# Alternative solution of the Beal conjecture including another proof of the Fermat's Last Theorem, without references to the other works in the main part. (elementary aspect)

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**Abstract.** The aim of this paper is to prove the following a possible solution of the Beal conjecture and another proof of the Fermat's Last Theorem, without references to the other works in the main part.

**Theorem.**

The equation

$$A^n + B^n = D^n \ [\mathbf{1}]$$

has no solutions in positive integers A, B, D if n is an integer greater than

**Proof.**

**1.1.** Let us call

$$A \quad B = D,$$

where $A, B$ - are arbitrary natural numbers, as

$$A^{\alpha x - p\ } \quad B^{\beta y - q\ z > 0} = D^{\gamma z - mxy > 0} \ [\mathbf{2}]$$

Multiplying [**2**] by

$$\quad qxz D^{mxy},$$

we obtain

$$(A^{\ \ z}D^{my})^x + (A^{pz}B^{\beta}D^{mx})^y \equiv (A^{py}B^{qx}D^{\gamma})^z \ [\mathbf{3}].$$

**1.2.** Suppose $x, y, z$ are arbitrary prime pairwise natural numbers. Then,

$$\begin{cases} \alpha x - pyz = 1 \\ \beta y - qxz = 1 \ [\mathbf{4}] \\ \gamma z - mxy = 1 \end{cases}$$

where $\alpha, \beta, \gamma, p, q, m -$ are natural numbers which corresponding to solutions of the equation [**4**].

**1.3.** If $x = z = n$. Then, from [**4**]

$$\begin{cases} \alpha n - pny = n & \alpha = py + 1 \\ \beta y - qn^2 = 1 & \beta = \dfrac{qn^2 + 1}{2} \quad [\mathbf{5}] \\ \gamma n - mny = n & \gamma = my + 1 \end{cases}$$

We claim that $y = 2$ without affecting the generality, consider $q = 1$ and we get:

$$n = 2k + 1 \quad \text{-is odd number}$$

$$\beta = \frac{n^2 + 1}{2} = 2k^2 + 2k + 1$$

$k$ –is arbitrary natural number.

Then, we obtain the identity:

$$(A^{2p+1} B^{2k+1} D^{2m})^{2k+1} + [A^{p(2k+1)} B^{2k^2+2k+1} D^{m(2k+1)}]^2 \equiv$$

$$\equiv (A^{2p} B^{2k+1} D^{2m+1})^{2k+1} \quad [6],$$

if

$$A^{2k+1} + B^1 = D^{2k+1} \quad [7]$$

$$A^{2p(2k+1)} B^{(2k+1)^2} D^{2m(2k+1)} (A^{2k+1} + B^1) \equiv$$

$$\equiv A^{2p(2k+1)} B^{(2k+1)^2} D^{2m(2k+1)} D^{2k+1}.$$

**1.4.** For

$$x = y = n = 2k + 1; \ z = 2; \ m = 1$$

by analogy with **1.3.**:

$$(A^{2p+1} B^{2q} D^{2k+1})^{2k+1} + (A^{2p} B^{2k+1} D^{2k+1})^{2k+1} \equiv$$

$$\equiv \left[ A^{p(2k+1)} B^{q(2k+1)} D^{2k+1} \right]^2 \quad [8],$$

if

$$A^{2k+1} + B^{2k+1} = D^1 \quad [9].$$

$$A^{2p(2k+1)} B^{2q(2k+1)} D^{(2k+1)^2} (A^{2k+1} + B^{2k+1}) \equiv$$

$$\equiv A^{2p(2k+1)} B^{2q(2k+1)} D^{(2k+1)^2} D^1.$$

**1.5.** The equations [7] and [9] allow us to compare their left side with the left side of the equation [1] as follows:

$$\begin{cases} D^{2k+1} - A^{2k+1} = B^1 \ [10] \\ D^{2k+1} - A^{2k+1} = B^{2k+1}[11] \end{cases} \quad \begin{cases} A^{2k+1} + B^{2k+1} = D^1 \ [12] \\ A^{2k+1} + B^{2k+1} = D^{2k+1} \ [13]. \end{cases}$$

**1.6.** Among the all conceivable solutions [10] and [12] in natural numbers from 1 to infinity (including all possible combinations of the values of all parameters:$A, B, D, K$) values should be left parts of solutions[11] and [13], if they exist.

**1.7.** Follows from [5] $y = 2$ (by analogy $z = 2$) cannot be an even numbers greater than two since in

$n = x = z = 2k + 1$ [6] ($n = x = y = 2k + 1$ [8])
$\beta = 2k^2 + 2k + 1$ ($\gamma = 2k^2 + 2k + 1$) -are odd numbers.

**1.8.** From these considerations, with respect to   [**6**]

$$2\beta = 2(2k^2 + 2k + 1) = (2k + 1)^2 + 1$$

([**8**] $2\gamma = 2(2k^2 + 2k + 1) = (2x + 1)^2 + 1$)

cannot be a multiplier $2k + 1$ , therefore, equations [**11**] and [**13**] for $n = 2k + 1$ - is odd cannot have a solution in positive integers (even for arbitrary "$p$" and "$q$" , equal $2k + 1$), since,

**1.9.** if $A, B, D$ be satisfied the equation

$$A^n + B^n = D^n$$

, then it will satisfy any triple of the form $(\delta A, \delta B, \delta D)$ , where $\delta$ –is a natural number. And, if triple $(\delta A, \delta B, \delta D)$ is the solution of the equation, then the solution will be triple $A, B, D$ . If Fermat's Last Theorem is true for the exponent "$n$" ,then it is automatically true for any exponent $vn$ ,folding "$n$" , because, if the equation

$$A^{vn} + B^{vn} = D^{vn}$$

has solution $(A, B, D)$, then it will have a solution $(A^v, B^v, D^v)$. Therefore, is sufficient to prove FLT for $n = 4$ (the way Fermat himself had made) and for the exponent, where $n \geq 3$ -is arbitrary prime number.

(M.M.Postnikov, "Fermat's theorem", "Science", Glavfizmatgiz, Moscow, 1978, p 18, 27-30).

**1.10.** Since primes are part (excluding the number 2) of odd numbers, and for odd numbers the theorem is proved above in .1.1.-1.8. this article, without loss of generality it can be assumed that, the equation

$$A^n + B^n = D^n$$

for $n > 2$ has no solutions in positive integers. This completes the proof of Fermat's Last Theorem.

**1.11.** But the odd solutions in positive integers equations

$$D^{2k+1} - A^{2k+1} = B^{2k^2+2k+1}$$
$$(A^{2k+1} + B^{2k+1} = D^{2x^2+2k+1})$$

and even the right-hand side of the equations

$$D^{2k+1} - A^{2k+1} = B^{2(2k^2+2k+1)}$$
$$(A^{2k+1} + B^{2k+1} = D^{2(2x^2+2k+1)}) -$$

- are countless.
Similary, for

$$k = 3, \quad 2k + 1 = 7; \ 2k^2 + 2k + 1 = 5^2 , p = 5, \quad q = 5$$

$$(A^{2\times5+1}B^{2\times5}D^{2\times3+1})^7 + (A^{2\times5}B^{2\times5+1}D^{2\times3+1})^7 =$$

$$= (A^{5\times(2\times3+1)}B^{5\times(2\times3+1)}D^{5\times5})^2,$$

if

$$A^7 + B^7 = D^1$$

,or

$$(A^{11}B^{10}D^7)^7 + (A^{10}B^{11}D^7)^7 = (A^{14}B^{14}D^{10})^5,$$

if

$$A^7 + B^7 = D^1,$$

and etc.

The alternative solutions of one of the Beal conjecture problems.

**References:**

[1] М.М.Постников, " Теорема Ферма", "Наука", Главфизматгиз, Москва, 1978, (199.)