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Exploring Prime Numbers and Modular Functions II: On the Prime Number via Elliptic Integral Function

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ABSTRACT. The main goal this paper is to develop an asymptotic formula for the prime number, using elliptic integral function.

1. INTRODUCTION

As consequence of the prime number theorem, I put the asymptotic formula for the *n*th prime number, denoted by p_n:

$$p_n \sim n \ln n. \tag{1}$$

In this paper, I prove that

$$p_n \sim 2nK\left(\frac{256n-4}{256n+4}\right) - \frac{\ln 2}{8}(64n+1) - \frac{\ln n}{64}.$$

2. THEOREM

THEOREM 1. *I have*

$$p_n \sim 2nK\left(\frac{256n-4}{256n+4}\right) - \frac{\ln 2}{8}(64n+1) - \frac{\ln n}{64},$$

where p_n *denotes the n*th prime number and $K(x)$ denotes the complete elliptic integral of first kind.

Proof. In [1], we encounter the identity

$$\operatorname{agm}(x, y) = \frac{\pi}{4} \cdot \frac{x+y}{K\left(\frac{x-y}{x+y}\right)}. \tag{2}$$

In [2], we encounter an alternative for extremely high precision calculation is the formula

$$\ln n \approx \frac{\pi}{2\operatorname{agm}(1, 4/s)} - m \ln 2,$$

where agm denotes the arithmetic-geometric mean of 1 and $4/s$, and

$$s = n \cdot 2^m > 2^{p/2},$$

with m chosen so that p bits of precision is attained. Now, the value of 8 for m is sufficient. Hence,

$$\ln n \approx \frac{\pi}{2\operatorname{agm}(1, 4/256n)} - 8 \ln 2. \tag{3}$$

Substituting (2) into (3), I get around

$$\ln n \approx \frac{512nK\left(\frac{256n-4}{256n+4}\right)}{256n+4} - 8 \ln 2 = \frac{128nK\left(\frac{256n-4}{256n+4}\right) - 8 \ln 2(64n+1)}{64n+1}. \tag{4}$$

Wherefore,

$$(64n+1)\ln n \approx 128nK\left(\frac{256n-4}{256n+4}\right) - 8\ln 2\,(64n+1),$$

$$n\ln n \approx \frac{128}{64}nK\left(\frac{256n-4}{256n+4}\right) - \frac{8}{64}\ln 2\,(64n+1) - \frac{\ln n}{64},$$

$$n\ln n \approx 2nK\left(\frac{256n-4}{256n+4}\right) - \frac{\ln 2}{8}(64n+1) - \frac{\ln n}{64}. \tag{5}$$

I take (5) in (1), and achieve

$$p_n \sim 2nK\left(\frac{256n-4}{256n+4}\right) - \frac{\ln 2}{8}(64n+1) - \frac{\ln n}{64}.$$

This completes the proof. □

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