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# Magic Graphoidal on Class of Trees

A. Nellai Murugan

Department of Mathematics, V.O.Chidambaram College, Tuticorin, Tamilnadu.

**Keywords:** Graphoidal Cover, Magic Graphoidal, Graphoidal Constant.

**Abstract:** B.D.Acharya and E. Sampathkumar [1] defined Graphoidal cover as partition of edge set of a graph G into internally disjoint paths (not necessarily open). The minimum cardinality of such cover is known as graphoidal covering number of G.

Let G = {V, E} be a graph and let $\psi$ be a graphoidal cover of G. Define f:

$V \cup E \rightarrow \{1, 2, \ldots, p+q\}$ such that for every path $P = (v_0, v_1, v_2, \ldots v_n)$ in $\psi$ with $f^*(P) = f(v_0) + f(v_n) + \sum_1^n f(v_{i-1} v_i) = k$, a constant, where $f^*$ is the induced labeling on $\psi$. Then, we say that G admits $\psi$ - magic graphoidal total labeling of G.

A graph G is called magic graphoidal if there exists a minimum graphoidal cover $\psi$ of G such that G admits $\psi$ - magic graphoidal total labeling.

In this paper, we proved that $[P_n;S_1]$, $[P_n;S_2]$, $T_{(n)}$, $P_m \odot K_{1,3}$, $P_m \odot 2K_1$ and $K_{1,n}$ $2\odot_1$ are magic graphoidal

## 1. INTRODUCTION

By a graph, we mean a finite simple and undirected graph. The vertex set and edge set of a graph G are denoted by V(G) and E(G) respectively. Terms and notations not used here are as in [3].

**1.1. Definition :** Let $S_1 = (v_0, v_1)$ be a star and let $[P_n ; S_1]$ be the graph obtained from n copies of $S_1$ and the path $P_n = (u_1, u_2, \ldots, u_n)$ by joining $u_j$ with the vertex $v_0$ of the $j^{th}$ copy of $S_1$ by means of an edge, for $1 \leq j \leq n$.

**1.2. Definition :** Let $S_2 = (v_0,v_1,v_2)$ be a star and let $[P_n ; S_2]$ be the graph obtained from n copies of $S_2$ and the path $P_n = (u_1, u_2, \ldots, u_n)$ by joining $u_j$ with the vertex $v_0$ of the $j^{th}$ copy of $S_2$ by means of an edge, for $1 \leq j \leq n$.

**1.3. Defintion :** Let T be any Tree. Denote the tree, obtained from T by considering two copies of T by adding an edge between them, by $T_{(2)}$ and in general the graph obtained from $T_{(n-1)}$ and T by adding an edge between them is denoted by $T_{(n)}$.

**1.4. Result [11] :** For a Tree T, $\gamma(T)$ = n-1 where n is the number of pendent vertices of G.

## 2. PRELIMINARIES

Let G = {V, E} be a graph with p vertices and q edges. A graphoidal cover $\psi$ of G is a collection of (open) paths such that

(i) every edge is in exactly one path of $\psi$

(ii) every vertex is an interval vertex of atmost one path in $\psi$.

We define $\gamma(G) = \min_{\psi \in \zeta} |\psi|$,

where $\zeta$ is the collec tion of graphoidal covers $\psi$ of G and

$\gamma$ is graphoidal covering number of G.

Let $\psi$ be a graphoidal cover of G. Then we say that G admits $\psi$ - magic graphoidal total labeling of G if there exists a bijection f: $V \cup E \rightarrow \{1, 2, \ldots, p+q\}$ such that for every path P $=(v_0 v_1 v_2 \ldots v_n)$ in $\psi$, then, $f^*(P) = f(v_0) + f(v_n) + \sum_1^n f\,(v_{1i-1}v\ )$ = k, a constant, where $f^*$ is the induced labeling of $\psi$. A graph G is called magic graphoidal if there exists a minimum graphoidal cover $\psi$ of G such that G admits $\psi$ - magic graphoidal total labeling.

## 3. Magical Graphoidal on Trees

**3.1. Theorem :** $[P_n ; S_1]$, (n - even) is magic graphoidal.

**Proof:** Let $G = [P_n ; S_1]$

Let $V(G) = \{u_i, v_i, w_i: \ 1 \le i \le n\}$ and

$E(G) = \{ [(u_iv_i) \cup (v_iw_i) : 1 \le i \le n\ ] \ \cup [(u_iu_{i+1}) : 1 \le i \le \ n-1]\}$

Define $f : V \cup E \rightarrow \{1, 2, \ldots, p+q\}$ by

$f(w_1) = 1$

$f(w_1v_1) = 2$

$f(v_1u_1) = 3$

$f(u_{i+1}) = 6 + i \qquad 1 \le i \le n-2$

$f(w_{i+1}) = 6n - i \qquad 1 \le i \le n-1$

$f(v_{i+1}w_{i+1}) = 4n + 1 + i \qquad 1 \le i \le n-1$

$f(u_{i+1}v_{i+1}) = 3n + 3 - 2i \qquad 1 \le i \le (n\backslash 2)-1$

$$f\left(u_{\frac{n}{2}+i}\, v_{\frac{n}{2}+i}\right) = 3n + 4 - 2i \quad 1 \le i \le \frac{n}{2}$$

$$f(u_iu_{i+1}) = \frac{3n}{2} + 4 + i \qquad 1 \le i \le \frac{n}{2} - 1$$

$$f\left(u_{\frac{n}{2}-1+i}\, u_{\frac{n}{2}+i}\right) = n + 4 + i \quad 1 \le i \le \frac{n}{2}$$

Let $\psi = \{P_1 = (w_1,v_1,u_1,u_2,v_2,w_2),\ P_2 = (u_i,u_{i+1},v_{i+1},w_{i+1}) : \ 2 \le i \le n-1\}$

$f^*[P_1] = f(w_1) + f(w_2) + f(w_1v_1) + f(v_1u_1) + f(u_1u_2) + f(u_2v_2) + f(v_2w_2)$

$$= 1 + 6n - 1 + 2 + 3 + \frac{3n}{2} + 4 + 1 \ + 3n + 3 - 2 + 4n + 2$$

$$= 13n + \frac{3n}{2} + 13 \text{ ---------- } \mathbf{(A)}$$

For $2 \le i \le (n\backslash 2)-1$,

$f^*[P_2] = f(u_i) + f(w_{i+1}) + f(u_iu_{i+1}) + f(u_{i+1}v_{i+1}) + f(v_{i+1}w_{i+1})$

$= 6 + i - 1 + 6n - i + (3n\backslash 2) + 4 + i + 3n + 3 - 2i + 4n + 1 + i$

$= 13n + (3n\backslash 2) + 13$ **---------- (B)**

For $(n\backslash 2) \le i \le n-1$,

$f^{*}[P_2] \;=\; f(u_i) + f(w_{i+1}) + f(u_iu_{i+1}) + f(u_{i+1}v_{i+1}) + f(v_{i+1}w_{i+1})$

$= \; 6+i-1+6n-i+n+4+i-(n\backslash 2)+1+3n+4-2(i+1-(n\backslash 2))+4n+1+i$

$= \; 13n+(3n\backslash 2)+13$ **---------- (C)**

From (A), (B) and (C), we conclude that G admits $\psi$ - magic graphoidal total labeling. Hence, $[P_n ; S_1]$, (n - even) is magic graphoidal.
For example, consider the graph $[P_6 ; S_1]$ shown in figure 1.

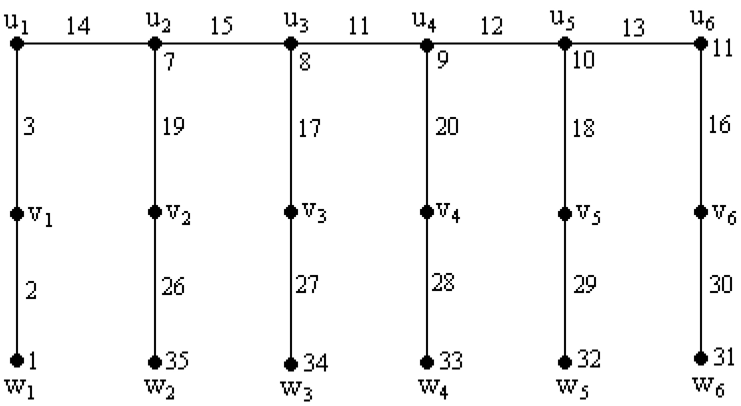


**Figure 1** $[P6 ; S1]$

Clearly, $\psi = \{(w_1,v_1,u_1,u_2,v_2,w_2),\ (u_2,u_3,v_3,w_3),\ (u_3,u_4,v_4,w_4),\ (u_4,u_5,v_5,w_5),\ (u_5,u_6,v_6,w_6)\}$ is a minimum graphoidal cover and $[P_6 ; S_1]$ is magic graphoidal. Here the constant K = 100

**3.2. Theorem :** $[P_n ; S_1]$, (n - odd) is magic graphoidal.

**Proof:** Let $G = [P_n ; S_1]$

Let $V(G) = \{u_i, v_i, w_i: \; 1 \le i \le n\}$ and

$$E(G) = \{ [ (u_iv_i) \cup (v_iw_i) : 1 \le i \le n ] \cup [(u_iu_{i+1}) : 1 \le i \le n-1]\}$$

Define $f : V \cup E \to \{1, 2, \ldots, p+q\}$ by

$f(w_1) = 1$

$f(w_1v_1) = 2$

$f(v_1u_1) = 3$

$f(u_{i+1}) = 5 + 2i \qquad 1 \le i \le (n-1)\backslash 2$

$f(w_{i+1}) = 6n - i \qquad 1 \le i \le n-1$

$f(v_{i+1}w_{i+1}) = 4n + 1 + i \qquad 1 \le i \le n-1$

$f(u_1u_2) = n + 5$

$$f(u_{i+1}u_{i+2}) = \frac{3n+11}{2} - i \qquad 1 \le i \le \frac{n-1}{2}$$

$$f(u_{n+1-i}u_{n-i}) = \frac{3n+9}{2} + i \qquad 1 \le i \le \frac{n-3}{2}$$

$f(u_{i+2}v_{i+2}) = 3n + 2 - i \qquad 1 \le i \le n-2$

$$f(u_2v_2) = \frac{7n+3}{2}$$

Let $\psi = \{P_1 = (w_1,v_1,u_1,u_2,v_2,w_2),\ P_2 = (u_i,u_{i+1},v_{i+1},w_{i+1}) :\ 2 \le i \le n-1\}$

$$f^*[P_1] = f(w_1) + f(w_2) + f(w_1v_1) + f(v_1u_1) + f(u_1u_2) + f(u_2v_2) + f(v_2w_2)$$

$$= 1 + 6n - 1 + 2 + 3 + n + 5 + \frac{7n+3}{2} + 4n + 1 + 1$$

$$= 14n + \frac{n+1}{2} + 13 \text{ ---------- } \mathbf{(A)}$$

For $2 \le i < \dfrac{n+1}{2}$

$$f^*[P_2] = f(u_i) + f(w_{i+1}) + f(u_iu_{i+1}) + f(u_{i+1}v_{i+1}) + f(v_{i+1}w_{i+1})$$

$$= 5 + 2(i-1) + 6n - i + \frac{3n+11}{2} - (i-1) + 3n + 2 - (i-1) + 4n + 1 + i$$

$$= 14n + \frac{n+1}{2} + 13 \text{ ---------- } \mathbf{(B)}$$

For $\dfrac{n+1}{2} \le i \le n-1$,

$$f^*[P_2] = f(u_i) + f(w_{i+1}) + f(u_iu_{i+1}) + f(u_{i+1}v_{i+1}) + f(v_{i+1}w_{i+1})$$

$$= 6 + 2\left(i - \frac{n+1}{2}\right) + 6n - i + \frac{3n+9}{2} + (n-i) + 3n + 2 - (i-1) + 4n + 1 + i$$

$$= 14n + \frac{n+1}{2} + 13 \text{ ---------- } \mathbf{(C)}$$

From (A), (B) and (C), we conclude that $\psi$ is minimum magic graphoidal cover
Hence, $[P_n ; S_1]$, (n - even) is magic graphoidal.
For example, consider the graph $[P_7 ; S_1]$ shown in figure 2.

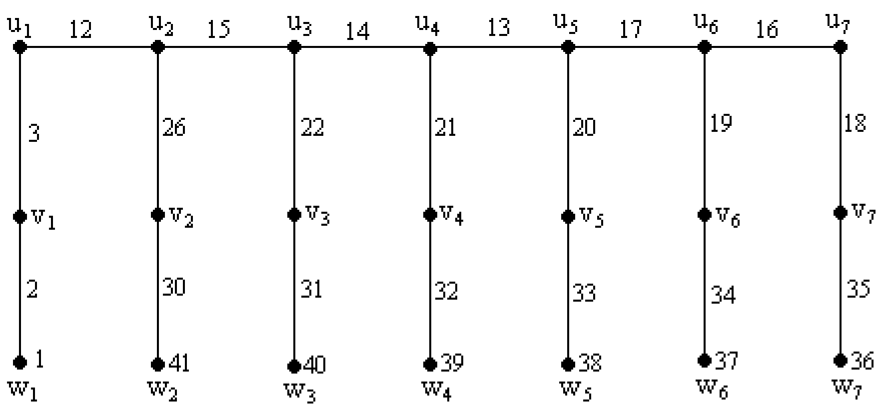


**Figure 2** $[P_7 ; S_1]$

Clearly, $\psi = \{(w_1,v_1,u_1,u_2,v_2,w_2),\ (u_2,u_3,v_3,w_3),\ (u_3,u_4,v_4,w_4),\ (u_4,u_5,v_5,w_5),\ (u_5,u_6,v_6,w_6),\ (u_6,u_7,v_7,w_7)\}$ is a minimum graphoidal cover and $[P_7 ; S_1]$ is magic graphoidal. Here the constant $K = 115$.

**3.3. Theorem :** $[P_n ; S_2]$ is magic graphoidal.

**Proof:** Let $G = [P_n ; S_2]$

$V(G) = \{(u_i, v_i) :\ 1 \le i \le n\}$

$E(G) = \{ [ (u_iu_{i+1}) : 1 \le i \le n\text{-}1] \cup [(u_iv_i) : 1 \le i \le n]$
$\cup [(v_iv_{i1}) \cup (v_iv_{i2}) : 1 \le i \le n] \}$

Define $f : V \cup E \to \{1, 2, \dots, p+q\}$ by

$f(v_1) = 1$
$f(u_1v_1) = 2$ f
$(u_1) = 3$
$f(u_{i+1}) = 3 + i \qquad 1 \le i \le n\text{-}2$
$f(u_n) = p + q$
$f(v_{i1}) = n + 1 + i \qquad 1 \le i \le n$
$f(v_{i+1}) = 8n - 1 - i \qquad 1 \le i \le n\text{-}1$
$f(v_{i2}) = 7n - i \qquad 1 \le i \le n$
$f(v_iv_{i2}) = 5n - 1 + i \qquad 1 \le i \le n$
$f(u_{i+1}v_{i+1}) = 4n + i \qquad 1 \le i \le n - 1$
$f(u_iu_{i+1}) = 4n + 1 - i \qquad 1 \le i \le n - 1$
$f(v_iv_{i1}) = 3n + 2 - i \qquad 1 \le i \le n$

Let $\psi = \{P_1 = [(v_{i1}, v_i, v_{i2}) : 1 \le i \le n], P_2 = (v_1, u_1, u_2, v_2), P_3 = [(u_i, u_{i+1}, v_{i+1}) : 2 \le i \le n-1]\}$

For $1 \le i \le n$,

$f^*[P_1] = f(v_{i1}) + f(v_{i2}) + f(v_{i1}v_i) + f(v_iv_{i2})$

$= n + 1 + i + 7n - i + 3n + 2 - i + 5n - 1 + i$

$= 16n + 2$ **---------- (A)**

$f^*[P_2] = f(v_1) + f(v_2) + f(v_1u_1) + f(u_1u_2) + f(u_2v_2)$

$= 1 + 8n - 2 + 2 + 4n + 4n + 1$

$= 16\,n + 2$ **---------- (B)**

For $2 \le i \le n-1$,

$f^*[P_3] = f(u_i) + f(v_{i+1}) + f(u_iu_{i+1}) + f(u_{i+1}v_{i+1})$

$= 3 + i - 1 + 8n - 1 - i + 4n + 1 - i + 4n + i$

$= 16\,n + 2$ **---------- (C)**

From (A), (B) and (C), we conclude that G admits $\psi$ - magic graphoidal total labeling. Hence, $[P_n ; S_2]$ is magic graphoidal.

For example, consider the graphs $[P_3 ; S_2]$ and $[P_4 ; S_2]$ shown in figure 3.1 and 3.2.

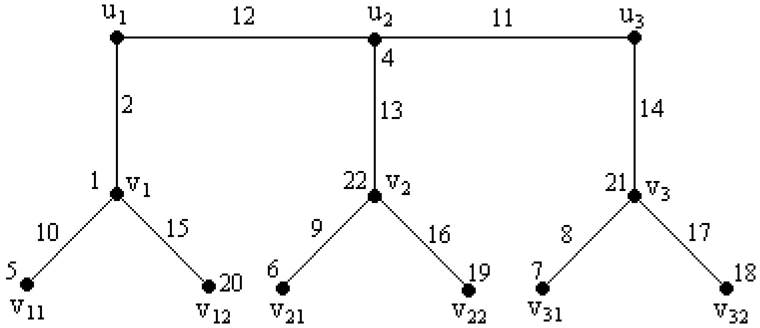


**Figure 3.1** $[P_3 ; S_2]$

Clearly, $\psi = \{(v_{11}, v_1, v_{12}), (v_{21}, v_2, v_{22}), (v_{31}, v_3, v_{32}), (v_1, u_1, u_2, v_2), (u_2, u_3, v_3)\}$ is a minimum graphoidal cover and $[P_3 ; S_2]$ is magic graphoidal. Here the constant K = 50

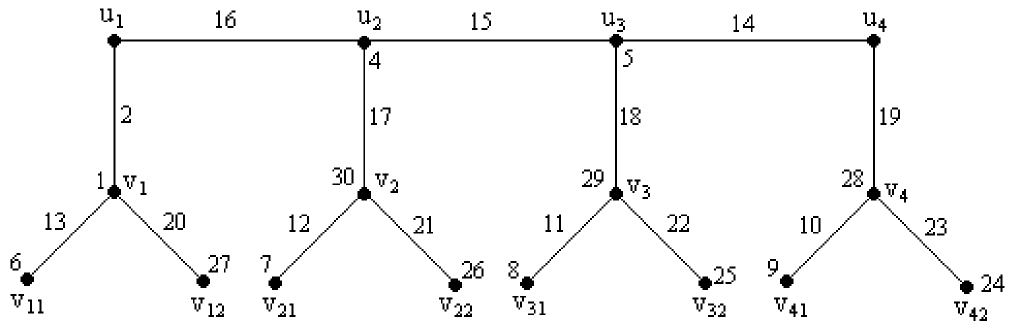


**Figure 3.2** [$P_4$ ; $S_2$]

Clearly, $\psi = \{(v_{11},v_1,v_{12}), (v_{21},v_2,v_{22}), (v_{31},v_3,v_{32}), (v_{41},v_4,v_{42}), (v_1,u_1,u_2,v_2), (u_2,u_3,v_3), (u_3,u_4,v_4)\}$ is a minimum graphoidal cover and [$P_4$ ; $S_2$] is magic graphoidal. Here the constant K = 66

**3.4. Theorem :** For a Tree, $T_{(n)}$ is magic graphoidal.

**Proof :**

Let $T_{(n)}$ be a graph such that

$V[T_{(n)}] = \{u_{i1}, u_{i2}, u_{i3}, u_{i4}, u_{i5} : 1 \le i \le n\}$ and

$E[T_{(n)}] = \{[(u_{i1}u_{i2}), (u_{i2}u_{i3}), (u_{i3}u_{i4}), (u_{i4}u_{i5}): 1 \le i \le n] \cup [(u_{in}u_{i+1n}) : 1 \le i \le n-1)]\}$

Define $f : V \cup E \to \{1, 2, \ldots, p+q\}$ by

| | | |
|---|---|---|
| $f(u_{i1})$ | $= i$ | $1 \le i \le n$ |
| $f(u_{i3})$ | $= n + 1$ | |
| $f(u_{i3}u_{i5})$ | $= n + 2$ | |
| $f(u_{i1}u_{i2})$ | $= 2n + 3 - i$ | $1 \le i \le n$ |
| $f(u_{i+2,5}u_{i+3,5})$ | $= 5n - 3 - 2i$ | $1 \le i \le n - 3$ |
| $f(u_{n+1-i,5}u_{n-i,5})$ | $= 4n - 1 + i$ | $n - 2 \le i \le n - 1$ |
| $f(u_{n+1-i,2}u_{n+1-i,3})$ | $= 5n - 2 + i$ | $1 \le i \le n$ |
| $f(u_{n+1-i,3}u_{n+1-i,4})$ | $= 6n - 2 + i$ | $1 \le i \le n$ |
| $f(u_{n+1-i,3}u_{n+1-i,5})$ | $= 7n - 2 + i$ | $1 \le i \le n - 1$ |
| $f(u_{i4})$ | $= 8n - 2 + 2(i-1)$ | $1 \le i \le n$ |
| $f(u_{i+1,3})$ | $= 8n - 1 + 2(i-1)$ | $1 \le i \le n - 1$ |

Let $\psi = \{P_1 = (u_{i1},u_{i2},u_{i3},u_{i4}), P_2 = (u_{i3},u_{i5},u_{25},u_{23}), P_3 = [(u_{i5},u_{i+15}, u_{i+13}) : 2 \le i \le n - 1]\}$

$f^*[P_1] = f(u_{i1}) + f(u_{i4}) + f(u_{i1}u_{12}) + f(u_{i2}u_{i3}) + f(u_{i3}u_{i4})$

$= i+8n-2+2(i-1)+2n+3-i+5n-2 + (n + 1- i ) + 6n - 2 + (n + 1- i )$

$= 23n - 3$ **---------- (A)**

$f^*[P_2] = f(u_{i5}) + f(u_{i+1,3}) + f(u_5\, u_{i+1,3}) + f(u_{i+1,5}\, u_{i+1,3})$

$= 2n+2+i-1+8n - 1 + 2( i - 1) + 5n - 3 - 2 (i - 2) + 7n-2+ n - i$

$= 23n - 3$ **---------- (B)**

$f^*[P_3] = f(u_{i3}) + f(u_{23}) + f(u_{i3}u_{15}) + f(u_{15}u_{25}) + f(u_{25}u_{23})$

$= n + 1 + 8n - 1 + 2(i - 1) + 5n - 3 - 2(i - 2) + 7n - 2 + n - i$

$= 23n - 3$ **---------- (C)**

From (A), (B) and (C), we conclude that G admits $\psi$ - magic graphoidal total labeling. Hence, $T_{(n)}$ is magic graphoidal.

For example, consider the graph $T_{(3)}$ shown in figure 4.

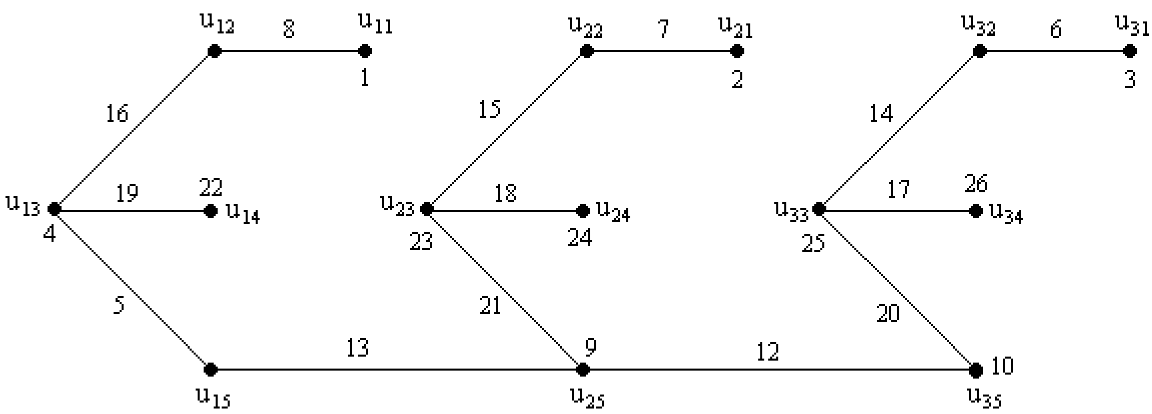


**Figure 4** $T_{(3)}$

Clearly, $\psi = \{(u_{11},u_{12},u_{13},u_{14}),\ (u_{21},u_{22},u_{23},u_{24}),\ (u_{31},u_{32},u_{33},u_{34}),\ (u_{13},u_{15},u_{25},u_{23}),\ (u_{25},u_{35},u_{33})\}$ is a minimum graphoidal cover and $T_{(3)}$ is magic graphoidal. Here the constant K = 66

**3.5. Theorem :** The graph Double Crowned Star $K_{1,n} \odot K_1$ is magic graphoidal.

**Proof :** Let $G = K_{1,n} \odot 2K_1$

$V(G) = \{u, [u_i : 1 \le i \le n], [(u_{i1},u_{i2}): 1 \le i \le n]\}$ and

$E(G) = \{ [(uu_i): 1 \le i \le n] \cup [ (u_iu_{i1}) \cup (u_iu_{i2}) : 1 \le i \le n ] \}$

Define $f : V \cup E \to \{1, 2, \ldots, p+q\}$ by

| | | |
|---|---|---|
| $f(u)$ | $= 2n + 3$ | |
| $f(u_{i1})$ | $= i$ | $1 \le i \le n$ |
| $f(u_1)$ | $= n + 1$ | |
| $f(u_2)$ | $= n + 2$ | |
| $f(u_{n+1-i,2})$ | $= n + 2 + i$ | $1 \le i \le n$ |
| $f(u_iu_{i,1})$ | $= 2n + 3 + i$ | $1 \le i \le n$ |
| $f(uu_1)$ | $= 3n + 4$ | |
| $f(u_{2+i})$ | $= 3n + 4 + i$ | $1 \le i \le n-2$ |
| $f(uu_{n+1-i})$ | $= 4n + 2 + i$ | $1 \le i \le n-1$ |
| $f(u_{n+1-i}\, u_{n+1-i,2})$ | $= 5n + 1 + i$ | $1 \le i \le n$ |

Let $\psi = \{P_1 = (u_1,u,u_2),\ P_2 = [(u,u_i) : 3 \le i \le n],\ P_3 = [(u_{i1},u_i,u_{i2}) : 1 \le i \le n]\}$

$f^*[P_1] = f(u_1) + f(u_2) + f(u_1u) + f(uu_2)$

$= n + 1 + n + 2 + 3n + 4 + 5n + 1$

$= 10n + 8$ **---------- (A)**

For $3 \le i \le n$,

$f^*[P_2] = f(u) + f(u_i) + f(uu_i)$

$= 2n + 3 + 3n + 4 + i - 2 + 4n + 2 + n + 1 - i$

$= 10n + 8$ **---------- (B)**

For $1 \le i \le n$,

$$f^*[P_3] = f(u_{i1}) + f(u_{i2}) + f(u_{i1}u_i) + f(u_iu_{i2})$$
$$= i + n + 2 + n + 1 - i + 2n + 3 + i + 5n + 1 + n + 1 - i$$
$$= 10n + 8 \text{ ---------- } \textbf{(C)}$$

From (A), (B) and (C), we conclude that G admits $\psi$ - magic graphoidal total labeling. Hence, Double Crowned Star $K_{1,n} \odot 2K_1$ is magic graphoidal.
For example, consider the graph $K_{1,5} \odot 2K_1$ shown in figure 5.

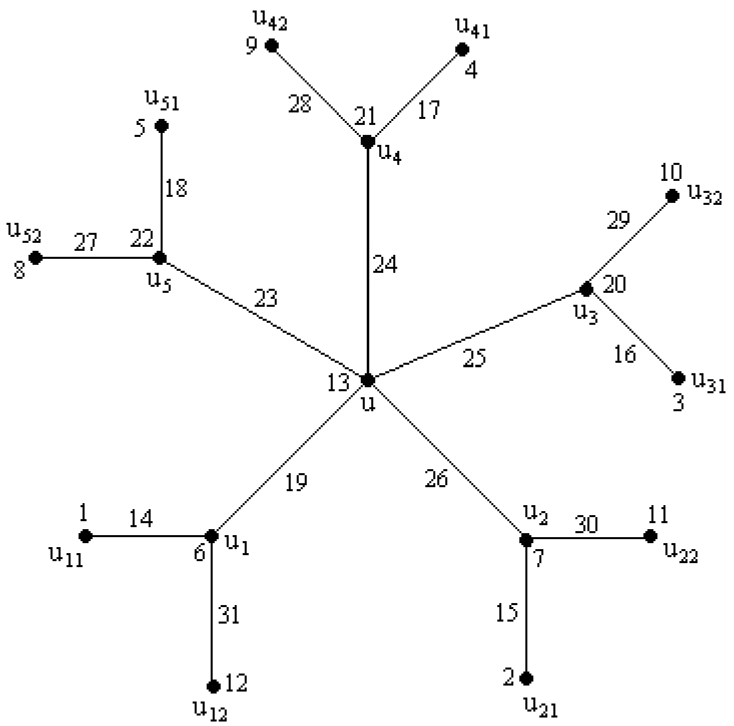


**Figure 5. $K_{1,5} \odot 2K_1$**

Clearly, $\psi$ = {$(u_{11},u_1,u_{12})$, $(u_{21},u_2,u_{22})$, $(u_{31},u_3,u_{32})$, $(u_{41},u_4,u_{42})$, $(u_{51},u_5,u_{52})$, $(u_1,u,v,u_2)$, $(u,u_3)$, $(u,u_4)$, $(u,u_5)$} is a minimum graphoidal cover and $K_{1,5} \odot 2K_1$ is magic graphoidal. Here the constant K = 58.

**3.6. Theorem :** $P_m \widetilde{\odot} 2K_1$ is magical graphoidal.

**Proof :** Let $G = P_m \widetilde{\odot} 2K_1$

$V(G) = \{ (u_i : 1 \le i \le m),\ (u_{ij} : 1 \le i \le m,\ 1 \le j \le 2) \}$ and
$E(G) = \{ [(u_i\, u_{i+1}) : 1 \le i \le m\text{-}1] \cup [(u_i\, u_{ij}) : 1 \le i \le m,\ 1 \le j \le 2] \}$

Let $\psi = \{P_1 = [(u_{i1},u_i,u_{i2}) : 1 \le i \le m],\ P_2 = [(u_i,u_{i+1}) : 1 \le i \le m\text{-}1]\}$
Define $f : V \to \{0, 1, 2, \ldots, 6m\text{-}1\}$ by

$f(u_i\, u_{i1}) = i \qquad 1 \le i \le m$
$f(u_i\, u_{i2}) = 2m+1-i \qquad 1 \le i \le m$

$$f(u_{2i}) = \begin{cases} 2m+i & 1 \le i \le \dfrac{m}{2} \quad \text{if } m \text{ is even} \\[2ex] 2m+i & 1 \le i \le \dfrac{m-1}{2} \quad \text{if } m \text{ is odd} \end{cases}$$

$$f(u_{i1}) = \begin{cases} \frac{5m}{2}+i & 1 \le i \le m \text{ if } m \text{ is even} \\ \frac{5m-1}{2}+i & 1 \le i \le m \text{ if } m \text{ is odd} \end{cases}$$

$$f(u_{2i-1}) = \begin{cases} \frac{7m}{2}+i & 1 \le i \le \frac{m}{2} & \text{if } m \text{ is even} \\ \frac{7m-1}{2}+i & 1 \le i \le \frac{m+1}{2} & \text{if } m \text{ is odd} \end{cases}$$

$$f(u_{m+1-i}\, u_{m-i}) = 4m + i \qquad 1 \le i \le m-1$$
$$f(u_{m+1-i,\,2}) = 5m - 1 + i \qquad 1 \le i \le m$$

**Case (i) :** when m is odd

For $1 \le i \le m$,

$$f^*[P_1] = f(u_{i1}) + f(u_{i2}) + f(u_{i1}u_i) + f(u_iu_{i2})$$
$$= \frac{5m-1}{2} + i + 5m - 1 + m + 1 - i + i + 2m + 1 - i$$
$$= \frac{21m+1}{2} \text{ --------- } \mathbf{(A)}$$

For $1 \le i \le m-1$, $i \equiv 1 \bmod 2$

$$f^*[P_2] = f(u_i) + f(u_{i+1}) + f(u_iu_{i+1})$$
$$= \frac{7m-1}{2} + \frac{i+1}{2} + 2m + \frac{i+1}{2} + 4m + m - i$$
$$= \frac{21m+1}{2} \text{ --------- } \mathbf{(B)}$$

For $1 \le i \le m-1$, $i \equiv 0 \bmod 2$

$$f^*[P_2] = f(u_i) + f(u_{i+1}) + f(u_iu_{i+1})$$
$$= 2m + \frac{i}{2} + \frac{7m-1}{2} + \frac{i+2}{2} + 4m + m - i$$
$$= \frac{21m+1}{2} \text{ --------- } \mathbf{(C)}$$

From (A), (B) and (C), we conclude that G admits $\psi$ - magic graphoidal total labeling. Hence, $P_m \odot 2K_1$ (m-odd) is magic graphoidal.

For example, consider the graph $P_5 \odot 2K_1$ shown in figure 6.1.

**Case (ii) :** when m is even

For $1 \le i \le m$,

$$f^*[P_1] = f(u_{i1}) + f(u_{i2}) + f(u_{i1}u_i) + f(u_iu_{i2})$$

$$\frac{5m}{2} =+ i + 5m - 1 + m + 1 - i + i + 2m + 1 - i$$

$$= \frac{21m+2}{2} \text{ --------- (A)}$$

For $1 \leq i \leq m-1$, $i \equiv 1 \bmod 2$

$f^*[P_2] = f(u_i) + f(u_{i+1}) + f(u_iu_{i+1})$

$$= \frac{7m}{2} + \frac{i+1}{2} + 2m + \frac{i+1}{2} + 4m + m - i$$

$$= \frac{21m+2}{2} \text{ --------- (B)}$$

For $1 \leq i \leq m-1$, $i \equiv 0 \bmod 2$

$f^*[P_2] = f(u_i) + f(u_{i+1}) + f(u_iu_{i+1})$

$$= 2m + \frac{i}{2} + \frac{7m}{2} + \frac{i+2}{2} + 4m + m - i$$

$$= \frac{21m+2}{2} \text{ --------- (C)}$$

From (A), (B) and (C), we conclude that G admits $\psi$ - magic graphoidal total labeling. Hence, $P_m \circledcirc 2K_1$ (m-even) is magic graphoidal.

For example, consider the graph $P_4 \circledcirc 2K_1$ shown in figure 6.2.

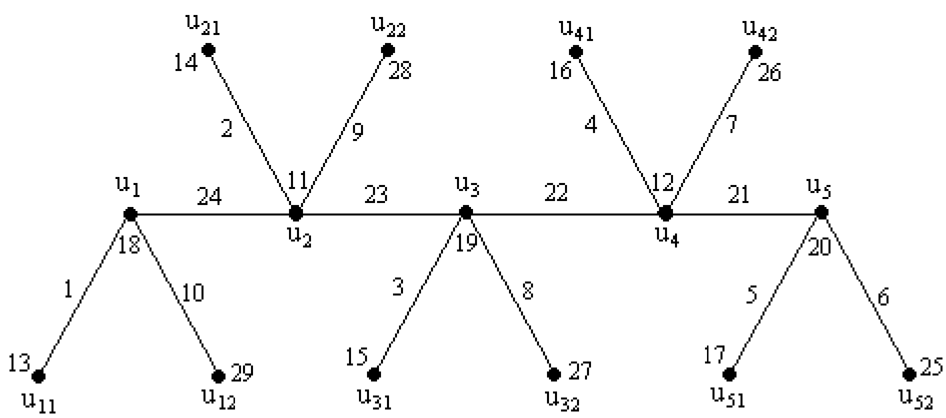


**Figure 6.1 $P_5 \circledcirc 2K_1$**

Clearly, $\psi = \{(u_{11},u_1,u_{12}), (u_{21},u_2,u_{22}), (u_{31},u_3,u_{32}), (u_{41},u_4,u_{42}), (u_{51},u_5,u_{52}), (u_1,u_2), (u_2,u_3), (u_3,u_4), (u_4,u_5)\}$ is a minimum graphoidal cover and $P_4 \circledcirc 2K_1$ is magic graphoidal. Here the constant $K = 53$.

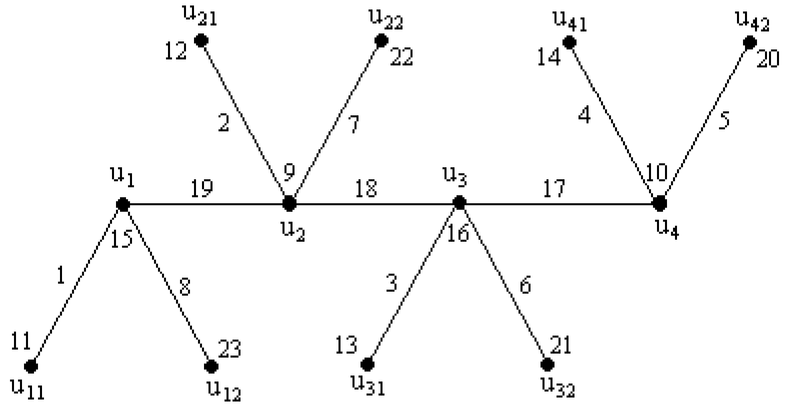


**Figure 6.2 $P_4 \widetilde{\odot} 2K_1$**

Clearly, $\psi = \{(u_{11},u_1,u_{12}),(u_{21},u_2,u_{22}),(u_{31},u_3,u_{32}),(u_{41},u_4,u_{42}),(u_1,u_2),(u_2,u_3),(u_3,u_4)\}$ is a minimum graphoidal cover and $P_4 \widetilde{\odot} 2K_1$ is magic graphoidal. Here the constant K = 43.

**3.7. Theorem :** $P_m \widetilde{\odot} K_{1,3}$ is magic graphoidal.

**Proof :** Let $G = P_m \widetilde{\odot} K_{1,3}$

$V(G) = \{[(u_iv_i): 1 \le i \le m], [(v_{ij}): 1 \le i \le m, 1 \le i \le 3]\}$

$E(G) = \{ [(u_iu_{i+1}): 1 \le i \le m\text{-}1] \cup [ (v_iv_{ij}) : 1 \le i \le m, 1 \le j \le 3 ] \}$

with $u_i = u_{i3}$, $1 \le i \le m$

Let $\psi = \{P_1= [(v_{i1},v_i,v_{i2}) : 1 \le i \le m], P_2=[(v_i,u_i,u_{i+1}): 1\le i \le m–2],$

$P_3=(v_{m-1},u_{m-1},u_m,v_m)\}$

Define $f : V \cup E \to \{1, 2, …., 8m–1\}$ by

| | | |
|---|---|---|
| $f(v_m)$ | $= 1$ | |
| $f(v_iv_{i1})$ | $= i+1$ | $1 \le i \le m$ |
| $f(u_iv_i)$ | $= m+1+ i$ | $1 \le i \le m–1$ |
| $f(u_{m+1-i}\, u_{m-i})$ | $= 2m + i$ | $1 \le i \le m –1$ |
| $f(v_{m+1-i}\, v_{m+1-i,2})$ | $= 3m–1+ i$ | $1 \le i \le m$ |
| $f(v_{i1})$ | $= 4m – 1 + i$ | $1 \le i \le m$ |
| $f(v_i)$ | $= 5m – 1 + i$ | $1 \le i \le m–1$ |
| $f(u_mv_m)$ | $= 6m – 1$ | |
| $f(u_{m-i})$ | $= 6m + i$ | $1 \le i \le m–2$ |
| $f(v_{m+1-i, 2})$ | $= 7m –2 + i$ | $1 \le i \le m$ |

For $1 \le i \le m$,

$f^*[P_1] = f(v_{i1}) + f(v_{i2}) + f(v_{i1}v_i) + f(v_iv_{i2})$

$= 4m – 1+ i + 7m – 2 + m + 1 – i + i + 1 + 3m – 1 + m + 1 – i$

$= 16m – 1$ **--------- (A)**

For $1 \le i \le m – 2$,

$f^*[P_2] = f(v_i) + f(u_{i+1}) + f(v_iu_i) + f(u_iu_{i+1})$

$= 5m – 1 + i+ 6m + (m – i – 1) + m + 1 + i + 2m + m – i$

$= 16m – 1$ **--------- (B)**

$f^*[P_3] = f(v_{m-1}) + f(v_m) + f(v_{m-1}u_{m-1}) + f(u_{m-1}u_m) + f(u_m v_m)$

$= 6m - 2 + 1 + 2m + 2m + 1 + 6m - 1$

$= 16m - 1$ **--------- (C)**

From (A), (B) and (C), we conclude that G admits $\psi$ - magic graphoidal total labeling. Hence, $P_m \odot K_{1,3}$ is magic graphoidal.

For example, consider the graph $P_4 \odot K_{1,3}$ shown in figure 7.

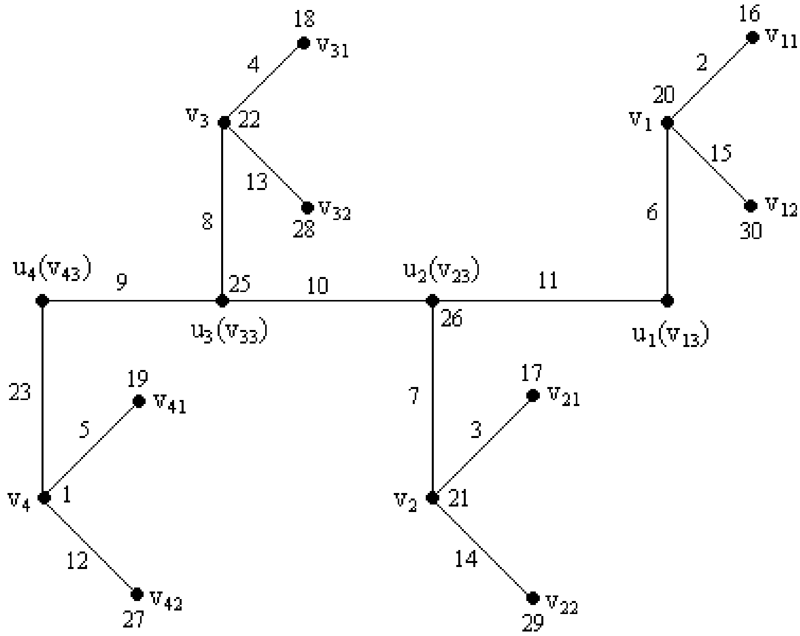


**Figure 7. $P_4 \odot K_{1,3}$**

Clearly, $\psi = \{(v_{11},v_1,v_{12}), (v_{21},v_2,v_{22}), (v_{31},v_3,v_{32}), (v_{41},v_4,v_{42}), (v_1,u_1,u_2), (v_1,u_2,u_3), (v_3,u_3,u_4,v_4)\}$ is magic graphoidal. Here the constant K = 63.

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