

## Exploring Prime Numbers and Modular Functions II: On the Prime Number via Elliptic Integral Function

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**ABSTRACT.** The main goal this paper is to develop an asymptotic formula for the prime number, using elliptic integral function.

### 1. INTRODUCTION

As consequence of the prime number theorem, I put the asymptotic formula for the  $n$ th prime number, denoted by  $p_n$ :

$$p_n \sim n \ln n. \quad (1)$$

In this paper, I prove that

$$p_n \sim 2nK \left( \frac{256n-4}{256n+4} \right) - \frac{\ln 2}{8} (64n+1) - \frac{\ln n}{64}.$$

### 2. THEOREM

THEOREM 1. *I have*

$$p_n \sim 2nK \left( \frac{256n-4}{256n+4} \right) - \frac{\ln 2}{8} (64n+1) - \frac{\ln n}{64},$$

where  $p_n$  denotes the  $n$ th prime number and  $K(x)$  denotes the complete elliptic integral of first kind.

*Proof.* In [1], we encounter the identity

$$\operatorname{agm}(x, y) = \frac{\pi}{4} \cdot \frac{x+y}{K\left(\frac{x-y}{x+y}\right)}. \quad (2)$$

In [2], we encounter an alternative for extremely high precision calculation is the formula

$$\ln n \approx \frac{\pi}{2\operatorname{agm}(1, 4/s)} - m \ln 2,$$

where  $\operatorname{agm}$  denotes the arithmetic-geometric mean of 1 and  $4/s$ , and

$$s = n \cdot 2^m > 2^{p/2},$$

with  $m$  chosen so that  $p$  bits of precision is attained. Now, the value of 8 for  $m$  is sufficient. Hence,

$$\ln n \approx \frac{\pi}{2\operatorname{agm}(1, 4/256n)} - 8 \ln 2. \quad (3)$$

Substituting (2) into (3), I get around

$$\ln n \approx \frac{512nK\left(\frac{256n-4}{256n+4}\right)}{256n+4} - 8 \ln 2 = \frac{128nK\left(\frac{256n-4}{256n+4}\right) - 8 \ln 2(64n+1)}{64n+1}. \quad (4)$$

Wherefore,

$$\begin{aligned}
 (64n + 1) \ln n &\approx 128nK \left( \frac{256n - 4}{256n + 4} \right) - 8 \ln 2 (64n + 1), \\
 n \ln n &\approx \frac{128}{64} nK \left( \frac{256n - 4}{256n + 4} \right) - \frac{8}{64} \ln 2 (64n + 1) - \frac{\ln n}{64}, \\
 n \ln n &\approx 2nK \left( \frac{256n - 4}{256n + 4} \right) - \frac{\ln 2}{8} (64n + 1) - \frac{\ln n}{64}. \tag{5}
 \end{aligned}$$

I take (5) in (1), and achieve

$$p_n \sim 2nK \left( \frac{256n - 4}{256n + 4} \right) - \frac{\ln 2}{8} (64n + 1) - \frac{\ln n}{64}.$$

This completes the proof.  $\square$

## REFERENCES

- [1] [http://en.wikipedia.org/wiki/Arithmetic\\_geometric\\_mean](http://en.wikipedia.org/wiki/Arithmetic_geometric_mean), available in November 16, 2013.
- [2] [http://en.wikipedia.org/wiki/Natural\\_logarithm](http://en.wikipedia.org/wiki/Natural_logarithm), available in November 16, 2013. καιδιζikas