Static Deformation Due to a Long Tensile Fault of Finite Width in an Isotropic Half-Space Welded with an Orthotropic Half-Space

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Abstract: Closed-form analytical expressions for displacements and stresses at any point of a two-phase medium consisting of a homogeneous, isotropic, perfectly elastic half-space in welded contact with a homogeneous, orthotropic, perfectly elastic half-space caused by a tensile fault of finite width located at an arbitrary distance from the interface in the isotropic half-space are obtained. The Airy stress function approach is used to obtain the expressions for the stresses and displacements. The vertical tensile fault is considered graphically. The variations of the displacements with the distance from the fault and with depth for various cases have been studied graphically. Also horizontal and vertical displacement of the surface are presented graphically.

1. INTRODUCTION

Dislocation theory is very useful to determine the static changes that accompany faulting within the earth, which has been discussed by Steketee (1958a, 1958b)[1,2]. As a mathematical model of a fault he uses a displacement dislocation surface, i.e., a surface across which there is a discontinuity in the displacement vector. Using results of Steketee, M.A. Chinnery (1961) [3] deduce the results for deformation of ground around surface faults. The calculation of the deformation caused by shear and tensile faults is necessary for the investigation of seismic and volcanic sources. The solution of the two-dimensional problem of a long inclined shear fault in two welded half-spaces is well known. Tensile fault representation has several very important geophysical applications, such as modelling of the deformation field due to a dyke injection in the volcanic region, mine collapse and fluid driven crack. Recent studies have shown that a large number of earthquake sources cannot be represented by the double-couple source mechanism which models a shear fault. According to Sipkin (1986)[4], the non double-couple mechanism might be due to tensile failure under high fluid pressure.

Maruyama (1964)[5] calculated the Green's functions for two-dimensional elastic dislocations in a semi-infinite medium and obtained surface displacements due to vertical and horizontal rectangular tensile faults in a semi-infinite Poisson solid. Freund and Barnett (1976)[6] developed a 2-D model of dip-slip faulting in a uniform half-space, using the theory of analytic functions of a complex variable and obtained the relationship between fault slip and surface deformation. Davis (1983)[7] modeled the crustal deformation associated with hydro fracture by a dipping rectangular tensile fault beneath the surface of an elastic half-space.

Singh and Garg (1986)[8] obtained the integral expressions for the Airy stress function in an unbounded medium due to various two-dimensional seismic sources. Beginning with these expressions, Rani et al. (1991)[9] obtained the integral expressions for the Airy stress function, displacements and stresses in a homogeneous, isotropic, perfectly elastic half-space due to various two-dimensional sources by applying the traction-free boundary conditions at the surface of the half-space. The integrals were then evaluated analytically, obtaining closed-form expressions for the Airy stress function, the displacements and the stresses at any point of the half-space caused by two-dimensional buried sources. Wu and Chou (1982)[10] applied the generalized method of images to obtain the elastic field of an in-plane line force acting in a two phase orthotropic medium. Singh (1986)[11], and Pan (1989a)[12] studied the static deformation of a transversely isotropic

multilayered half-space by surface loads. The problem of the static deformation of a transversely isotropic multilayered half-space by buried sources has been discussed by Pan (1989b)[13]. Static deformation of an orthotropic multilayered elastic half-space by two-dimensional surface loads has been investigated by Garg et al. (1991)[14].

Singh et al. (1991)[15] followed a similar approach to obtain closed-form analytic expressions for the displacements and the stresses at any point of either of two homogeneous, isotropic, perfectly elastic half-spaces in welded contact due to two-dimensional sources. Kumar et al. (2005)[16] obtained closed-form analytical expressions for the Airy Stress function for a tensile line source in two-welded half-spaces which are integrated analytically to derive the Airy stress function for a tensile fault of finite width.

In the present paper, our aim is to study the two-dimensional deformation of an isotropic half-space in welded contact with an orthotropic half-space due to a long inclined tensile fault of finite width. Beginning with the closed-form expression for the Airy stress function for an arbitrary line source in isotropic half-space in welded contact with an orthotropic half-space given by Singh, J. and Rani, S. (1991)[17] and following Singh and Singh (2000)[18], we obtained Airy stress function for a long tensile fault of arbitrary dip and finite width by analytic integration over the width of the fault. The expressions for the stresses and displacements at any point of the half-space caused by a long vertical tensile fault follow immediately. Two orthotropic materials namely Topaz and Barytes have been considered for numerical computations in case of vertical tensile fault. Numerical results show that the effect of anisotropy on the displacement field is more pronounced when the observer is in the orthotropic half-space.

2. THEORY

Let the Cartesian co-ordinates be denoted by (x_1, x_2, x_3) with x_3 -axis vertically upwards. Consider two homogeneous, perfectly elastic half-spaces which are welded along the plane (x_1, x_2, x_3) . The upper half-space ($x_3 > 0$) is assumed to be isotropic with stress-strain relation

$$p_{ij} = 2\mu \left[e_{ij} + \frac{\sigma}{1-2\sigma} \delta_{ij} e_{kk} \right], \quad (i, j = 1, 2, 3) \quad (1)$$

where, p_{ij} are the components of stress tensor, e_{ij} are the components of strain tensor, μ is the shear modulus and σ is Poisson's ratio. The lower half-space ($x_3 < 0$) is assumed to be orthotropic with stress-strain relation.

$$\begin{bmatrix} p_{11} \\ p_{22} \\ p_{33} \\ p_{23} \\ p_{31} \\ p_{12} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{13} & c_{23} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix} \begin{bmatrix} e'_{11} \\ e'_{22} \\ e'_{33} \\ 2e'_{23} \\ 2e'_{31} \\ 2e'_{12} \end{bmatrix} \quad (2)$$

We consider a two dimensional approximation in which displacement component u_1, u_2, u_3 are independent of x_1 so that $\partial/\partial x_1 = 0$. Under this assumption the plane strain problem ($u_1 = 0$) and anti-strain problem ($u_2 = 0$ and $u_3 = 0$) are decoupled and therefore, can be solved separately. The plane strain problem for an isotropic medium can be solved in terms of Airy stress function U such that

$$p_{22} = \partial^2 U / \partial x_3^2, \quad p_{33} = \partial^2 U / \partial x_2^2, \quad p_{23} = -\partial^2 U / \partial x_2 \partial x_3 \quad (3)$$

$$\nabla^2 \nabla^2 U = 0 \quad (4)$$

The plane strain problem for an orthotropic medium can be solved in terms of the Airy Stress function U^* such that Garg et al (1991)

$$p'_{22} = \partial^2 U^* / \partial x_3^2, \quad p'_{33} = \partial^2 U^* / \partial x_2^2, \quad p'_{23} = -\partial^2 U^* / \partial x_2 \partial x_3, \quad (5)$$

$$\left(a^2 \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2}\right) \left(b^2 \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2}\right) U^* = 0 \quad (6)$$

$$a^2 + b^2 = \frac{(c_{22}c_{33} - c_{23}^2 - 2c_{23}c_{44})}{c_{33}c_{44}}, \quad a^2b^2 = \frac{c_{22}}{c_{33}} \quad (7)$$

For an isotropic medium,

$$\begin{aligned} c_{11} &= c_{22} = c_{33} = \frac{2\mu(1-\sigma)}{1-2\sigma} \\ c_{12} &= c_{13} = c_{23} = \frac{2\mu\sigma}{1-2\sigma} \\ c_{44} &= c_{55} = c_{66} = \mu \end{aligned} \quad (8)$$

This yields $a^2 = b^2 = 1$ and equation (6) reduces to equation (4).

As given by Rani et al. (1991), we have the Airy stress function for an arbitrary line source parallel to the x_1 -axis and passing through the point (y_2, y_3) located in the isotropic half-space welded with the orthotropic half-space.

For the isotropic half-space,

$$\begin{aligned} U = & L_0 \tan^{-1} \left(\frac{x_2 - y_2}{|x_3 - y_3|} \right) + M_0 \frac{(x_2 - y_2)|x_3 - y_3|}{R^2} - P_0 \ln R + Q_0 \frac{(x_3 - y_3)^2}{R^2} \\ & + L^- \left\{ X_1 \tan^{-1} \left(\frac{x_2 - y_2}{x_3 + y_3} \right) + \frac{2X_2(x_2 - y_2)x_3}{S^2} \right\} \\ & + M^- \left\{ 2(D - C) \tan^{-1} \left(\frac{x_2 - y_2}{x_3 + y_3} \right) + \frac{(X_1 y_3 + X_2 x_3)(x_2 - y_2)}{S^2} + \frac{4X_2 y_3 (x_2 - y_2)x_3(x_3 + y_3)}{S^4} \right\} \\ & + P^- \left\{ -X_1 \ln S + \frac{2X_2 x_3 (x_3 + y_3)}{S^2} \right\} \\ & + Q^- \left\{ -2(D - C) \ln S + \frac{(X_1 y_3 + X_2 x_3)(x_3 + y_3) - 2X_2 x_3 y_3}{S^2} + \frac{4X_2 y_3 x_3 (x_3 + y_3)^2}{S^4} \right\} \end{aligned} \quad (9)$$

and for orthotropic half-space,

$$\begin{aligned} U^* = & 2L^- \left\{ A \tan^{-1} \left(\frac{x_2 - y_2}{y_3 - ax_3} \right) + B \tan^{-1} \left(\frac{x_2 - y_2}{y_3 - bx_3} \right) \right\} \\ & + 2M^- \left\{ -C \tan^{-1} \left(\frac{x_2 - y_2}{y_3 - ax_3} \right) + D \tan^{-1} \left(\frac{x_2 - y_2}{y_3 - bx_3} \right) + (x_2 - y_2)y_3 \left(\frac{A}{T^2} + \frac{B}{H^2} \right) \right\} \\ & - 2P^- \{ A \ln T + B \ln H \} \\ & + 2Q^- \left\{ C \ln T + \frac{Ay_3(y_3 - ax_3)}{T^2} - D \ln H + \frac{By_3(y_3 - bx_3)}{H^2} \right\} \end{aligned} \quad (10)$$

Where, $(x_2, x_3) =$ receiver location,

$$\begin{aligned} R^2 &= (x_2 - y_2)^2 + (x_3 - y_3)^2 & S^2 &= (x_2 - y_2)^2 + (x_3 + y_3)^2 \\ T^2 &= (x_2 - y_2)^2 + (y_3 - ax_3)^2 & H^2 &= (x_2 - y_2)^2 + (y_3 - bx_3)^2 \\ x_3 &\neq y_3, \quad x_2 \neq y_2, \quad ax_3 \neq y_3, \quad bx_3 \neq y_3, \\ X_1 &= 2(A + B) - 1, & X_2 &= A(1 + a) + B(1 + b) - 1, \\ X_3 &= 2D(1 + b) - 2C(1 + a) + 1, \\ A &= \frac{\alpha(1 - b + 2\mu s_2 - 2\mu r_2)}{W}, & B &= \frac{\alpha(a - 1 + 2\mu r_1 - 2\mu s_1)}{W} \\ C &= \frac{(1 + b - ba + 2\mu as_2)}{W}, & D &= \frac{(1 + a - aa + 2\mu as_1)}{W} \\ r_1 &= \frac{c_{33}a^2 + c_{23}}{\Delta}, & r_2 &= \frac{c_{33}b^2 + c_{23}}{\Delta}, \\ s_1 &= \frac{c_{23}a^2 + c_{22}}{a\Delta}, & s_2 &= \frac{c_{23}b^2 + c_{22}}{b\Delta}, \quad \Delta = (c_{22}c_{33} - c_{23}^2), \\ W &= (1 + a - \alpha + 2\mu ar_1)(1 + b - ba + 2\mu as_2) \\ &\quad - (1 + a - aa + 2\mu as_1)(1 + b - \alpha + 2\mu ar_2), \end{aligned} \quad \begin{matrix} L_0, M_0, P_0, Q_0 \\ x_3 < y_3 \end{matrix} \quad (11)$$

And in equations (9), (10), L_0, M_0, P_0, Q_0 are the source coefficients and L^-, M^-, P^-, Q^- are the values of these source coefficients valid for $x_3 < y_3$. Singh and Garg (1986) and Singh and Rani (1991) have given these source coefficients for various seismic sources. For ready reference, the relevant source coefficients are given in Appendix. Let U_i denote the Airy stress function at an arbitrary point P (x_2, x_3) for a unit concentrated force acting at the point Q (y_2, y_3) in the x_i -direction. Then, the Airy stress function for a long fault can be expressed as a line integral (Maruyama, 1964).

$$U = \int_0^L \Delta u_i U_{ij} n_j ds \quad (12)$$

where, the summation convention has been used (the suffices can assume the values 2 and 3 only). In equation (12), Δu_i is the displacement dislocation vector; n_j is the unit normal to the fault; and

$$U_{ij} = U_{ji} = \lambda \delta_{ij} \frac{\partial}{\partial y_k} U_k + \mu \left(\frac{\partial}{\partial y_i} U_j + \frac{\partial}{\partial y_j} U_i \right)$$

For a tensile fault, vector Δu_i is parallel to the normal n_i to the fault. Therefore, if b' is the magnitude of Δu_i is δ is dip angle (figure 1(a)), we have

$$\Delta u_2 = -b' \sin \delta, \quad \Delta u_3 = b' \cos \delta \quad (13)$$

$$n_2 = -\sin \delta, \quad n_3 = \cos \delta, \quad (14)$$

Using equations (13) and (14) in equation (12), we get the following expression for the Airy stress function for a line source.

$$U = b' ds [U_{22} \sin^2 \delta - U_{23} \sin 2\delta + U_{33} \cos^2 \delta] \quad (15)$$

ds is the width of the line fault. Therefore, the Airy stress function for a long tensile fault of arbitrary dip can be expressed as a linear combination of

- i) $b' ds U_{22}$, the Airy stress function for a vertical tensile fault ($\delta = 90^\circ$) with dislocation in the x_2 -direction;
- ii) $b' ds U_{33}$, the Airy stress function for a horizontal tensile fault ($\delta = 0^\circ$) with dislocation in the x_3 -direction;
- iii) $b' ds U_{23}$, the Airy stress function for a vertical dip-slip fault.

Using the values of the source coefficients L_0, M_0, P_0, Q_0 , L^-, M^-, P^-, Q^- given in Appendix, equations (9), (10), (15) yield the Airy stress function due to a long tensile line source of arbitrary dip parallel to x_1 -axis and acting at the point (y_2, y_3) located in the isotropic half-space welded with the orthotropic half-space in the form

For isotropic half-space

$$U = \frac{\alpha \mu b' ds}{\pi} \left[-\ln R - X_1 \ln S + \frac{2X_2 x_3 (x_3 + y_3)}{S^2} + \cos 2\delta \left\{ \frac{(x_3 - y_3)^2}{R^2} - 2(D - C) \ln S + \frac{(X_1 y_3 + X_3 x_3)(x_3 + y_3) - 2X_2 x_3 y_3}{S^2} + \frac{4X_2 x_3 y_3 (x_3 + y_3)^2}{S^4} \right\} - \sin 2\delta \left\{ \frac{(x_2 - y_2)(x_3 - y_3)}{R^2} - \frac{(X_1 y_3 + X_3 x_3)(x_2 - y_2)}{S^2} - 2(D - C) \tan^{-1} \left(\frac{x_2 - y_2}{x_3 + y_3} \right) - \frac{4X_2 x_3 y_3 (x_2 - y_2)(x_3 + y_3)}{S^4} \right\} \right] \quad (16)$$

and for the orthotropic half space

$$U^* = \frac{2\alpha \mu b' ds}{\pi} \left[-(A \ln T + B \ln H) + \cos 2\delta \left\{ C \ln T - D \ln H + \frac{A y_3 (y_3 - a x_3)}{T^2} + \frac{B y_3 (y_3 - b x_3)}{H^2} \right\} - \sin 2\delta \left\{ C \tan^{-1} \left(\frac{x_2 - y_2}{y_3 - a x_3} \right) - D \tan^{-1} \left(\frac{x_2 - y_2}{y_3 - b x_3} \right) - (x_2 - y_2) y_3 \left(\frac{A}{T^2} + \frac{B}{H^2} \right) \right\} \right] \quad (17)$$

where, $\alpha = \frac{1}{2(1 - \sigma)}$,

Now, using the polar coordinates (s, δ) , see figure1(a)

$$y_2 = s \cos \delta,$$

$$y_3 = d + s \sin \delta$$

and s (L), we will obtain the following expressions for the Airy stress function for a long tensile fault of width L and infinite length with lower edge of the fault at distance d from the interface:

$$U = \frac{\alpha \mu b'}{\pi} \left[\{(1 + X_1) \cos^2 \delta + 2(D - C) \cos 2\delta\} s + (x_2 \cos \delta + X \sin \delta - s) \ln R \right. \\ + \{2(D - C)(x_2 \cos \delta + X' \sin \delta - s \cos 2\delta) + X_1(x_2 \cos \delta - d \sin \delta - s) - X_2 x_3 \sin \delta\} \ln S \\ + 2(D - C) s \sin 2\delta \tan^{-1} \left(\frac{x_2 - s \cos \delta}{X' + s \sin \delta} \right) - 2X_2 x_3 (x_2 \cos \delta - X' \sin \delta - s)(d + s \sin \delta) \frac{1}{S^2} \\ + \{x_3 \cos \delta (-X_1 + X_2 + 2X_2)\} \\ \left. - 2(D - C)(-x_2 \sin \delta + X' \cos \delta) \tan^{-1} \left(\frac{s - x_2 \cos \delta + X' \sin \delta}{x_2 \sin \delta + X' \cos \delta} \right) \right] \Big|_0^L \quad (18)$$

$$U^* = \frac{2\alpha \mu b'}{\pi} \left[\{(D - C) \cos 2\delta + (A + B) \cos^2 \delta\} s \right. \\ + \{A(x_2 \cos \delta - d \sin \delta - s) - C(x_2 \cos \delta + Y \sin \delta - s \cos 2\delta)\} \ln T \\ + \{B(x_2 \cos \delta - d \sin \delta - s) + D(x_2 \cos \delta + Y' \sin \delta - s \cos 2\delta)\} \ln H \\ - Cs \sin 2\delta \tan^{-1} \left(\frac{x_2 - s \cos \delta}{Y + s \sin \delta} \right) \\ + Ds \sin 2\delta \tan^{-1} \left(\frac{x_2 - s \cos \delta}{Y' + s \sin \delta} \right) \\ + \{Aax_3 \cos \delta - C(x_2 \sin \delta - Y \cos \delta)\} \tan^{-1} \left(\frac{s - x_2 \cos \delta + Y \sin \delta}{x_2 \sin \delta + Y \cos \delta} \right) \\ \left. + \{Bbx_3 \cos \delta + D(x_2 \sin \delta - Y' \cos \delta)\} \tan^{-1} \left(\frac{s - x_2 \cos \delta + Y' \sin \delta}{x_2 \sin \delta + Y' \cos \delta} \right) \right] \Big|_0^L \quad (19)$$

Where now,

$$R^2 = (x_2 - s \cos \delta)^2 + (X - s \sin \delta)^2, \quad S^2 = (x_2 - s \cos \delta)^2 + (X' + s \sin \delta)^2,$$

$$T^2 = (x_2 - s \cos \delta)^2 + (Y + s \sin \delta)^2,$$

$$H^2 = (x_2 - s \cos \delta)^2 + (Y' + s \sin \delta)^2,$$

$$X = x_2 - d, \quad X' = x_2 + d, \quad Y = d - ax_3, \quad Y' = d - bx_3, \quad f(s) \Big|_0^L = f(L) - f(0)$$

(a)

Figure 1 Geometry of a tensile fault of width L having lower edge at a distance d from the interface in the isotropic half-space in welded contact with an orthotropic half-space (a) δ is dip angle and s is the distance from the lower edge of the fault, measured in dip direction; (b) $\delta = 90^\circ$.

3. STRESSES

Using equations (3) and (18), we will obtain the following expressions stresses for a long tensile fault of width L and infinite length with lower edge of the fault at distance d from the interface as for isotropic half-space,

$$\begin{aligned}
 p_{22} = \frac{\alpha \mu b'}{\pi} & \left[(x_2 \cos \delta + 3X \sin \delta + s \cos 2\delta) \frac{1}{R^2} - 2(x_2 \cos \delta + X \sin \delta - s)(X - s \sin \delta)^2 \frac{1}{R^4} \right. \\
 & + \{2(X_3 - D + C)(x_2 \cos \delta - X' \sin \delta - s) + (4X_2 - X_1)(x_2 \cos \delta + d \sin \delta - s \cos 2\delta) \\
 & - X_3 x_2 \sin \delta\} \frac{1}{S^2} \\
 & - 2\{X_1(x_2 \cos \delta - X' \sin \delta - s)(X' + s \sin \delta)^2 + 2X_2 x_3^2 \cos \delta (x_2 - s \cos \delta) \\
 & - (X_1 - X_2)x_3(X' + s \sin \delta)(x_2 \cos \delta - X' \sin \delta - s) \\
 & - 2X_2(d + s \sin \delta)(X' + s \sin \delta)[2(x_2 \cos \delta - X' \sin \delta - s) + 3x_3 \sin \delta]\} \frac{1}{S^4} \\
 & \left. - 16X_2 x_3(x_2 \cos \delta - X' \sin \delta - s)(d + s \sin \delta)(X' + s \sin \delta)^2 \frac{1}{S^6} \right] \Bigg|_0^L
 \end{aligned} \tag{20}$$

$$\begin{aligned}
 p_{23} = \frac{\alpha \mu b'}{\pi} & \left[-(x_2 \sin \delta - X \cos \delta) \frac{1}{R^2} + 2(X - s \sin \delta)^2(x_2 \sin \delta - X \cos \delta) \frac{1}{R^4} \right. \\
 & + \{-2(D - C)(x_2 \sin \delta + X' \cos \delta) + (-X_1 + 4X_2 + X_3)x_3 \cos \delta + X_3(x_2 \sin \delta + X' \cos \delta)\} \frac{1}{S^2} \\
 & + \{2x_3(X_1 - 2X_2)(X' + s \sin \delta)^2 \cos \delta \\
 & - 2X_1(X' + s \sin \delta)[X' \cos \delta(X' + s \sin \delta) + d \sin \delta(x_2 - s \cos \delta) + s \cos \delta(x_2 \sin \delta + X' \cos \delta)] \\
 & + 4X_2(d + s \sin \delta)[(x_3 + X' + s \sin \delta)(x_2 \sin \delta + X' \cos \delta) + 2x_3 \cos \delta(X' + s \sin \delta)] \\
 & - 2X_3 x_3(X' + s \sin \delta)(x_2 \sin \delta + X' \cos \delta)\} \frac{1}{S^4} \\
 & \left. - 16X_2 x_3(d + s \sin \delta)(X' + s \sin \delta)^2(x_2 \sin \delta + X' \cos \delta) \frac{1}{S^6} \right] \Bigg|_0^L
 \end{aligned} \tag{21}$$

$$\begin{aligned}
p_{33} = \frac{\alpha\mu b'}{\pi} & \left[(x_2 \cos \delta - X \sin \delta - s \cos 2\delta) \frac{1}{R^2} \right. \\
& + 2(x_2 \cos \delta + X \sin \delta - s)(X - s \sin \delta)^2 \frac{1}{R^4} \\
& + \{2(D - C)(x_2 \cos \delta - X' \sin \delta - s) + X_1(x_2 \cos \delta + d \sin \delta - s \cos 2\delta) + X_2 x_3 \sin \delta\} \frac{1}{S^2} \\
& + \{2X_1(X' + s \sin \delta)[(X' + s \sin \delta)(x_2 \cos \delta - d \sin \delta - s) - x_3 \cos \delta (x_2 - s \cos \delta)] \\
& + 4X_2 x_3 [x_3 \cos \delta (x_2 - s \cos \delta) + 3 \sin \delta (X' + s \sin \delta)(d + s \sin \delta)] \\
& + 2X_3 x_3 (X' + s \sin \delta)(x_2 \cos \delta - X' \sin \delta - s)\} \frac{1}{S^4} \\
& \left. + 16X_2 x_3 (x_2 \cos \delta - X' \sin \delta - s)(d + s \sin \delta)(X' + s \sin \delta)^2 \right] \Big|_0^L
\end{aligned} \quad (22)$$

Using equations (5) and (19), we will obtain the following expressions stresses for a long tensile fault of width Land infimite length with lower edge of the fault at distance d from the interface as for orthotropic half-space,

$$\begin{aligned}
p'_{22} = \frac{2\alpha\mu b'}{\pi} & \left[a^2 \{A(s \cos 2\delta - x_2 \cos \delta - d \sin \delta) + C(x_2 \cos \delta - Y \sin \delta - s)\} \frac{1}{T^2} \right. \\
& + b^2 \{B(s \cos 2\delta - x_2 \cos \delta - d \sin \delta) - D(x_2 \cos \delta - Y' \sin \delta - s)\} \frac{1}{H^2} \\
& - 2a^2 A(d + s \sin \delta)(x_2 \cos \delta - Y \sin \delta - s)(Y + s \sin \delta) \frac{1}{T^4} \\
& \left. - 2b^2 B(d + s \sin \delta)(x_2 \cos \delta - Y' \sin \delta - s)(Y' + s \sin \delta) \frac{1}{H^4} \right] \Big|_0^L
\end{aligned} \quad (23)$$

$$\begin{aligned}
p'_{23} = \frac{2\alpha\mu b'}{\pi} & \left[-a \{C(x_2 \sin \delta + Y \cos \delta) + aAx_3 \cos \delta\} \frac{1}{T^2} + b \{D(x_2 \sin \delta + Y' \cos \delta) - bBx_3 \cos \delta\} \frac{1}{H^2} \right. \\
& - 2aA(d + s \sin \delta)(-x_2 \sin \delta - Y \cos \delta)(Y + s \sin \delta) \frac{1}{T^4} \\
& \left. - 2bB(d + s \sin \delta)(-x_2 \sin \delta - Y' \cos \delta)(Y' + s \sin \delta) \frac{1}{H^4} \right] \Big|_0^L
\end{aligned} \quad (24)$$

$$\begin{aligned}
p'_{33} = \frac{2\alpha\mu b'}{\pi} & \left[\{A(x_2 \cos \delta + d \sin \delta - s \cos 2\delta) - C(x_2 \cos \delta - Y \sin \delta - s)\} \frac{1}{T^2} \right. \\
& + \{B(x_2 \cos \delta + d \sin \delta - s \cos 2\delta) + D(x_2 \cos \delta - Y' \sin \delta - s)\} \frac{1}{H^2} \\
& + 2A(d + s \sin \delta)(x_2 \cos \delta - Y \sin \delta - s)(Y + s \sin \delta) \frac{1}{T^4} \\
& \left. + 2B(d + s \sin \delta)(x_2 \cos \delta - Y' \sin \delta - s)(Y' + s \sin \delta) \frac{1}{H^4} \right] \Big|_0^L
\end{aligned} \quad (25)$$

4. DISPLACEMENTS

The displacements, for the isotropic half-space, are given by the expressions (Singh and Rani (1991))

$$2\mu u_2 = -\frac{\partial U}{\partial x_2} + \frac{1}{2\alpha} \int (p_{22} + p_{33}) dx_2, \quad (26a)$$

$$2\mu u_3 = -\frac{\partial U}{\partial x_3} + \frac{1}{2\alpha} \int (p_{22} + p_{33}) dx_3, \quad (26b)$$

The displacements, for the orthotropic half-space, are given by the expressions (Singh and Rani (1991))

$$u'_2 = \frac{1}{\Delta} \int (c_{33} p'_{22} - c_{23} p'_{33}) dx_2, \quad (27b)$$

$$u'_3 = \frac{1}{\Delta} \int (c_{22} p'_{33} - c_{23} p'_{22}) dx_3, \quad (27b)$$

$$\text{where, } \Delta = c_{22}c_{33} - c_{23}^2$$

Using equations (20), (22), (26a), and (26b), we will obtain the following expressions stresses for a long tensile fault of width L and infinite length with lower edge of the fault at distance d from the interface as for isotropic half-space,

$$\begin{aligned} u_2 = \frac{\alpha b'}{2\pi} & \left[\left(\frac{1}{\alpha} - 1 \right) \cos \delta \ln R - \left\{ 2(D - C) + X_1 - \frac{(X_2 + 2X_3)}{\alpha} \right\} \cos \delta \ln S \right. \\ & + \frac{\sin \delta}{\alpha} \left\{ \tan^{-1} \left(\frac{x_2 - s \cos \delta}{X - s \sin \delta} \right) - X_3 \tan^{-1} \left(\frac{x_2 - s \cos \delta}{X' + s \sin \delta} \right) \right\} \\ & - 2(D - C) \sin \delta \tan^{-1} \left(\frac{s - x_2 \cos \delta + X' \sin \delta}{x_2 \sin \delta + X' \cos \delta} \right) - (X - s \sin \delta)(x_2 \sin \delta - X \cos \delta) \frac{1}{R^2} \\ & + \left\{ \left(X_1 - \frac{2X_2}{\alpha} \right) (d + s \sin \delta)(x_2 \sin \delta + X' \cos \delta) + x_3 [2X_2 x_3 \cos \delta + X_3 (x_2 \sin \delta + X' \cos \delta)] \right\} \frac{1}{S^2} \\ & \left. + 4X_2 x_3 (x_2 \sin \delta + X' \cos \delta)(d + s \sin \delta)(X' + s \sin \delta) \frac{1}{S^4} \right] \Bigg|_0^L \end{aligned} \quad (28)$$

$$\begin{aligned} u_3 = \frac{\alpha b'}{2\pi} & \left[\left(\frac{1}{\alpha} - 1 \right) \sin \delta \ln R - \left\{ 2(D - C) + X_2 \left(\frac{1}{\alpha} - 1 \right) \right\} \sin \delta \ln S \right. \\ & + \frac{\cos \delta}{\alpha} \left\{ \tan^{-1} \left(\frac{X - s \sin \delta}{x_2 - s \cos \delta} \right) + (2X_2 + X_3) \tan^{-1} \left(\frac{X' + s \sin \delta}{x_2 - s \cos \delta} \right) \right\} \\ & + \{ 2(D - C) + X_1 - 2X_2 - X_3 \} \cos \delta \tan^{-1} \left(\frac{s - x_2 \cos \delta + X' \sin \delta}{x_2 \sin \delta + X' \cos \delta} \right) \\ & - (X - s \sin \delta)(x_2 \cos \delta + X \sin \delta - s) \frac{1}{R^2} \\ & + \left\{ \left(-X_1 + 2X_2 - \frac{2X_3}{\alpha} \right) (d + s \sin \delta)(x_2 \cos \delta - X' \sin \delta - s) \right. \\ & + 2X_2 x_3 (s \cos 2\delta - x_2 \cos \delta - d \sin \delta) - X_3 x_3 ((x_2 \cos \delta - X' \sin \delta - s)) \left. \right\} \frac{1}{S^2} \\ & \left. - 4X_2 x_3 (x_2 \cos \delta - X' \sin \delta - s)(d + s \sin \delta)(X' + s \sin \delta) \frac{1}{S^4} \right] \Bigg|_0^L \end{aligned} \quad (29)$$

Using equations (23), (25), (27a), and (27b), we will obtain the following expressions stresses for a long tensile fault of width L and infinite length with lower edge of the fault at distance d from the interface as for orthotropic half-space,

$$\begin{aligned} u'_2 = -\frac{2\alpha\mu b'}{\pi} & \left[(A - C)r_1 \cos \delta \ln T + (B + D)r_2 \cos \delta \ln H \right. \\ & - Ar_1 (d + s \sin \delta)(x_2 \sin \delta + Y \cos \delta) \frac{1}{T^2} - Br_2 (d + s \sin \delta)(x_2 \sin \delta + Y' \cos \delta) \frac{1}{H^2} \\ & \left. + Cr_1 \sin \delta \tan^{-1} \left(\frac{x_2 - s \cos \delta}{Y + s \sin \delta} \right) - Dr_2 \sin \delta \tan^{-1} \left(\frac{x_2 - s \cos \delta}{Y' + s \sin \delta} \right) \right] \Bigg|_0^L \end{aligned} \quad (30)$$

$$\begin{aligned} u'_3 = \frac{2\alpha\mu b'}{\pi} & \left[-Cs_1 \sin \delta \ln T + Ds_2 \sin \delta \ln H + As_1 (d + s \sin \delta)(x_2 \cos \delta - Y \sin \delta - s) \frac{1}{T^2} \right. \\ & + Bs_2 (d + s \sin \delta)(x_2 \cos \delta - Y' \sin \delta - s) \frac{1}{H^2} \\ & \left. - (A - C)s_1 \cos \delta \tan^{-1} \left(\frac{Y + s \sin \delta}{x_2 - s \cos \delta} \right) - (B + D)s_2 \cos \delta \tan^{-1} \left(\frac{Y' + s \sin \delta}{x_2 - s \cos \delta} \right) \right] \Bigg|_0^L \end{aligned} \quad (31)$$

5. NUMERICAL RESULTS AND DISCUSSIONS

We compare the displacement field due to a long vertical tensile fault of width L its edge at the distance d from the interface located in the isotropic half-space welded with orthotropic half-space along the horizontal plane with the corresponding displacement field when both the half-spaces are isotropic. Therefore, we take $\delta = 90^\circ$ (figure 1b). We assume the isotropic half-space to be poissonian so that $\sigma = 0.25$. For the orthotropic half-space, we use the values of elastic constants given by Love (1944). For Topaz,

$$\begin{aligned} c_{11} &= 2870, & c_{22} &= 3560, & c_{33} &= 3000, \\ c_{12} &= 1280, & c_{23} &= 900, & c_{13} &= 860, \\ c_{44} &= 1100, & c_{55} &= 1350, & c_{66} &= 1330, \end{aligned}$$

In terms of a unit of $10^6 \text{ grammes wt/cm}^2$, this yields $a = 1.2992$ and $b = 0.8385$.

For Barytes,

$$\begin{aligned} c_{11} &= 907, & c_{22} &= 800, & c_{33} &= 1074, \\ c_{12} &= 468, & c_{23} &= 273, & c_{13} &= 275, \\ c_{44} &= 122, & c_{55} &= 293, & c_{66} &= 283, \end{aligned}$$

In terms of a unit of $10^6 \text{ grammes wt/cm}^2$, this yields $a = 2.3118$ and $b = 0.3735$.

When the lower half-space is also isotropic,

$$\begin{aligned} c_{11} &= c_{22} = c_{33} = \frac{2\mu'(1-\sigma')}{1-2\sigma'} \\ c_{12} &= c_{13} = c_{23} = \frac{2\mu'\sigma'}{1-2\sigma'} \\ c_{44} &= c_{55} = c_{66} = \mu' \end{aligned}$$

We take $\sigma' = 0.25$, $c_{44}/\mu =$ and 0.5 for numerical computations.

Figure 2 Variation of horizontal displacement (u'_1/b') with distance from the fault (x_2/L) due to interface breaking tensile fault ($d = 0$) for (a) $x_3 = 0$, (b) $x_3 = -L/10$ and (c) $x_3 = -L/2$.

Figure 2a-c display horizontal displacement (u'_2/b') with distance from the fault (x_2/L) due to interface breaking tensile fault ($d = 0$). The horizontal displacement is anti-symmetric about the line $x_2 = 0$. In figure 2a, the observer is at the interface, the horizontal displacement attains maximum value at origin. In figure 2b- c, the observer is in the orthotropic half-space at $x_3 = -L/10$, $x_3 = -L/2$ respectively. The horizontal displacement for Barytes varies more significantly in magnitude rather than Topaz from the corresponding one for the isotropic case. It is observed that magnitude of horizontal displacement decreases when distance from the fault in orthotropic half-space increases.

Figure 3a-c display horizontal displacement (u'_2/b') with distance from the fault (x_2/L) located at a distance $d = L/2$ from the interface for (a) $x_3 = 0$, (b) $x_3 = -L/10$ and (c) $x_3 = -L/2$.

In figure 3a, the observer is at the interface but In figure 3b-c, the observer is in the orthotropic half-space at $x_3 = -L/10$, $x_3 = -L/2$ respectively. We observe that magnitude of the displacement for Barytes is affected significantly, but the pattern is not affected much. It is also observed that the magnitude of horizontal displacement is maximum at the interface.

Figure 4 Variation of the horizontal displacement (u'_1/b') with depth from the fault (x_2/L) at $x_2 = L/10$ at a distance $d = -L$ for different values of ratio of rigidities $\frac{c_{44}}{\mu} = 0.1, 0.5, 2$ and 10 taking the orthotropic material as (a)Topaz, (b) Barytes. The magnitude of displacement is affected.

Figures 4a-c show the variation of the horizontal displacement with depth (x_2/L) for four values of rigidities $\frac{c_{44}}{\mu} = 0.1, 0.5, 2$ and 10 for Topaz and Barytes respectively due to tensile fault located at a distance $d = -L$ from the interface. For all values of ratio of rigidities, the horizontal displacement attains maximum value at the lower end of the fault and minimum value at the upper end of the fault. It is observed that with increase in the value of ratio of rigidities, there is decrease in horizontal displacement. Also, for the same value of ratio of rigidities horizontal displacement for Topaz is more than barytes in magnitude.

(c)

Figure 5 Variation of vertical displacement (u'_3/b') with distance from the fault (x_2/L) due to interface breaking tensile fault ($d=0$) for (a) $x_3 = 0$, (b) $x_3 = -L/10$ and (c) $x_3 = -L/2$.

Figure 5a-c display vertical displacement (u'_3/b') with distance from the fault (x_2/L) due to interface breaking tensile fault ($d = 0$). The vertical displacement is symmetric about the line $x_2 = 0$. In figure 5a, the observer is at the interface, the vertical displacement attains minimum value at origin. In figure 5b-c, the observer is in the orthotropic half-space at $x_3 = -L/10$, $x_3 = -L/2$ respectively. The vertical displacement for Barytes varies more significantly in magnitude rather than Topaz from the corresponding one for the isotropic case.

(a)

(b)

(c)

Figure 6 Variation of vertical displacement (u'_3/b') with distance from the fault (x_2/L) due to a long vertical tensile fault at a distance $d = L/2$, for (a) $x_3 = 0$, (b) $x_3 = -L/10$ and (c) $x_3 = -L/2$.

Figure 6a-c display vertical displacement (u'_3/b') with distance from the fault (x_2/L) located at a distance $d = L/2$ from the interface. In figure 6a, the observer is at the interface but In figure 2b-c, the observer is in the orthotropic half-space at $x_3 = -L/10$, $x_3 = -L/2$ respectively. We observe that magnitude of the displacement for Barytes is affected significantly, but the pattern is not affected much. It is also observed that the magnitude of vertical displacement is maximum at the interface.

Figure 7 Variation of the vertical displacement (u'_3/b') with depth from the fault (x_2/L) at $x_2 = L/2$ at a distance $d = -L/2$ for different values of ratio of rigidities $\frac{c_{44}}{\mu} = 0.1, 0.5, 2$ and 10 taking the orthotropic material as (a) Topaz (b) Barytes. The magnitude of displacement is affected.

Figures 7a-c show the variation of the vertical displacement with depth (x_3/L) for four values of rigidities $\frac{c_{44}}{\mu} = 0.1, 0.5, 2$ and 10 for Topaz and Barytes respectively due to tensile fault located at a distance $d = -L/2$ from the interface. For all values of ratio of rigidities, the vertical displacement attains maximum value at the lower end of the fault and minimum value at the upper end of the fault. It is observed that with increase in the value of ratio of rigidities, there is decrease in vertical displacement. Also, for the same value of ratio of rigidities vertical displacement for Topaz is more than barytes in magnitude.

Now, we also considering surface plots for horizontal and vertical displacement of surface due to a long vertical tensile fault of width L .

Figure 8 Horizontal displacement of the surface for Topaz due to vertical tensile fault located at (a) interface breaking fault (b) $d = L/2$.

Figure 9 Horizontal displacement of the surface for Barytes due to vertical tensile fault located at (a) interface breaking fault (b) $d = L/2$.

Figure 10 Horizontal displacement of the surface for Isotropic half-space due to vertical tensile fault located at (a) interface breaking fault (b) $d = L/2$

Figure 11 Vertical displacement of the surface for Topaz due to vertical tensile fault located at (a) interface breaking fault (b) $d = L/2$.

Figure 12 Horizontal displacement of the surface for Topaz due to vertical tensile fault located at (a) interface breaking fault (b) $d = L/2$.

Figure 13 Horizontal displacement of the surface for Topaz due to vertical tensile fault located at (a) interface breaking fault (b) $d = L/2$.

From all figures, we observe that horizontal and vertical displacement of the surface at the interface breaking fault is more pronounced than when tensile fault is at depth $d = L/2$.

6. CONCLUSION

We have obtained the stresses and displacements for a long tensile fault of finite width L located in the isotropic half-space overlying the orthotropic half-space. The results obtained here satisfy the necessary continuity conditions

$$\begin{aligned} p_{23} &= p'_{23}, & p_{33} &= p'_{33} \\ u_2 &= u'_2, & u_3 &= u'_3 \end{aligned} \quad (32)$$

at $x_3 = 0$ for the two half-spaces to be in welded contact along the plane $x_3 = 0$. Moreover, when the orthotropic half-space is replaced by isotropic one, the results of the present paper, in the limit, coincide with the corresponding results of Kumar et al. (2005) for two half-spaces to be in welded contact. Numerical results presented the variation of horizontal displacement, vertical displacement with distance and depth from the fault for different distances of fault from the interface. Also displacements of the surface presented graphically. Numerical computations indicate that the deformation field due to a source in an isotropic half-space in welded contact with an anisotropic half-space may differ substantially from the deformation field when both the half-spaces are isotropic.

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