admits ψ - magic graphoidal total labeling. Hence, $[P_n ; S_1]$, (n - even) is magic graphoidal.
For example, consider the graph $[P_6 ; S_1]$ shown in figure 1.

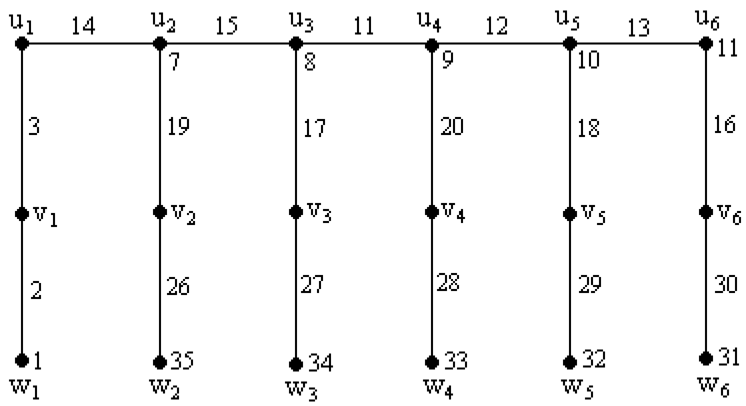

Figure 1 $[P6 ; S1]$

Clearly, $\psi = \{(w_1,v_1,u_1,u_2,v_2,w_2), (u_2,u_3,v_3,w_3), (u_3,u_4,v_4,w_4), (u_4,u_5,v_5,w_5), (u_5,u_6,v_6,w_6)\}$ is a minimum graphoidal cover and $[P_6 ; S_1]$ is magic graphoidal. Here the constant K = 100

3.2. Theorem : $[P_n ; S_1]$, (n - odd) is magic graphoidal.

Proof: Let $G = [P_n ; S_1]$

Let $V(G) = \{u_i, v_i, w_i: 1 \le i \le n\}$ and

$E(G) = \{ [(u_iv_i) \cup (v_iw_i) : 1 \le i \le n] \cup [(u_iu_{i+1}) : 1 \le i \le n-1]\}$

Define $f : V \cup E \rightarrow \{1, 2, \ldots, p+q\}$ by

$f(w_1) = 1$

$f(w_1v_1) = 2$

$f(v_1u_1) = 3$

$f(u_{i+1}) = 5 + 2i \qquad 1\le i \le(n-1)\backslash 2$

$f(w_{i+1}) = 6n - i \qquad 1\le i \le n-1$

$f(v_{i+1}w_{i+1}) = 4n + 1 + i \qquad 1\le i \le n-1$

$f(u_1u_2) = n + 5$

$$f(u_{i+1}u_{i+2}) = \frac{3n+11|}{2} - i \qquad 1\le i \le \frac{n-1}{2}$$

$$f(u_{n+1-i}u_{n-i}) = \frac{3n+9}{2} + i \qquad 1\le i \le \frac{n-3}{2}$$

$f(u_{i+2}v_{i+2}) = 3n + 2 - i \qquad 1\le i \le n-2$

$$f(u_2v_2) = \frac{7n+3}{2}$$

Let $\psi = \{P_1 = (w_1,v_1,u_1,u_2,v_2,w_2),\ P_2 = (u_i,u_{i+1},v_{i+1},w_{i+1}) : 2 \le i \le n-1\}$

$$f^*[P_1] = f(w_1) + f(w_2) + f(w_1v_1) + f(v_1u_1) + f(u_1u_2) + f(u_2v_2) + f(v_2w_2)$$

$$= 1 + 6n - 1 + 2 + 3 + n + 5 + \frac{7n+3}{2} + 4n + 1 + 1$$

$$= 14n + \frac{n+1}{2} + 13 \text{ ---------- } \mathbf{(A)}$$

For $2 \le i < \frac{n+1}{2}$

$$f^*[P_2] = f(u_i) + f(w_{i+1}) + f(u_iu_{i+1}) + f(u_{i+1}v_{i+1}) + f(v_{i+1}w_{i+1})$$

$$= 5 + 2(i-1) + 6n - i + \frac{3n+11}{2} - (i-1) + 3n + 2 - (i-1) + 4n + 1 + i$$

$$= 14n + \frac{n+1}{2} + 13 \text{ ---------- } \mathbf{(B)}$$

For $\frac{n+1}{2} \le i \le n-1$,

$$f^*[P_2] = f(u_i) + f(w_{i+1}) + f(u_iu_{i+1}) + f(u_{i+1}v_{i+1}) + f(v_{i+1}w_{i+1})$$

$$= 6 + 2\left(i - \frac{n+1}{2}\right) + 6n - i + \frac{3n+9}{2} + (n-i) + 3n + 2 - (i-1) + 4n + 1 + i$$

$$= 14n + \frac{n+1}{2} + 13 \text{ ---------- } \mathbf{(C)}$$

From (A), (B) and (C), we conclude that ψ is minimum magic graphoidal cover
Hence, $[P_n ; S_1]$, (n - even) is magic graphoidal.
For example, consider the graph $[P_7 ; S_1]$ shown in figure 2.

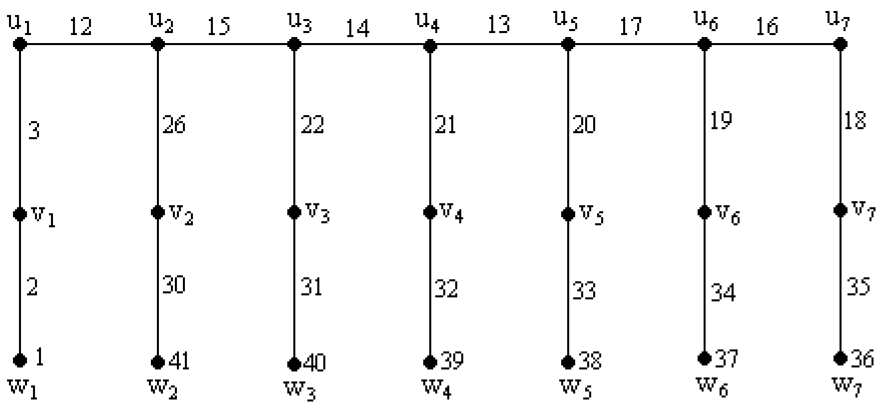

Figure 2 $[P_7 ; S_1]$

Clearly, $\psi = \{(w_1,v_1,u_1,u_2,v_2,w_2), (u_2,u_3,v_3,w_3), (u_3,u_4,v_4,w_4), (u_4,u_5,v_5,w_5), (u_5,u_6,v_6,w_6), (u_6,u_7,v_7,w_7)\}$ is a minimum graphoidal cover and $[P_7 ; S_1]$ is magic graphoidal. Here the constant $K = 115$.

3.3. Theorem : $[P_n ; S_2]$ is magic graphoidal.

Proof: Let $G = [P_n ; S_2]$
$V(G) = \{(u_i, v_i) : 1 \le i \le n\}$

$E(G) = \{ [(u_i u_{i+1}) : 1 \le i \le n\text{-}1] \cup [(u_i v_i) : 1 \le i \le n]$
$\cup [(v_i v_{i1}) \cup (v_i v_{i2}) : 1 \le i \le n] \}$

Define $f : V \cup E \rightarrow \{1, 2, \ldots, p+q\}$ by

$f(v_1) = 1$
$f(u_1 v_1) = 2$ f
$(u_1) = 3$
$f(u_{i+1}) = 3 + i \quad 1 \le i \le n - 2$
$f(u_n) = p + q$
$f(v_{i1}) = n + 1 + i \quad 1 \le i \le n$
$f(v_{i+1}) = 8n - 1 - i \quad 1 \le i \le n - 1$
$f(v_{i2}) = 7n - i \quad 1 \le i \le n$
$f(v_i v_{i2}) = 5n - 1 + i \quad 1 \le i \le n$
$f(u_{i+1} v_{i+1}) = 4n + i \quad 1 \le i \le n - 1$
$f(u_i u_{i+1}) = 4n + 1 - i \quad 1 \le i \le n - 1$
$f(v_i v_{i1}) = 3n + 2 - i \quad 1 \le i \le n$

Let $\psi = \{P_1 = [(v_{i1}, v_i, v_{i2}) : 1 \le i \le n], P_2 = (v_1, u_1, u_2, v_2), P_3 = [(u_i, u_{i+1}, v_{i+1}) : 2 \le i \le n-1]\}$

For $1 \le i \le n$,

$f^*[P_1] = f(v_{i1}) + f(v_{i2}) + f(v_{i1} v_i) + f(v_i v_{i2})$

$= n + 1 + i + 7n - i + 3n + 2 - i + 5n - 1 + i$

$= 16n + 2$ ---------- **(A)**

$f^*[P_2] = f(v_1) + f(v_2) + f(v_1 u_1) + f(u_1 u_2) + f(u_2 v_2)$

$= 1 + 8n - 2 + 2 + 4n + 4n + 1$

$= 16 n + 2$ ---------- **(B)**

For $2 \le i \le n-1$,

$f^*[P_3] = f(u_i) + f(v_{i+1}) + f(u_i u_{i+1}) + f(u_{i+1} v_{i+1})$

$= 3 + i - 1 + 8n - 1 - i + 4n + 1 - i + 4n + i$

$= 16 n + 2$ ---------- **(C)**

From (A), (B) and (C), we conclude that G admits ψ - magic graphoidal total labeling. Hence, $[P_n ; S_2]$ is magic graphoidal.

For example, consider the graphs $[P_3 ; S_2]$ and $[P_4 ; S_2]$ shown in figure 3.1 and 3.2.

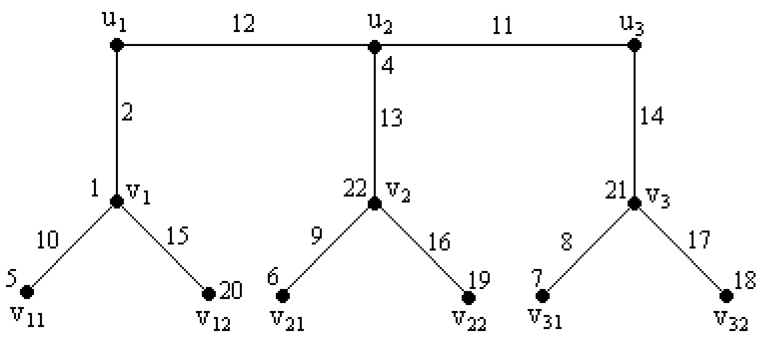

Figure 3.1 $[P_3 ; S_2]$

Clearly, $\psi = \{(v_{11}, v_1, v_{12}), (v_{21}, v_2, v_{22}), (v_{31}, v_3, v_{32}), (v_1, u_1, u_2, v_2), (u_2, u_3, v_3)\}$ is a minimum graphoidal cover and $[P_3 ; S_2]$ is magic graphoidal. Here the constant K = 50

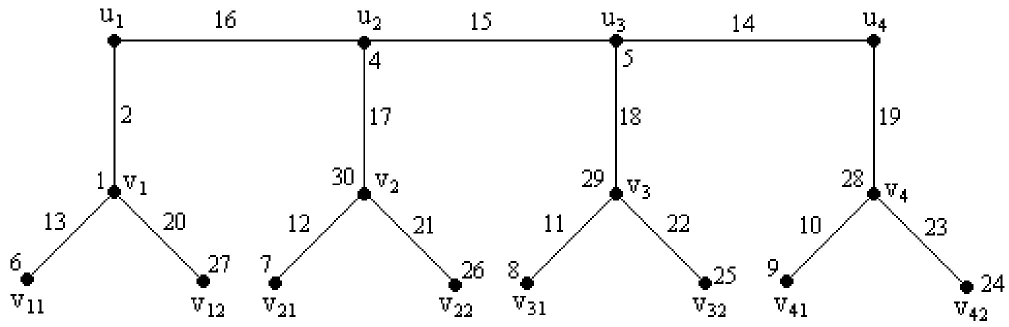

Figure 3.2 $[P_4 ; S_2]$

Clearly, $\psi = \{(v_{11},v_1,v_{12}), (v_{21},v_2,v_{22}), (v_{31},v_3,v_{32}), (v_{41},v_4,v_{42}), (v_1,u_1,u_2,v_2), (u_2,u_3,v_3), (u_3,u_4,v_4)\}$ is a minimum graphoidal cover and $[P_4 ; S_2]$ is magic graphoidal. Here the constant K = 66

3.4. Theorem : For a Tree, $T_{(n)}$ is magic graphoidal.

Proof :

Let $T_{(n)}$ be a graph such that
$V[T_{(n)}] = \{u_{i1}, u_{i2}, u_{i3}, u_{i4}, u_{i5} : 1 \le i \le n\}$ and
$E[T_{(n)}] = \{[(u_{i1}u_{i2}), (u_{i2}u_{i3}), (u_{i3}u_{i4}), (u_{i4}u_{i5}): 1 \le i \le n] \cup [(u_{in}u_{i+1n}) : 1\le i \le n\text{-}1)]\}$
Define $f : V \cup E \rightarrow \{1, 2, \ldots, p+q\}$ by

$f(u_{i1})$	$= i$	$1 \le i \le n$
$f(u_{i3})$	$= n + 1$	
$f(u_{i3}u_{i5})$	$= n + 2$	
$f(u_{i1}u_{i2})$	$= 2n + 3 - i$	$1 \le i \le n$
$f(u_{i+2,5}u_{i+3,5})$	$= 5n - 3 - 2i$	$1 \le i \le n - 3$
$f(u_{n+1-i,5}u_{n-i,5})$	$= 4n - 1 + i$	$n - 2 \le i \le n - 1$
$f(u_{n+1-i,2}u_{n+1-i,3})$	$= 5n - 2 + i$	$1 \le i \le n$
$f(u_{n+1-i,3}u_{n+1-i,4})$	$= 6n - 2 + i$	$1 \le i \le n$
$f(u_{n+1-i,3}u_{n+1-i,5})$	$= 7n - 2 + i$	$1 \le i \le n - 1$
$f(u_{i4})$	$= 8n - 2 + 2(i\text{-}1)$	$1 \le i \le n$
$f(u_{i+1,3})$	$= 8n - 1 + 2(i\text{-}1)$	$1 \le i \le n - 1$

Let $\psi = \{P_1= (u_{i1},u_{i2},u_{i3},u_{i4}), P_2=(u_{i3},u_{i5},u_{25},u_{23}), P_3= [(u_{i5},u_{i+15}, u_{i+13}) : 2 \le i \le n - 1]\}$

$f^*[P_1] = f(u_{i1}) + f(u_{i4}) + f(u_{i1}u_{12}) + f(u_{i2}u_{i3}) + f(u_{i3}u_{i4})$

$= i+8n-2+2(i\text{-}1)+2n+3-i+5n-2 + (n + 1- i) + 6n - 2 + (n + 1- i)$

$= 23n - 3$ **---------- (A)**

$f^*[P_2] = f(u_{i5}) + f(u_{i+1,3}) + f(u_5\, u_{i+1,3}) + f(u_{i+1,5}\, u_{i+1,3})$

$= 2n+2+i-1+8n - 1 + 2(i - 1) + 5n - 3 - 2 (i - 2) + 7n-2+ n - i$

$= 23n - 3$ **---------- (B)**

$f^*[P_3] = f(u_{i3}) + f(u_{23}) + f(u_{i3}u_{15}) + f(u_{15}u_{25}) + f(u_{25}u_{23})$

$= n + 1 + 8n - 1 + 2(i - 1) + 5n - 3 - 2(i - 2) + 7n - 2 + n - i$

$= 23n - 3$ **---------- (C)**

From (A), (B) and (C), we conclude that G admits ψ - magic graphoidal total labeling. Hence, $T_{(n)}$ is magic graphoidal.

For example, consider the graph $T_{(3)}$ shown in figure 4.

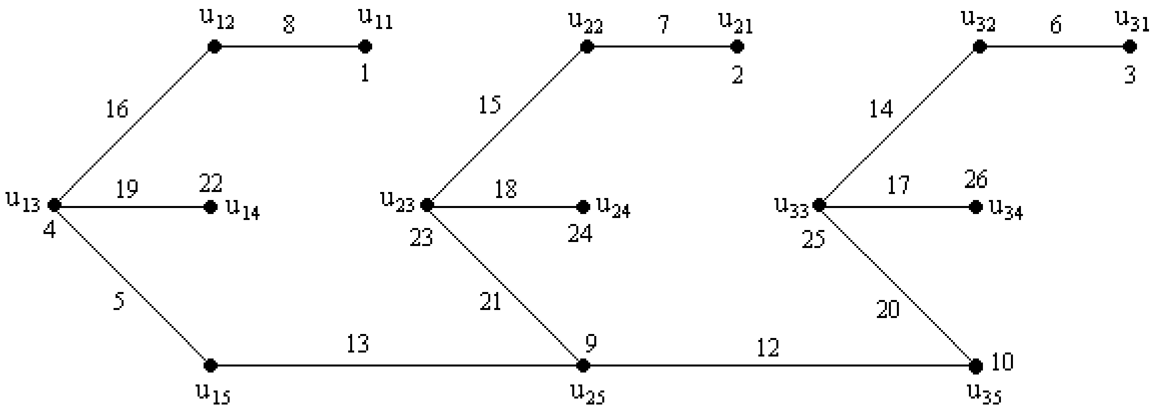

Figure 4 $T_{(3)}$

Clearly, $\psi = \{(u_{11},u_{12},u_{13},u_{14}), (u_{21},u_{22},u_{23},u_{24}), (u_{31},u_{32},u_{33},u_{34}), (u_{13},u_{15},u_{25},u_{23}), (u_{25},u_{35},u_{33})\}$ is a minimum graphoidal cover and $T_{(3)}$ is magic graphoidal. Here the constant K = 66

3.5. Theorem : The graph Double Crowned Star $K_{1,n} \odot K_1$ is magic graphoidal.

Proof : Let $G = K_{1,n} \odot 2K_1$

$V(G) = \{u, [u_i : 1 \le i \le n], [(u_{i1},u_{i2}): 1 \le i \le n] \}$ and

$E(G) = \{ [(uu_i): 1 \le i \le n] \cup [(u_iu_{i1}) \cup (u_iu_{i2}) : 1 \le i \le n] \}$

Define $f : V \cup E \to \{1, 2, \ldots, p+q\}$ by

$f(u) = 2n + 3$
$f(u_{i1}) = i \quad 1 \le i \le n$
$f(u_1) = n + 1$
$f(u_2) = n + 2$
$f(u_{n+1-i,2}) = n + 2 + i \quad 1 \le i \le n$
$f(u_iu_{i,1}) = 2n + 3 + i \quad 1 \le i \le n$
$f(uu_1) = 3n + 4$
$f(u_{2+i}) = 3n + 4 + i \quad 1 \le i \le n - 2$
$f(uu_{n+1-i}) = 4n + 2 + i \quad 1 \le i \le n - 1$
$f(u_{n+1-i}\, u_{n+1-i,\,2}) = 5n + 1 + i \quad 1 \le i \le n$

Let $\psi = \{P_1 = (u_1,u,u_2), P_2 = [(u,u_i) : 3 \le i \le n], P_3 = [(u_{i1},u_i,u_{i2}) : 1 \le i \le n]\}$

$f^*[P_1] = f(u_1) + f(u_2) + f(u_1u) + f(uu_2)$

$= n + 1 + n + 2 + 3n + 4 + 5n + 1$

$= 10n + 8$ **---------- (A)**

For $3 \le i \le n$,

$f^*[P_2] = f(u) + f(u_i) + f(uu_i)$

$= 2n + 3 + 3n + 4 + i - 2 + 4n + 2 + n + 1 - i$

$= 10n + 8$ **---------- (B)**

For $1 \le i \le n$,

$f^*[P_3] = f(u_{i1}) + f(u_{i2}) + f(u_{i1}u_i) + f(u_iu_{i2})$

$= i + n + 2 + n + 1 - i + 2n + 3 + i + 5n + 1 + n + 1 - i$

$= 10n + 8$ **---------- (C)**

From (A), (B) and (C), we conclude that G admits ψ - magic graphoidal total labeling. Hence, Double Crowned Star $K_{1,n} \odot 2K_1$ is magic graphoidal.
For example, consider the graph $K_{1,5} \odot 2K_1$ shown in figure 5.

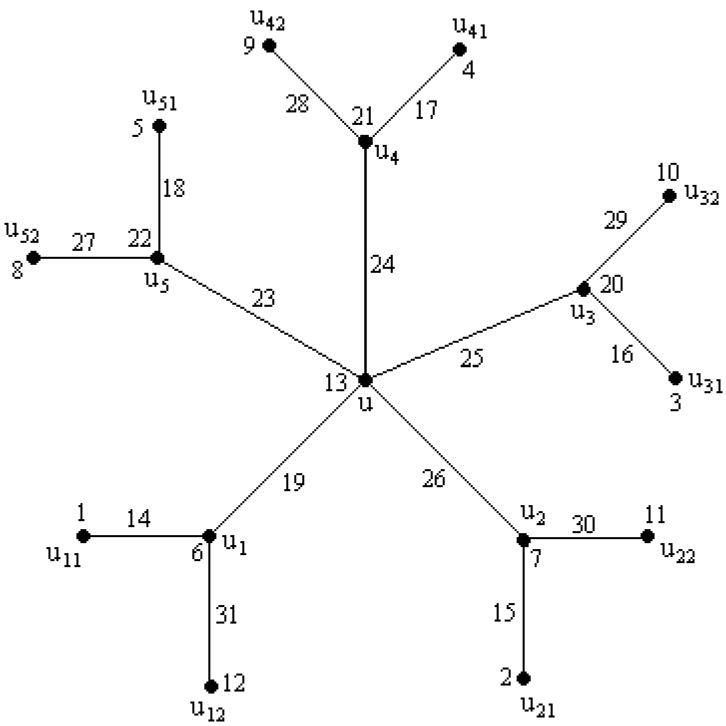

Figure 5. $K_{1,5} \odot 2K_1$

Clearly, $\psi = \{(u_{11},u_1,u_{12}), (u_{21},u_2,u_{22}), (u_{31},u_3,u_{32}), (u_{41},u_4,u_{42}), (u_{51},u_5,u_{52}), (u_1,u,v,u_2), (u,u_3), (u,u_4), (u,u_5)\}$ is a minimum graphoidal cover and $K_{1,5} \odot 2K_1$ is magic graphoidal. Here the constant $K = 58$.

3.6. Theorem : $P_m \tilde{\odot} 2K_1$ is magical graphoidal.

Proof : Let $G = P_m \tilde{\odot} 2K_1$

$V(G) = \{ (u_i : 1 \le i \le m), (u_{ij} : 1 \le i \le m, 1 \le j \le 2) \}$ and
$E(G) = \{ [(u_i u_{i+1}) : 1 \le i \le m\text{-}1] \cup [(u_i u_{ij}) : 1 \le i \le m, 1 \le j \le 2] \}$
Let $\psi = \{P_1 = [(u_{i1},u_i,u_{i2}) : 1 \le i \le m], P_2 = [(u_i,u_{i+1}) : 1 \le i \le m\text{-}1]\}$
Define $f : V \to \{0, 1, 2, \ldots, 6m\text{-}1\}$ by

$f(u_i u_{i1}) = i \qquad 1 \le i \le m$

$f(u_i u_{i2}) = 2m+1-i \qquad 1 \le i \le m$

$$f(u_{2i}) = \begin{cases} 2m+i & 1 \le i \le \dfrac{m}{2} \quad \text{if } m \text{ is even} \\[2ex] 2m+i & 1 \le i \le \dfrac{m-1}{2} \quad \text{if } m \text{ is odd} \end{cases}$$

$$f(u_{i1}) = \begin{cases} \dfrac{5m}{2}+i & 1 \le i \le m \text{ if } m \text{ is even} \\ \dfrac{5m-1}{2}+i & 1 \le i \le m \text{ if } m \text{ is odd} \end{cases}$$

$$f(u_{2i-1}) = \begin{cases} \dfrac{7m}{2}+i & 1 \le i \le \dfrac{m}{2} & \text{if } m \text{ is even} \\ \dfrac{7m-1}{2}+i & 1 \le i \le \dfrac{m+1}{2} & \text{if } m \text{ is odd} \end{cases}$$

$f(u_{m+1-i}\, u_{m-i}) = 4m + i \qquad 1 \le i \le m - 1$

$f(u_{m+1-i,\, 2}) = 5m - 1 + i \qquad 1 \le i \le m$

Case (i) : when m is odd

For $1 \le i \le m$,

$f^*[P_1] = f(u_{i1}) + f(u_{i2}) + f(u_{i1}u_i) + f(u_iu_{i2})$

$= \dfrac{5m-1}{2} + i + 5m - 1 + m + 1 - i + i + 2m + 1 - i$

$= \dfrac{21m+1}{2}$ --------- **(A)**

For $1 \le i \le m - 1$, $i \equiv 1 \bmod 2$

$f^*[P_2] = f(u_i) + f(u_{i+1}) + f(u_iu_{i+1})$

$= \dfrac{7m-1}{2} + \dfrac{i+1}{2} + 2m + \dfrac{i+1}{2} + 4m + m - i$

$= \dfrac{21m+1}{2}$ --------- **(B)**

For $1 \le i \le m - 1$, $i \equiv 0 \bmod 2$

$f^*[P_2] = f(u_i) + f(u_{i+1}) + f(u_iu_{i+1})$

$= 2m + \dfrac{i}{2} + \dfrac{7m-1}{2} + \dfrac{i+2}{2} + 4m + m - i$

$= \dfrac{21m+1}{2}$ --------- **(C)**

From (A), (B) and (C), we conclude that G admits ψ - magic graphoidal total labeling. Hence, $P_m \odot 2K_1$ (m-odd) is magic graphoidal.

For example, consider the graph $P_5 \odot 2K_1$ shown in figure 6.1.

Case (ii) : when m is even

For $1 \le i \le m$,

$f^*[P_1] = f(u_{i1}) + f(u_{i2}) + f(u_{i1}u_i) + f(u_iu_{i2})$

$$\frac{5m}{2} = + i + 5m - 1 + m + 1 - i + i + 2m + 1 - i$$

$$= \frac{21m+2}{2} \quad \text{---------} \ \textbf{(A)}$$

For $1 \leq i \leq m - 1$, $i \equiv 1 \bmod 2$

$f^*[P_2] = f(u_i) + f(u_{i+1}) + f(u_i u_{i+1})$

$$= \frac{7m}{2} + \frac{i+1}{2} + 2m + \frac{i+1}{2} + 4m + m - i$$

$$= \frac{21m+2}{2} \quad \text{---------} \ \textbf{(B)}$$

For $1 \leq i \leq m - 1$, $i \equiv 0 \bmod 2$

$f^*[P_2] = f(u_i) + f(u_{i+1}) + f(u_i u_{i+1})$

$$= 2m + \frac{i}{2} + \frac{7m}{2} + \frac{i+2}{2} + 4m + m - i$$

$$= \frac{21m+2}{2} \quad \text{---------} \ \textbf{(C)}$$

From (A), (B) and (C), we conclude that G admits ψ - magic graphoidal total labeling. Hence, $P_m \tilde{\odot} 2K_1$ (m-even) is magic graphoidal.
For example, consider the graph $P_4 \tilde{\odot} 2K_1$ shown in figure 6.2.

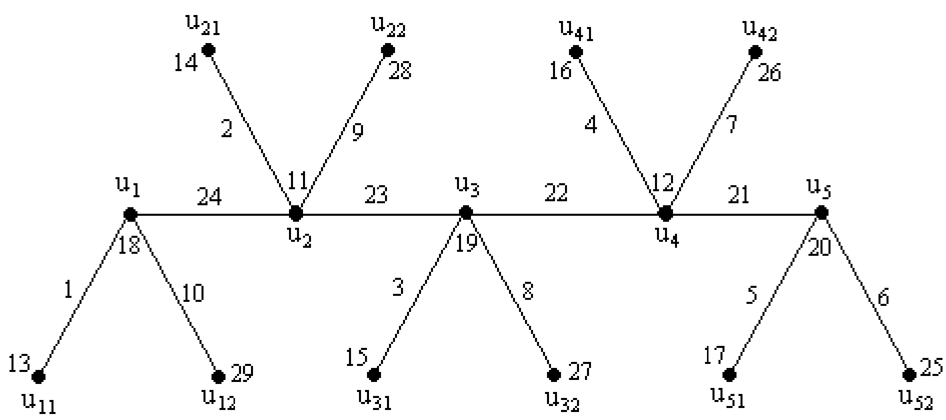

Figure 6.1 $P_5 \tilde{\odot} 2K_1$

Clearly, $\psi = \{(u_{11},u_1,u_{12}), (u_{21},u_2,u_{22}), (u_{31},u_3,u_{32}), (u_{41},u_4,u_{42}), (u_{51},u_5,u_{52}), (u_1,u_2), (u_2,u_3), (u_3,u_4), (u_4,u_5)\}$ is a minimum graphoidal cover and $P_4 \tilde{\odot} 2K_1$ is magic graphoidal. Here the constant $K = 53$.

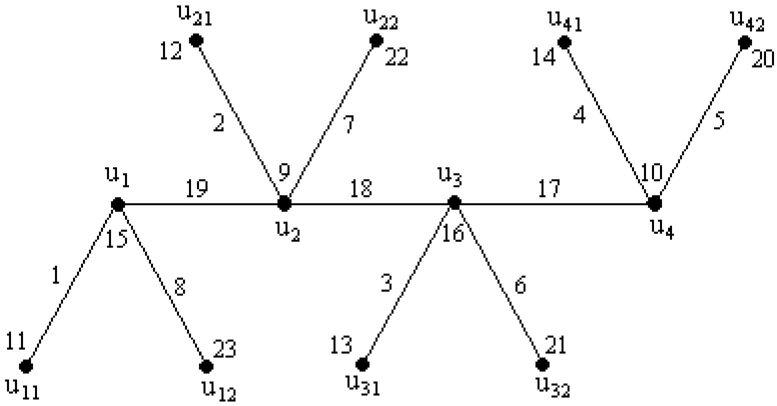

Figure 6.2 $P_4 \tilde{\odot} 2K_1$

Clearly, $\psi = \{(u_{11},u_1,u_{12}),(u_{21},u_2,u_{22}),(u_{31},u_3,u_{32}),(u_{41},u_4,u_{42}),(u_1,u_2),(u_2,u_3),(u_3,u_4)\}$ is a minimum graphoidal cover and $P_4 \tilde{\odot} 2K_1$ is magic graphoidal. Here the constant $K = 43$.

3.7. Theorem : $P_m \tilde{\odot} K_{1,3}$ is magic graphoidal.

Proof : Let $G = P_m \tilde{\odot} K_{1,3}$

$V(G) = \{[(u_iv_i): 1 \le i \le m], [(v_{ij}): 1 \le i \le m, 1 \le i \le 3]\}$

$E(G) = \{ [(u_iu_{i+1}): 1 \le i \le m\text{-}1] \cup [(v_iv_{ij}) : 1 \le i \le m, 1 \le j \le 3] \}$

with $u_i = u_{i3}$, $1 \le i \le m$

Let $\psi = \{P_1= [(v_{i1},v_i,v_{i2}) : 1 \le i \le m], P_2=[(v_i,u_i,u_{i+1}): 1\le i \le m-2],$

$P_3=(v_{m-1},u_{m-1},u_m,v_m)\}$

Define $f : V \cup E \to \{1, 2, \ldots, 8m-1\}$ by

$f(v_m)$	$= 1$	
$f(v_iv_{i1})$	$= i+1$	$1 \le i \le m$
$f(u_iv_i)$	$= m+1+ i$	$1 \le i \le m-1$
$f(u_{m+1-i}\, u_{m-i})$	$= 2m + i$	$1 \le i \le m -1$
$f(v_{m+1-i}\, v_{m+1-i,2})$	$= 3m-1+ i$	$1 \le i \le m$
$f(v_{i1})$	$= 4m - 1 + i$	$1 \le i \le m$
$f(v_i)$	$= 5m - 1 + i$	$1 \le i \le m-1$
$f(u_mv_m)$	$= 6m - 1$	
$f(u_{m-i})$	$= 6m + i$	$1 \le i \le m-2$
$f(v_{m+1-i,\, 2})$	$= 7m -2 + i$	$1 \le i \le m$

For $1 \le i \le m$,

$f^*[P_1] = f(v_{i1}) + f(v_{i2}) + f(v_{i1}v_i) + f(v_iv_{i2})$

$= 4m - 1+ i + 7m - 2 + m + 1 - i + i + 1 + 3m - 1 + m + 1 - i$

$= 16m - 1$ **--------- (A)**

For $1 \le i \le m - 2$,

$f^*[P_2] = f(v_i) + f(u_{i+1}) + f(v_iu_i) + f(u_iu_{i+1})$

$= 5m - 1 + i+ 6m + (m - i - 1) + m + 1 + i + 2m + m - i$

$= 16m - 1$ **--------- (B)**

$f^*[P_3] = f(v_{m-1}) + f(v_m) + f(v_{m-1}u_{m-1}) + f(u_{m-1}u_m) + f(u_mv_m)$

$= 6m - 2 + 1 + 2m + 2m + 1 + 6m - 1$

$= 16m - 1$ **--------- (C)**

From (A), (B) and (C), we conclude that G admits ψ - magic graphoidal total labeling. Hence, $P_m \widetilde{\odot} K_{1,3}$ is magic graphoidal.

For example, consider the graph $P_4 \widetilde{\odot} K_{1,3}$ shown in figure 7.

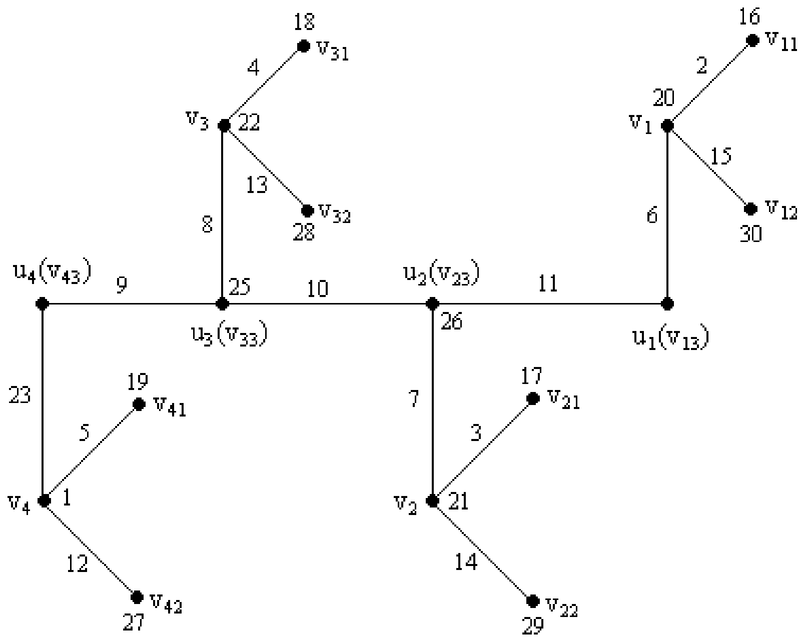

Figure 7. $P_4 \widetilde{\odot} K_{1,3}$

Clearly, $\psi = \{(v_{11},v_1,v_{12}), (v_{21},v_2,v_{22}), (v_{31},v_3,v_{32}), (v_{41},v_4,v_{42}), (v_1,u_1,u_2), (v_1,u_2,u_3), (v_3,u_3,u_4,v_4)\}$ is magic graphoidal. Here the constant K = 63.

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