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Alternative solution of the Beal conjecture including another proof of the Fermat's Last Theorem, without references to the other works in the main part. (elementary aspect)

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Abstract. The aim of this paper is to prove the following a possible solution of the Beal conjecture and another proof of the Fermat's Last Theorem, without references to the other works in the main part.

Theorem.

The equation

$$A^n + B^n = D^n \ [\mathbf{1}]$$

has no solutions in positive integers A, B, D if n is an integer greater than 2.

Proof.

1.1. Let us call

$$A + B = D,$$

where A, B - are arbitrary natural numbers, as

$$A^{\alpha x-pyz>0} + B^{\beta y-qxz>0} = D^{\gamma z-mxy>0} \ [\mathbf{2}]$$

Multiplying $[\mathbf{2}]$ by

$$A^{pyz} B^{qxz} D^{mxy},$$

we obtain

$$(A^{\alpha} B^{qz} D^{my})^x + (A^{pz} B^{\beta} D^{mx})^y \equiv (A^{py} B^{qx} D^{\gamma})^z \ [\mathbf{3}].$$

1.2. Suppose x, y, z are arbitrary coprime pairwise natural numbers. Then,

$$\begin{cases} \alpha x - pyz = 1 \\ \beta y - qxz = 1 \\ \gamma z - mxy = 1 \end{cases} [\mathbf{4}]$$

, where $\alpha, \beta, \gamma, p, q, m -$ are natural numbers which corresponding to solutions of the equations $[\mathbf{4}]$.

1.3. If $x = z = n$. Then, from $[\mathbf{4}]$

$$\begin{cases} \alpha n - pny = n & \alpha = py + 1 \\ \beta y - qn^2 = 1 & \beta = \dfrac{qn^2 + 1}{2} \\ \gamma n - mny = n & \gamma = my + 1 \end{cases} \quad [\mathbf{5}]$$

We claim that $y = 2$ without affecting the generality, consider $q = 1$ and we get:

$$n = 2k + 1 \quad \text{-is odd number}$$

$$\beta = \frac{n^2+1}{2} = 2k^2 + 2k + 1$$

k –is arbitrary natural number.

Then, we obtain the identity:

$$(A^{2p+1}B^{2k+1}D^{2m})^{2k+1} + [A^{p(2k+1)}B^{2k^2+2k+1}D^{m(2k+1)}]^2 \equiv$$
$$\equiv (A^{2p}B^{2k+1}D^{2m+1})^{2k+1} \quad [\mathbf{6}],$$

if

$$A^{2k+1} + B^1 = D^{2k+1} \ [\mathbf{7}]$$
$$A^{2p(2k+1)}B^{(2k+1)^2}D^{2m(2k+1)}(A^{2k+1}+B^1) \equiv$$
$$\equiv A^{2p(2k+1)}B^{(2k+1)^2}D^{2m(2k+1)}D^{2k+1}.$$

1.4. For

$$x = y = n = 2k+1; \ z = 2; \ m = 1$$

by analogy with **1.3.**:

$$(A^{2p+1}B^{2q}D^{2k+1})^{2k+1} + (A^{2p}B^{2q}D^{2k+1})^{2k+1} \equiv$$
$$\equiv \left[A^{p(2k+1)}B^{q(2k+1)}D^{2k^2+2k+1}\right]^2 \ [\mathbf{8}],$$

if

$$A^{2k+1} + B^{2k+1} = D^1 \ [\mathbf{9}].$$
$$A^{2p(2k+1)}B^{2q(2k+1)}D^{(2k+1)^2}(A^{2k+1}+B^{2k+1}) \equiv$$
$$\equiv A^{2p(2k+1)}B^{2q(2k+1)}D^{(2k+1)^2}D^1.$$

1.5. The equations [**7**] and [**9**] allow us to compare their left side with the left side of the equation [**1**] as follows:

$$\begin{cases} D^{2k+1} - A^{2k+1} = B^1 \ [\mathbf{10}] \\ D^{2k+1} - A^{2k+1} = B^{2k+1} [\mathbf{11}] \end{cases} \quad \begin{cases} A^{2k+1} + B^{2k+1} = D^1 \ [\mathbf{12}] \\ A^{2k+1} + B^{2k+1} = D^{2k+1} \ [\mathbf{13}]. \end{cases}$$

1.6. Among the all conceivable solutions [**10**] and [**12**] in natural numbers from 1 to infinity (including all possible combinations of the values of all parameters:$\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{D}, \boldsymbol{K}$) values should be left parts of solutions[**11**] and [**13**], if they exist.

1.7. Follows from [**5**] $y = 2$ (by analogy $z = 2$) cannot be an even numbers greater than two since in

$n = x = z = 2k+1$ [**6**] ($n = x = y = 2k+1$ [**8**])
$\beta = 2k^2 + 2k + 1$ ($\gamma = 2k^2 + 2k + 1$) -are odd numbers.

1.8. From these considerations, with respect to [**6**]

$$2\beta = 2(2k^2 + 2k + 1) = (2k + 1)^2 + 1$$

([**8**] $2\gamma = 2(2k^2 + 2k + 1) = (2x + 1)^2 + 1$)

cannot be a multiplier $2k + 1$, therefore, equations [**11**] and [**13**] for $n = 2k + 1$ - is odd cannot have a solution in positive integers (even for arbitrary "p" and "q", equal $2k + 1$), since,

1.9. if A, B, D be satisfied the equation

$$A^n + B^n = D^n$$

, then it will satisfy any triple of the form $(\delta A, \delta B, \delta D)$, where δ –is a natural number. Also, if triple $(\delta A, \delta B, \delta D)$ is the solution of the equation, then the solution will be triple (A, B, D). If Fermat's Last Theorem is true for the exponent "n", then it is automatically true for any exponent νn, folding "n", because, if the equation

$$A^{\nu n} + B^{\nu n} = D^{\nu n}$$

has solution (A, B, D), then it will have a solution (A^ν, B^ν, D^ν). Therefore, is sufficient to prove FLT for $n = 4$ (the way Fermat himself had made) and for the exponent, where $n \geq 3$ -is arbitrary prime number.

(M.M.Postnikov, "Fermat's theorem", "Science", Glavfizmatgiz, Moscow, 1978, p 18, 27-30).

1.10. Since primes are part (excluding the number 2) of odd numbers, and for odd numbers the theorem is proved above in .1.1.-1.8. this article without loss of generality it can be assumed that, the equation

$$A^n + B^n = D^n$$

for $n > 2$ has no solutions in positive integers. This completes the proof of Fermat's Last Theorem.

1.11. But the odd solutions in positive integers equations

$$D^{2k+1} - A^{2k+1} = B^{2k^2+2k+1}$$
$$(A^{2k+1} + B^{2k+1} = D^{2x^2+2k+1})$$

and even the right-hand side of the equations

$$D^{2k+1} - A^{2k+1} = B^{2(2k^2+2k+1)}$$
$$(A^{2k+1} + B^{2k+1} = D^{2(2x^2+2k+1)}) -$$

- are countless.
Similary, for

$$k = 3, \quad 2k + 1 = 7; \;\; 2k^2 + 2k + 1 = 5^2, p = 5, \quad q = 5$$

$$(A^{2\times5+1}B^{2\times5}D^{2\times3+1})^7 + (A^{2\times5}B^{2\times5+1}D^{2\times3+1})^7 =$$

$$= (A^{5\times(2\times3+1)}B^{5\times(2\times3+1)}D^{5\times5})^2,$$

if

$$A^7 + B^7 = D^1$$

,or

$$(A^{11}B^{10}D^7)^7 + (A^{10}B^{11}D^7)^7 = (A^{14}B^{14}D^{10})^5,$$

if

$$A^7 + B^7 = D^1,$$

and etc.

The alternative solutions of one of the Beal conjecture problems.

References:

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