

## Alternative solution of the Beal conjecture including another proof of the Fermat's Last Theorem, without references to the other works in the main part. (elementary aspect)

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**Abstract.** The aim of this paper is to prove the following a possible solution of the Beal conjecture and another proof of the Fermat's Last Theorem, without references to the other works in the main part.

### Theorem.

The equation

$$A^n + B^n = D^n \quad [1]$$

has no solutions in positive integers A, B, D if n is an integer greater than 2.

### Proof.

1.1. Let us call

$$A = a^p, B = b^q, D = d^r,$$

where A, B - are arbitrary natural numbers, as

$$A^{ax-pyz} + B^{\beta y-qxz} = D^{\gamma z-mxy} > 0 \quad [2]$$

Multiplying [2] by

$$d^{qxz} D^{mxy},$$

we obtain

$$(A^{ax-pyz} D^{mxy})^x + (B^{\beta y-qxz} D^{mxy})^y = (D^{\gamma z-mxy})^z \quad [3].$$

1.2. Suppose x, y, z are arbitrary pairwise natural numbers. Then,

$$\begin{cases} \alpha x - pyz = 1 \\ \beta y - qxz = 1 \\ \gamma z - mxy = 1 \end{cases} \quad [4]$$

where  $\alpha, \beta, \gamma, p, q, m$  - are natural numbers which corresponding to solutions of the equations [4].

1.3. If  $x = z = n$ . Then, from [4]

$$\begin{cases} \alpha n - pny = n & \alpha = py + 1 \\ \beta y - qn^2 = 1 & \beta = \frac{qn^2 + 1}{2} \\ \gamma n - mny = n & \gamma = my + 1 \end{cases} \quad [5]$$

We claim that  $y = 2$  without affecting the generality, consider  $q = 1$  and we get:

$$n = 2k + 1 \quad \text{-is odd number}$$

$$\beta = \frac{n^2 + 1}{2} = 2k^2 + 2k + 1$$

$k$  –is arbitrary natural number.

Then, we obtain the identity:

$$\begin{aligned} (A^{2p+1}B^{2k+1}D^{2m})^{2k+1} + [A^{p(2k+1)}B^{2k^2+2k+1}D^{m(2k+1)}]^2 &\equiv \\ &\equiv (A^{2p}B^{2k+1}D^{2m+1})^{2k+1} \quad [6], \end{aligned}$$

if

$$\begin{aligned} A^{2k+1} + B^1 &= D^{2k+1} \quad [7] \\ A^{2p(2k+1)}B^{(2k+1)^2}D^{2m(2k+1)}(A^{2k+1} + B^1) &\equiv \\ &\equiv A^{2p(2k+1)}B^{(2k+1)^2}D^{2m(2k+1)}D^{2k+1}. \end{aligned}$$

1.4. For

$$x = y = n = 2k + 1; z = 2; m = 1$$

by analogy with 1.3.:

$$\begin{aligned} (A^{2p+1}B^{2q}D^{2k+1})^{2k+1} + (A^{2p}B^{2k^2+2k+1}D^{2k+1})^{2k+1} &\equiv \\ &\equiv [A^{p(2k+1)}B^{q(2k+1)}D^{2k+1}]^2 \quad [8], \end{aligned}$$

if

$$\begin{aligned} A^{2k+1} + B^{2k+1} &= D^1 \quad [9]. \\ A^{2p(2k+1)}B^{2q(2k+1)}D^{(2k+1)^2}(A^{2k+1} + B^{2k+1}) &\equiv \\ &\equiv A^{2p(2k+1)}B^{2q(2k+1)}D^{(2k+1)^2}D^1. \end{aligned}$$

1.5. The equations [7] and [9] allow us to compare their left side with the left side of the equation [1] as follows.

$$\begin{cases} D^{2k+1} - A^{2k+1} = B^1 \quad [10] \\ D^{2k+1} - A^{2k+1} = B^{2k+1} \quad [11] \end{cases} \quad \begin{cases} A^{2k+1} + B^{2k+1} = D^1 \quad [12] \\ A^{2k+1} + B^{2k+1} = D^{2k+1} \quad [13]. \end{cases}$$

1.6. Among all conceivable solutions [10] and [12] in natural numbers from 1 to infinity (including all possible combinations of the values of all parameters:  $A, B, D, K$ ) values should be left parts of solutions [11] and [13], if they exist.

1.7. Follows from [5]  $y = 2$  (by analogy  $z = 2$ ) cannot be an even numbers greater than two since in

$$\begin{aligned} n = x = z = 2k + 1 \quad [6] \quad (n = x = y = 2k + 1 \quad [8]) \\ \beta = 2k^2 + 2k + 1 \quad (\gamma = 2k^2 + 2k + 1) \text{ -are odd numbers.} \end{aligned}$$

1.8. From these considerations, with respect to [6]

$$2\beta = 2(2k^2 + 2k + 1) = (2k + 1)^2 + 1$$

$$([8] \ 2\gamma = 2(2k^2 + 2k + 1) = (2x + 1)^2 + 1)$$

cannot be a multiplier  $2k + 1$ , therefore, equations [11] and [13] for  $n = 2k + 1$  - is odd cannot have a solution in positive integers (even for arbitrary "p" and "q", equal  $2k + 1$ ), since,

1.9. if  $A, B, D$  be satisfied the equation

$$A^n + B^n = D^n$$

, then it will satisfy any triple of the form  $(\delta A, \delta B, \delta D)$ , where  $\delta$  - is a natural number. And, if triple  $(\delta A, \delta B, \delta D)$  is the solution of the equation, then the solution will be triple  $(A, B, D)$ . If Fermat's Last Theorem is true for the exponent "n", then it is automatically true for any exponent  $vn$ , folding "n", because, if the equation

$$A^{vn} + B^{vn} = D^{vn}$$

has solution  $(A, B, D)$ , then it will have a solution  $(A^v, B^v, D^v)$ . Therefore, is sufficient to prove FLT for  $n = 4$  (the way Fermat himself had made) and for the exponent, where  $n \geq 3$  - is arbitrary prime number.

(M.M.Postnikov, "Fermat's theorem", "Science", Glavfizmatgiz, Moscow, 1978, p 18, 27-30).

1.10. Since primes are part (excluding the number 2) of odd numbers, and for odd numbers the theorem is proved above in 1.1.-1.8. this article, without loss of generality it can be assumed that, the equation

$$A^n + B^n = D^n$$

for  $n > 2$  has no solutions in positive integers. This completes the proof of Fermat's Last Theorem.

1.11. But the odd solutions in positive integers equations

$$\begin{aligned} D^{2k+1} - A^{2k+1} &= B^{2k^2+2k+1} \\ (A^{2k+1} + B^{2k+1} &= D^{2x^2+2k+1}) \end{aligned}$$

are given the right-hand side of the equations

$$\begin{aligned} D^{2k+1} - A^{2k+1} &= B^{2(2k^2+2k+1)} \\ (A^{2k+1} + B^{2k+1} &= D^{2(2x^2+2k+1)}) - \end{aligned}$$

- are countless.

Similary, for

$$k = 3, \quad 2k + 1 = 7; \quad 2k^2 + 2k + 1 = 5^2, \quad p = 5, \quad q = 5$$

$$\begin{aligned} (A^{2 \times 5 + 1} B^{2 \times 5} D^{2 \times 3 + 1})^7 + (A^{2 \times 5} B^{2 \times 5 + 1} D^{2 \times 3 + 1})^7 &= \\ &= (A^{5 \times (2 \times 3 + 1)} B^{5 \times (2 \times 3 + 1)} D^{5 \times 5})^2, \end{aligned}$$

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if

$$A^7 + B^7 = D^1$$

,or

$$(A^{11}B^{10}D^7)^7 + (A^{10}B^{11}D^7)^7 = (A^{14}B^{14}D^{10})^5,$$

if

$$A^7 + B^7 = D^1,$$

and etc.

The alternative solutions of one of the Beal conjecture problems.

**References:**

- [1] М.М.Постников, "Теорема Ферма", "Наука", Главфизматгиз, Москва, 1978, (199)

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