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A PROOF OF THE VERACITY OF BOTH GOLDBACH AND DE POLIGNAC CONJECTURES

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Abstract
The present algebraic development begins by an exposition of the data of the problem. The definition of the primal radius $r > 0$ is : For all positive integer $x \geq 3$ exists a finite number of integers called the primal radius $r > 0$, for which $x + r$ and $x - r$ are prime numbers. The corollary is that $2x = (x + r) + (x - r)$ is always the sum of a finite number of primes. Also, for all positive integer $x \geq 0$, exists an infinity of integers $r > 0$, for which $x + r$ and $r - x$ are prime numbers. The conclusion is that $2x = (x + r) - (r - x)$ is always an infinity of differences of primes.

Introduction

There is a similarity between the assertion : "an even number is always the sum of two primes" and the assertion : "an even number is always the difference of two primes". The present article gives the proof that the two assertions are the consequences of the same concept by the introduction of the notion of the primal radius.

The proof

Let us suppose that exists an integer $x \geq 3$ for which $2x$ is never the sum of two primes, then for all

p_1 and p_2 primes, $3 \leq p_2 < p_1$, $2x \neq p_1 + p_2$ or $2x = p_1 + p_2 + 2b_{p_1,p_2} = p_1 + p_2 + 2b$

then $x = \frac{p_1 + p_2}{2} + b$.

But for all p_1 , p_2 exists y for which $y = \frac{p_1 - p_2}{2} + b$

Let

$x_1 = p_1 + 2b$, $x_2 = p_2 - 2b$, $x_3 = p_2 + 2b$, $x_4 = p_1 - 2b$

We deduce that

$$x = \frac{p_1 + p_2}{2} [illegible] \frac{p_1 + x_2}{2} [illegible] = \frac{x_{[illegible]} + p_2}{2} = \frac{x_1 + x_2}{2} + b$$

$$= \frac{p_1 [illegible]}{2} [illegible] \frac{x_1 + [illegible]}{2} [illegible] \frac{x_4 + x_3}{2} + b = \frac{x_4 + x_2}{2} + 3b$$

$$y = \frac{p_1 - p_2}{2} + b [illegible] \frac{p_{[illegible]} - x_2}{2} = \frac{x_1 - p_2}{2} = \frac{x_1 - x_2}{2} - b$$

$$= \frac{p_1 - x_3}{2} + 2b = \frac{x_1 - x_3}{2} + b = \frac{x_4 - x_3}{2} + 3b = \frac{x_4 - x_2}{2} + b$$

$$x_1 + x_2 = p_1 + p_2$$

$$x_1 - x_3 = p_1 - p_2$$

Lemma 1

The following formula

$$x = \frac{p_1 + p_2}{2} + b = \frac{p_1 + x_2}{2} + 2b = \frac{x_1 + p_2}{2} = \frac{x_1 + x_2}{2} + b$$

$$= \frac{p_1 + x_3}{2} = \frac{x_1 + x_3}{2} - b = \frac{x_4 + x_3}{2} + b = \frac{x_4 + x_2}{2} + 3b$$

$$y = \frac{p_1 - p_2}{2} + b = \frac{p_1 - x_2}{2} = \frac{x_1 - p_2}{2} = \frac{x_1 - x_2}{2} - b$$

$$= \frac{p_1 - x_3}{2} + 2b = \frac{x_1 - x_3}{2} + b = \frac{x_4 - x_3}{2} + 3b = \frac{x_4 - x_2}{2} + b$$

Imply that exist p_1 and p_2 prime numbers for which b = 0

Proof of lemma 1

If x is prime 2x = x + x is the sum of two primes, then $p1 - p2 \neq 0$

We will suppose firstly that $(x_1 - x_2)(x_1 + x_3) \neq 0$

Let

$$\frac{x_1 - x_2}{p_1 - p_2} = \frac{p_1 - p_2 + 4b}{p_1 - p_2} = 1 + \frac{4b}{p_1 - p_2}$$

$$\frac{p_1 - p_2}{x_1 - x_2} = \frac{x_1 - x_2 - 4b}{x_1 - x_2} = 1 - \frac{4b}{x_1 - x_2}$$

We pose $k = \frac{2b}{p_1 - p_2}$, $k' = -\frac{2b}{x_1 - x_2}$

$kk' = 0 \Rightarrow b = 0$, we have supposed [illegible] $\neq 0$

$\forall (x,y); \exists \varphi \mid x = \varphi y$

$x + y = (\varphi + 1)y = x_1 \neq 0$, $x - y = [illegible] 1)y [illegible] \neq 0$

$\forall (k,k'); \exists \alpha \mid k = \alpha k'$

$$\Rightarrow \frac{2b}{p_1 - p_2} = -\alpha \frac{2b}{x_1 - x_2}$$

$$\Rightarrow x_1 - x_2 = -\alpha(p_1 - p_2) \Rightarrow x_1 - x_2 - p_1 + p_2 = 4b = -(\alpha + 1)(p_1 - p_2)$$

$$\Rightarrow b = -\frac{\alpha + 1}{4}(p_1 - p_2)$$

$$\Rightarrow x = \frac{p_1 + p_2}{2} + b = \frac{p_1 + p_2}{2} - \frac{\alpha + 1}{4}(p_1 - p_2) = \frac{(1-\alpha)p_1 + (3+\alpha)p_2}{4} = \frac{\varphi}{\varphi - 1} p_2$$

$$y = \frac{p_1 - p_2}{2} + b = \frac{p_1 - p_2}{2} - \frac{\alpha + 1}{4}(p_1 - p_2) = \frac{(1-\alpha)(p_1 - p_2)}{4} = \frac{1}{\varphi - 1} p_2$$

Let

$$\frac{x_1 + x_3}{p_1 + p_2} = \frac{p_1 + p_2 + 4b}{p_1 + p_2} = 1 + \frac{4b}{p_1 + p_2}$$

$$\frac{p_1 + p_2}{x_1 + x_3} = \frac{x_1 + x_3 - 4b}{x_1 + x_3} = 1 - \frac{4b}{x_1 + x_3}$$

We pose $m=\frac{2b}{p_1+p_2}$, $m'=-\frac{2b}{x_1+x_3}$

$mm'=0 \Rightarrow b=0$, we have supposed $mm'\neq 0$

$\forall(m,m');\exists\beta \mid m=\beta m'$

$$\Rightarrow \frac{2b}{p_1+p_2}=-\beta\frac{2b}{x_1+x_3}$$

$$\Rightarrow x_1+x_3=-\beta(p_1+p_2) \Rightarrow x_1+x_3-p_1-p_2=4b=-(\beta+1)(p_1+p_2)$$

$$\Rightarrow b=-\frac{\beta+1}{4}(p_1+p_2)$$

$$\Rightarrow x=\frac{p_1+p_2}{2}+b=\frac{p_1+p_2}{2}-\frac{\beta+1}{4}(p_1+p_2)=\frac{(1-\beta)(p_1+p_2)}{4}=\frac{\varphi}{\varphi-1}p_2$$

$$y=\frac{p_1-p_2}{2}+b=\frac{p_1-p_2}{2}-\frac{\beta+1}{4}(p_1+p_2)=\frac{(1-\beta)p_1-(\beta+3)p_2}{4}=\frac{1}{\varphi-1}p_2$$

$$b=-\frac{\alpha+1}{4}(p_1-p_2)=-\frac{\beta+1}{4}(p_1+p_2) \Rightarrow (\beta-\alpha)p_1=(-2-\alpha-\beta)p_2$$

But : $(2k+1)(2k'+1)=1 \Rightarrow 2kk'+k+k'=0 \Rightarrow (2k+1)k'=-k$

$$2k+1=\frac{-k}{k'}=\frac{k+1}{k'+1}=\frac{a(k+1)+a'(2k+1)}{a(k'+1)+a'}$$

$$2k'+1==\frac{-k'}{k}=\frac{k'+1}{k+1}=\frac{c(k'+1)+c'(2k'+1)}{c(k+1)+c'},\quad \forall(a,a',c,c')$$

$$\frac{2k+1}{2k'+1}=\frac{((a+2a')k+a+a')(ck+c+c')}{((c+2c')k'+c+c')(ak'+a+a')}$$

$$=\frac{(ac+2a'c)k^2+(2ac+ac'+3a'c+2a'c')k+ac+ac'+a'c+a'c'}{(ac+2ac')k'^2+(2ac+3ac'+a'c+2a'c')k'+ac+ac'+a'c+a'c'}$$

$$\Rightarrow \frac{2k+1}{2k'+1}-1=(k-k')\frac{2}{2k'+1}$$

$$=\frac{ac(k^2-k'^2)+2a'ck^2-2ac'k'^2+(2ac+ac'+a'c+2a'c')(k-k')+2a'ck-2ac'k'}{(ac+2ac')k'^2+(2ac+3ac'+a'c+2a'c')k'+ac+ac'+a'c+a'c'}$$

$\forall(a,a',c,c')$, particularly $(a,a',c,c') \mid ac'=\delta k$, $a'c=\delta k'$

$$\Rightarrow (k-k')\frac{2}{2k'+1}$$

$$=\frac{ac(k+k')(k-k')+(2ac+ac'+a'c+2a'c')(k-k')+2\delta kk'^2-2\delta k^2k'}{(ac+2ac')k'^2+(2ac+3ac'+a'c+2a'c')k'+ac+ac'+a'c+a'c'}$$

$$=(k-k')\frac{-2ackk'+2ac+ac'+a'c+2a'c'-2\delta kk'}{(ac+2ac')k'^2+(2ac+3ac'+a'c+2a'c')k'+ac+ac'+a'c+a'c'}$$

If $k=k' \Rightarrow \alpha=1 \Rightarrow \frac{(1-\alpha)p_1-(3+\alpha)p_2}{4}=p_2$, trivial solution : it is impossible.

$$k-k'\neq 0 \Rightarrow \frac{2}{2k'+1}=\frac{-2ackk'+2ac+ac'+a'c+2a'c'-2\delta kk'}{(ac+2ac')k'^2+(2ac+3ac'+a'c+2a'c')k'+ac+ac'+a'c+a'c'}$$

Also $(2m+1)(2m'+1)=1 \Rightarrow 2mm'+m+m'=0 \Rightarrow (2m+1)m'=-m$

$$2m+1=\frac{-m}{m'}=\frac{m+1}{m'+1}=\frac{a(m+1)+a'(2m+1)}{a(m'+1)+a'}$$

$$2m'+1==\frac{-m'}{m}=\frac{m'+1}{m+1}=\frac{c(m'+1)+c'(2m'+1)}{c(m+1)+c'},\quad \forall(a,a',c,c')$$

$$\frac{2m+1}{2m'+1}=\frac{((a+2a')m+a+a')(cm+c+c')}{((c+2c')m'+c+c')(am'+a+a')}$$

$$=\frac{(ac+2a'c)m^2+(2ac+ac'+3a'c+2a'c')m+ac+ac'+a'c+a'c'}{(ac+2ac')m'^2+(2ac+3ac'+a'c+2a'c')m'+ac+ac'+a'c+a'c'}$$

$$\Rightarrow \frac{2m+1}{2m'+1}-1=(m-m')\frac{2}{2m'+1}$$

$$=\frac{ac(m^2-m'^2)+2a'cm^2-2ac'm'^2+(2ac+ac'+a'c+2a'c')(m-m')+2a'cm-2ac'm'}{(ac+2ac')m'^2+(2ac+3ac'+a'c+2a'c')m'+ac+ac'+a'c+a'c'}$$

$\forall(a,a',c,c')$, particularly $(a,a',c,c')\mid ac'=\delta k^2=\gamma m^2$, $a'c=\delta k'^2=\gamma' m'^2$

$$\Rightarrow (m-m')\frac{2}{2m'+1}$$

$$=\frac{ac(m+m')(m-m')+2(\gamma'-\gamma)m^2m'^2+(2ac+ac'+a'c+2a'c')(m-m')+2\gamma' mm'^2-2\gamma m^2m'}{(ac+2ac')m'^2+(2ac+3ac'+a'c+2a'c')m'+ac+ac'+a'c+a'c'}$$

$$=(m-m')\frac{-2acmm'+2(\frac{\gamma'-\gamma}{m-m'})m^2m'^2+2ac+ac'+a'c+2a'c'-2(\frac{\gamma m-\gamma' m'}{m-m'})mm'}{(ac+2ac')m'^2+(2ac+3ac'+a'c+2a'c')m'+ac+ac'+a'c+a'c'}$$

$m=m'\Rightarrow\beta=1\Rightarrow x=\frac{(1-\beta)(p_1+p_2)}{4}=0$, it is impossible

But $\frac{2}{2k'+1}=\frac{-2ackk'+2ac+ac'+a'c+2a'c'-2\delta kk'}{(ac+2ac')k'^2+(2ac+3ac'+a'c+2a'c')k'+ac+ac'+a'c+a'c'}$

For $(a,a',c,c')\mid ac'=\delta k^2=\gamma m^2$, $a'c=\delta k'^2=\gamma' m'^2$

$$\Rightarrow\frac{2}{2m'+1}=\frac{-2acmm'+2ac+ac'+a'c+2a'c'-2\gamma mm'}{(ac+2ac')m'^2+(2ac+3ac'+a'c+2a'c')m'+ac+ac'+a'c+a'c'}$$

$$=\frac{-2acmm'+2ac+ac'+a'c+2a'c'-2\gamma' mm'}{(ac+2ac')m'^2+(2ac+3ac'+a'c+2a'c')m'+ac+ac'+a'c+a'c'}$$

$$\Rightarrow\frac{2}{2m'+1}=\frac{-2acmm'+2(\frac{\gamma'-\gamma}{m-m'})m^2m'^2+2ac+ac'+a'c+2a'c'-2(\frac{\gamma m-\gamma' m'}{m-m'})mm'}{(ac+2ac')m'^2+(2ac+3ac'+a'c+2a'c')m'+ac+ac'+a'c+a'c'}$$

$$=\frac{-2acmm'+2ac+ac'+a'c+2a'c'-2\gamma mm'}{(ac+2ac')m'^2+(2ac+3ac'+a'c+2a'c')m'+ac+ac'+a'c+a'c'}$$

$$=\frac{-2acmm'+2(\frac{\gamma-\gamma}{m-m'})m^2m'^2+2ac+ac'+a'c+2a'c'-2\gamma mm'}{(ac+2ac')k'^2+(2ac+3ac'+a'c+2a'c')k'+ac+ac'+a'c+a'c'}$$

$\Rightarrow\gamma=\gamma'\Rightarrow ac'=\delta k^2=\gamma m^2$, $a'c=\delta k'^2=\gamma' m'^2=\gamma m'^2$

$\Rightarrow\frac{\delta}{\gamma}=\frac{m^2}{k^2}=\frac{m'^2}{k'^2}\Rightarrow\frac{k^2}{k'^2}=(2k+1)^2=\alpha^2=\frac{[illegible]}{m}=(2m+1)^2=\beta^2$

If $\alpha=-\beta\Rightarrow(\beta-\alpha)p_1=-2\alpha p_1=[illegible]=(-2-\alpha-\beta)p_2=-2p_2\Rightarrow\alpha=-\beta=\frac{p_2}{p_1}$

$b=-(\alpha+1)\frac{p_1-p_2}{4}=-(\beta+1)\frac{p_1+p_2}{4}=-(\frac{p_1+p_2}{4p_1})(p_1-p_2)=\frac{p_2^2-p_1^2}{4p_1}$

$\Rightarrow 4bp_1=p_2^2-p_1^2\Rightarrow(4b+p_1)p_1=p_2^2\Rightarrow 4b+p_1=\frac{p_2^2}{p_1}$ and it is impossible because p_1 and p_2 are primes and $4b+p_1=\frac{p_2^2}{p_1}$ can not be an integer.

$\Rightarrow\alpha-\beta=0\Rightarrow(\beta-\alpha)p_1=(-2-\alpha-\beta)p_2=-2(1+\alpha)p_2=-2(1+\beta)p_2=0\Rightarrow\alpha=\beta=-1$

$\Rightarrow b=-\frac{(\alpha+1)}{4}(p_1-p_2)=-\frac{(\beta+1)}{4}(p_1+p_2)=0$

$\Rightarrow x=\frac{p_1+p_2}{2}$, $y=\frac{p_1-p_2}{2}$

Another proof : let u,u’,v,v’ verifying

$$(\frac{1}{up_1-vp_2}-\frac{1}{u'p_1+v'p_2})p_1=(\frac{1}{up_1-vp_2}+\frac{1}{u'p_1+v'p_2})p_2$$

$$\Rightarrow(u'-u)p_1^2+(v'+v)p_1p_2=(u+u')p_1p_2+(v'-v)p_2^2$$

Thus

$$(u'+u-v'-v)p_1p_2+(u-u')p_1^{\,2}=(v-v')p_2^{\,2}$$

We pose with a different of zero

$$u'+u-v'-v=a(\alpha-\beta)$$

$$\Rightarrow((v-v')-a(2+\alpha+\beta))p_2^{\,2}=(u-u')p_1^{\,2}$$

Let

$$v-v'-a(2+\alpha+\beta)=(2+\alpha+\beta)^2$$

$$u-u'=(\alpha-\beta)^2$$

$$\Rightarrow u'p_1+v'p_2=(u-(\alpha-\beta)^2)p_1+(v-a(2+\alpha+\beta)-(2+\alpha+\beta)^2)p_2$$

$$=up_1+vp_2-(\alpha-\beta)^2p_1-a(\alpha-\beta)p_1-(2+\alpha+\beta)(\alpha-\beta)p_1$$

Let $up_1=-vp_2$ hence

$$\left(\frac{1}{up_1-vp_2}-\frac{1}{u'p_1+v'p_2}\right)p_1=\left(\frac{1}{up_1-vp_2}+\frac{1}{u'p_1+v'p_2}\right)p_2$$

$$\Rightarrow\left(\frac{1}{2u}+\frac{1}{(\alpha-\beta)^2+a(\alpha-\beta)+(2+\alpha+\beta)(\alpha-\beta)}\right)p_1$$

$$=\left(\frac{1}{2u}-\frac{1}{(\alpha-\beta)^2+a(\alpha-\beta)+(2+\alpha+\beta)(\alpha-\beta)}\right)p_2$$

$$\Rightarrow\frac{p_1-p_2}{2u}=-p_1\left(\frac{1}{(2+\alpha+\beta)(2\alpha+a+2)}+\frac{1}{(\alpha-\beta)(2\alpha+a+2)}\right)$$

Let $2u=\alpha-\beta$ hence

$$\frac{1}{\alpha-\beta}-\frac{1}{\alpha+\beta+2}=-\frac{1}{(2+\alpha+\beta)(2\alpha+a+2)}-\frac{1}{(\alpha-\beta)(2\alpha+a+2)}$$

$$\Rightarrow\frac{2\alpha+a+3}{\alpha-\beta}=\frac{2\alpha+a+1}{2+\alpha+\beta};\forall$$ [illegible] $\Rightarrow$ [illegible] $= \beta$

It means b=0 thus

$x+y=p_1$, $x-y=p_2$ [illegible] are prim[illegible] y = r is the primal radius. As there is the condition $p_2 < x < p_1$, there is not an infinity of p_2, p_1

If $(x_1-x_2)(x_1+x_3)$ [illegible] $\Rightarrow(x_4-x_2)(x_4-x_3)$ [illegible] 0

Let $\frac{x_4+x_2}{p_1+p_2}=\frac{p_1+p_2-4b}{p_1+p_2}=1-\frac{4b}{p_1+p_2}$

$$\frac{p_1+p_2}{x_4+x_2}=\frac{x_4+x_2+4b}{x_4+x_2}=1+\frac{4b}{x_4+x_2}$$

$$k=-\frac{2b}{p_1+p_2},\ k'=\frac{2b}{x_4+x_2}$$

$$\frac{x_4-x_3}{p_1-p_2}=\frac{p_1-p_2-4b}{p_1-p_2}=1-\frac{4b}{p_1-p_2}$$

$$\frac{p_1-p_2}{x_4-x_3}=\frac{x_4-x_3+4b}{x_4-x_3}=1+\frac{4b}{x_4-x_3}$$

$$m=-\frac{2b}{p_1-p_2},\ m'=\frac{2b}{x_4-x_3}$$

With the same reasoning and calculus ⇒b =0 But b can not be equal to zero in all cases, it means there is an impossibility related to the fact that the conjecture is indecidable and if it is so, it is true ! Because, we would find in the case it is indecidable and false with the computer the 2x different of all sum of primes and it is contradictory !

Now, if we suppose that for all p_2, p_1 primes, exixts $x \mid 2x \neq p_1 - p_2$

$x = \frac{p_1 - p_2}{2} + b$, $y = \frac{p_1 + p_2}{2} + b$, with the same reasoning, the same calculus but replacing x by y and y by x, we prove that b = 0 , which means that for all positive integer x, exists p_1, p_2 for which $x = \frac{p_1 - p_2}{2}$, if we pose $y = \frac{p_1 + p_2}{2}$, $x + y = p_1$, $y - x = p_2$ y is the primal radius. As there is no condition on x, y, p_1, p_2 there is an infinity of couples of primes (p_1, p_2,) For x = 1 , p_1, and p_2 are twin primes. Let us prove it. Let us suppose that exists an integer x ≥ 0 for which $2x$ is never the difference of two primes, then for all p_1 and p_2 primes, $p_2 <$ [illegible] $\neq p_1 - p_2$, or $2x = p_1 - p_2 + 2b_{p_1, p_2} = p_1 - p_2 + 2b$, then $x = \frac{p_1 - p_2}{2} + b$.

But for all p_1, p_2 exists y, for which $y = \frac{p_1 + p_2}{2} + b$

Let

$x_1 = p_1 + 2b$, $x_2 = p_2 - 2b$, $x_3 = p_2 + 2b$, $x_4 = p_1 - 2b$

We deduce that

$$y = \frac{p_1 + p_2}{2} + b = \frac{p_1 + x_2}{2} + 2b = \frac{x_1 + p_2}{2} = \frac{x_1 + x_2}{2} + b$$

$$= \frac{p_1 + x_3}{2} = \frac{x_1 + x_3}{2} - b = \frac{x_4 + x_3}{2} + b = \frac{x_4 + x_2}{2} + 3b$$

$$x = \frac{p_1 - p_2}{2} + b = \frac{p_1 - x_2}{2} = \frac{x_1 - p_2}{2} = \frac{x_1 - x_2}{2} - b$$

$$= \frac{p_1 - x_3}{2} + 2b = \frac{x_1 - x_3}{2} + b = \frac{x_4 - x_3}{2} + 3b = \frac{x_4 - x_2}{2} + b$$

$$x_1 + x_2 = p_1 + p_2$$

$$x_1 - x_3 = p_1 - p_2$$

Lemma 2

The following formula

$$y = \frac{p_1 + p_2}{2} + b = \frac{p_1 + x_2}{2} + 2b = \frac{x_1 + p_2}{2} = \frac{x_1 + x_2}{2} + b$$

$$= \frac{p_1 + x_3}{2} = \frac{x_1 + x_3}{2} - b = \frac{x_4 + x_3}{2} + b = \frac{x_4 + x_2}{2} + 3b$$

$$x = \frac{p_1 - p_2}{2} + b = \frac{p_1 - x_2}{2} = \frac{x_1 - p_2}{2} = \frac{x_1 - x_2}{2} - b$$

$$= \frac{p_1 - x_3}{2} + 2b = \frac{x_1 - x_3}{2} + b = \frac{x_4 - x_3}{2} + 3b = \frac{x_4 - x_2}{2} + b$$

Imply that exist p_1 and p_2 prime numbers for which $b = 0$

Proof of lemma 2

If x is prime 0= $x - x$ is the sum of two primes, then $p_1 - p_2 \neq 0$

We will suppose firstly that $(x_1 - x_2)(x_1 + x_3) \neq 0$

Let

$$\frac{x_1 - x_2}{p_1 - p_2} = \frac{p_1 - p_2 + 4b}{p_1 - p_2} = 1 + \frac{4b}{p_1 - p_2}$$

$$\frac{p_1 - p_2}{x_1 - x_2} = \frac{x_1 - x_2 - 4b}{x_1 - x_2} = 1 - \frac{4b}{x_1 - x_2}$$

We pose $k = \frac{2b}{p_1 - p_2}$, $k' = -\frac{2b}{x_1 - x_2}$

$kk' = 0 \Rightarrow b = 0$, we have supposed $kk' \neq 0$

$\forall (x, y); \exists \varphi \mid y = \varphi x$

$x + y = (\varphi + 1)x = x_1 \neq 0$, $y - x = (\varphi - 1)x = p_2 \neq 0$

$\forall (k, k'); \exists \alpha \mid k = \alpha k'$

$$\Rightarrow \frac{2b}{p_1 - p_2} = -\alpha \frac{2b}{x_1 - x_2}$$

$$\Rightarrow x_1 - x_2 = -\alpha (p_1 - p_2) \Rightarrow x_1 - x_2 - p_1 + p_2 = 4b = -(\alpha + 1)(p_1 - p_2)$$

$$\Rightarrow b = -\frac{\alpha + 1}{4}(p_1 - p_2)$$

$$\Rightarrow y = \frac{p_1 + p_2}{2} + b = \frac{p_1 + p_2}{2} - \frac{\alpha + 1}{4}(p_1 - p_2) = \frac{(1 - \alpha) p_1 + (3 + \alpha) p_2}{4} = \frac{\varphi}{\varphi - 1} p_2$$

$$x = \frac{p_1 - p_2}{2} + b = \frac{p_1 - p_2}{2} - \frac{\alpha + 1}{4}(p_1 - p_2) = \frac{(1 - \alpha)(p_1 - p_2)}{4} = \frac{1}{\varphi - 1} p_2$$

Let

$$\frac{x_1 + x_3}{p_1 + p_2} = \frac{p_1 + p_2 + 4b}{p_1 + p_2} = 1 + \frac{4b}{p_1 + p_2}$$

$$\frac{p_1 + p_2}{x_1 + x_3} = \frac{x_1 + x_3 - 4b}{x_1 + x_3} = 1 - \frac{4b}{x_1 + x_3}$$

We pose $m = \frac{2b}{p_1 + p_2}$, $m' = $ [illegible] $\frac{2b}{x_1 + x_3}$

$mm' = 0 \Rightarrow b = 0$, we have supposed $mm' \neq 0$

$\forall (m, m'); \exists \beta \mid m = \beta m'$

$$\Rightarrow \frac{2b}{p_1 + p_2} = -\beta \frac{2b}{x_1 + x_3}$$

$$\Rightarrow x_1 + x_3 = -\beta (p_1 + p_2) \Rightarrow x_1 + x_3 - p_1 - p_2 = 4b = -(\beta + 1)(p_1 + p_2)$$

$\Rightarrow b = -$ [illegible] $+1$ $(p_1 +$ [illegible]

$$\Rightarrow y = \frac{p_1 + p_2}{2} + b = \frac{p_1 + p_2}{2} - \frac{\beta + 1}{4}(p_1 + p_2) = \frac{(1 - \beta)(p_1 + p_2)}{4} = \frac{\varphi}{\varphi - 1} p_2$$

$$x = \frac{p_1 - p_2}{2} + b = \frac{p_1 - p_2}{2} - \frac{\beta + 1}{4}(p_1 + p_2) = \frac{(1 - \beta) p_1 - (\beta + 3) p_2}{4} = \frac{1}{\varphi - 1} p_2$$

$$b = -\frac{\alpha + 1}{4}(p_1 - p_2) = -\frac{\beta + 1}{4}(p_1 + p_2) \Rightarrow (\beta - \alpha) p_1 = (-2 - \alpha - \beta) p_2$$

But : $(2k+1)(2k'+1)=1 \Rightarrow 2kk'+k+k'=0 \Rightarrow (2k+1)k'=-k$

$$2k+1=\frac{-k}{k'}=\frac{k+1}{k'+1}=\frac{a(k+1)+a'(2k+1)}{a(k'+1)+a'}$$

$$2k'+1=\frac{-k'}{k}=\frac{k'+1}{k+1}=\frac{c(k'+1)+c'(2k'+1)}{c(k+1)+c'},\ \forall(a,a',c,c')$$

$$\frac{2k+1}{2k'+1}=\frac{((a+2a')k+a+a')(ck+c+c')}{((c+2c')k'+c+c')(ak'+a+a')}$$

$$=\frac{(ac+2a'c)k^2+(2ac+ac'+3a'c+2a'c')k+ac+ac'+a'c+a'c'}{(ac+2ac')k'^2+(2ac+3ac'+a'c+2a'c')k'+ac+ac'+a'c+a'c'}$$

$$\Rightarrow\frac{2k+1}{2k'+1}-1=(k-k')\frac{2}{2k'+1}$$

$$=\frac{ac(k^2-k'^2)+2a'ck^2-2ac'k'^2+(2ac+ac'+a'c+2a'c')(k-k')+2a'ck-2ac'k'}{(ac+2ac')k'^2+(2ac+3ac'+a'c+2a'c')k'+ac+ac'+a'c+a'c'}$$

$\forall(a,a',c,c')$, particularly $(a,a',c,c')\,|\,ac'=\delta k^2,\ a'c=\delta k'^2$

$$\Rightarrow(k-k')\frac{2}{2k'+1}$$

$$=\frac{ac(k+k')(k-k')+(2ac+ac'+a'c+2a'c')(k-k')+2\delta kk'^2-2\delta k^2k'}{(ac+2ac')k'^2+(2ac+3ac'+a'c+2a'c')k'+ac+ac'+a'c+a'c'}$$

$$=(k-k')\frac{-2ackk'+2ac+ac'+a'c+2a'c'-2\delta kk'}{(ac+2ac')k'^2+(2ac+3ac'+a'c+2a'c')k'+ac+ac'+a'c+a'c'}$$

$k=k'\Rightarrow\alpha=1\Rightarrow y=\frac{(1-\alpha)p_1+(3+\alpha)p_2}{4}=p_2$ it is impossible.

$$k-k'\neq0\Rightarrow\frac{2}{2k'+1}=\frac{-2ackk'+2ac+ac'+a'c+2a'c'-2\delta kk'}{(ac+2ac')k'^2+(2ac+3ac'+a'c+2a'c')k'+ac+ac'+a'c+a'c'}$$

Also $(2m+1)(2m'+1)=1\Rightarrow 2mm'+m+m'=0\Rightarrow(2m+1)m'=-m$

$$2m+1=\frac{-m}{m'}=\frac{m+1}{m'+1}=\frac{a(m+1)+a'(2m+1)}{a(m'+1)+a'}$$

$$2m'+1=\frac{-m'}{m}=\frac{m'+1}{m+1}=\frac{c(m'+1)+c'(2m'+1)}{c(m+1)+c'},\ \forall(a,a',c,c')$$

$$\frac{2m+1}{2m'+1}=\frac{((a+2a')m+a+a')(cm+c+c')}{((c+2c')m'+c+c')(am'+a+a')}$$

$$=\frac{(ac+2a'c)m^2+(2ac+ac'+3a'c+2a'c')m+ac+ac'+a'c+a'c'}{(ac+2ac')m'^2+(2ac+3ac'+a'c+2a'c')m'+ac+ac'+a'c+a'c'}$$

$$\Rightarrow\frac{2m+1}{2m'+1}-1=(m-m')\frac{2}{2m'+1}$$

$$=\frac{ac(m^2-m'^2)+2a'cm^2-2ac'm'^2+(2ac+ac'+a'c+2a'c')(m-m')+2a'cm-2ac'm'}{(ac+2ac')m'^2+(2ac+3ac'+a'c+2a'c')m'+ac+ac'+a'c+a'c'}$$

$\forall(a,a',c,c')$, particularly $(a,a',c,c')\,|\,ac'=\delta k^2=\gamma m^2,\ a'c=\delta k'^2=\gamma' m'^2$

$$\Rightarrow(m-m')\frac{2}{2m'+1}$$

$$=\frac{ac(m+m')(m-m')+2(\gamma'-\gamma)m^2m'^2+(2ac+ac'+a'c+2a'c')(m-m')+2\gamma'mm'^2-2\gamma m^2m'}{(ac+2ac')m'^2+(2ac+3ac'+a'c+2a'c')m'+ac+ac'+a'c+a'c'}$$

$$=(m-m')\frac{-2acmm'+2(\frac{\gamma'-\gamma}{m-m'})m^2m'^2+2ac+ac'+a'c+2a'c'-2(\frac{\gamma m-\gamma'm'}{m-m'})mm'}{(ac+2ac')m'^2+(2ac+3ac'+a'c+2a'c')m'+ac+ac'+a'c+a'c'}$$

$m=m'\Rightarrow\beta=1\Rightarrow y=\frac{(1-\beta)(p_1+p_2)}{4}=0$, it is impossible

But $\frac{2}{2k'+1}=\frac{-2ackk'+2ac+ac'+a'c+2a'c'-2\delta kk'}{(ac+2ac')k'^2+(2ac+3ac'+a'c+2a'c')k'+ac+ac'+a'c+a'c'}$

For $(a,a',c,c')\,|\,ac'=\delta k^2=\gamma m^2\ ,\quad a'c=\delta k'^2=\gamma' m'^2$

$$\Rightarrow\frac{2}{2m'+1}=\frac{-2acmm'+2ac+ac'+a'c+2a'c'-2\gamma mm'}{(ac+2ac')m'^2+(2ac+3ac'+a'c+2a'c')m'+ac+ac'+a'c+a'c'}$$

$$=\frac{-2acmm'+2ac+ac'+a'c+2a'c'-2\gamma' mm'}{(ac+2ac')m'^2+(2ac+3ac'+a'c+2a'c')m'+ac+ac'+a'c+a'c'}$$

$$\Rightarrow\frac{2}{2m'+1}=\frac{-2acmm'+2(\frac{\gamma'-\gamma}{m-m'})m^2m'^2+2ac+ac'+a'c+2a'c'-2(\frac{\gamma m-\gamma' m'}{m-m'})mm'}{(ac+2ac')m'^2+(2ac+3ac'+a'c+2a'c')m'+ac+ac'+a'c+a'c'}$$

$$=\frac{-2acmm'+2ac+ac'+a'c+2a'c'-2\gamma mm'}{(ac+2ac')m'^2+(2ac+3ac'+a'c+2a'c')m'+ac+ac'+a'c+a'c'}$$

$$=\frac{-2acmm'+2(\frac{\gamma-\gamma}{m-m'})m^2m'^2+2ac+ac'+a'c+2a'c'-2\gamma mm'}{(ac+2ac')k'^2+(2ac+3ac'+a'c+2a'c')k'+ac+ac'+a'c+a'c'}$$

$$\Rightarrow\gamma=\gamma'\Rightarrow ac'=\delta k^2=\gamma m^2\ ,\quad a'c=\delta k'^2=\gamma' m'^2=\gamma m'^2$$

$$\Rightarrow\frac{\delta}{\gamma}=\frac{m^2}{k^2}=\frac{m'^2}{k'^2}\Rightarrow\frac{k^2}{k'^2}=(2k+1)^2=\alpha^2=\frac{m^2}{m'^2}=(2m+1)^2=\beta^2$$

If $\alpha=-\beta\Rightarrow(\beta-\alpha)p_1=-2\alpha p_1=2\beta p_1=(-2-\alpha-\beta)p_2=-2p_2\Rightarrow\alpha=-\beta=\frac{p_2}{p_1}$

$$b=-(\alpha+1)\frac{p_1-p_2}{4}=-(\beta+1)\frac{p_1+p_2}{4}=-(\frac{p_1+p_2}{4p_1})(p_1-p_2)=\frac{p_2^2-p_1^2}{4p_1}$$

$\Rightarrow 4bp_1=p_2^2-p_1^2\Rightarrow(4b+p_1)p_1=p_2^2\Rightarrow 4b+p_1=\frac{p_2^2}{p_1}$ and it is impossible because p_1 and p_2 are primes and $4b+p_1=\frac{p_2^2}{p_1}$ can not be an integer.

$$\Rightarrow\alpha-\beta=0\Rightarrow(\beta-\alpha)p_1=(-2-\alpha-\beta)p_2=-(2+2\alpha)p_2=-2(1+\beta)p_2=0\Rightarrow\alpha=\beta=-1$$

$$\Rightarrow b=-\frac{(\alpha+1)}{4}(p_1-p_2)=-\frac{(\beta+1)}{4}(p_1+p_2)=0$$

$$\Rightarrow y=\frac{p_1+p_2}{2}\ ,\quad x=\frac{p_1-p_2}{2}$$

Another proof : let u,u',v,v' verifying

$$(\frac{1}{up_1-vp_2}-\frac{1}{u'p_1+v'p_2})p_1=(\frac{1}{up_1-vp_2}+\frac{1}{u'p_1+v'p_2})p_2$$

$$\Rightarrow(u'-u)p_1^2+(v+v')p_1p_2=(u+u')p_1p_2+(v'-v)p_2^2$$

Thus

$$(u'+u-v'-v)p_1p_2+(u-u')p_1^2=(v-v')p_2^2$$

We pose with a different of zero

$$u'+u-v'-v=a(\alpha-\beta)$$

$$\Rightarrow((v-v')-a(2+\alpha+\beta))p_2^2=(u-u')p_1^2$$

Let

$$v-v'-a(2+\alpha+\beta)=(2+\alpha+\beta)^2$$

$$u-u'=(\alpha-\beta)^2$$

$$\Rightarrow u'p_1+v'p_2=(u-(\alpha-\beta)^2)p_1+(v-a(2+\alpha+\beta)-(2+\alpha+\beta)^2)p_2$$

$$=up_1+vp_2-(\alpha-\beta)^2p_1-a(\alpha-\beta)p_1-(2+\alpha+\beta)(\alpha-\beta)p_1$$

Let $up_1 = -vp_2$ hence

$$\left(\frac{1}{up_1 - vp_2} - \frac{1}{u'p_1 + v'p_2}\right)p_1 = \left(\frac{1}{up_1 - vp_2} + \frac{1}{u'p_1 + v'p_2}\right)p_2$$

$$\Rightarrow \left(\frac{1}{2u} + \frac{1}{(\alpha-\beta)^2 + a(\alpha-\beta) + (2+\alpha+\beta)(\alpha-\beta)}\right)p_1$$

$$= \left(\frac{1}{2u} - \frac{1}{(\alpha-\beta)^2 + a(\alpha-\beta) + (2+\alpha+\beta)(\alpha-\beta)}\right)p_2$$

$$\Rightarrow \frac{p_1 - p_2}{2u} = -p_1\left(\frac{1}{(2+\alpha+\beta)(2\alpha+a+2)} + \frac{1}{(\alpha-\beta)(2\alpha+a+2)}\right)$$

Let $2u = \alpha - \beta$ hence

$$\frac{1}{\alpha-\beta} - \frac{1}{\alpha+\beta+2} = -\frac{1}{(2+\alpha+\beta)(2\alpha+a+2)} - \frac{1}{(\alpha-\beta)(2\alpha+a+2)}$$

$$\Rightarrow \frac{2\alpha+a+3}{\alpha-\beta} = \frac{2\alpha+a+1}{2+\alpha+\beta}; \forall a \Rightarrow \alpha = \beta = -1$$

It means b=0 thus

$x+y=p_1$, $y-x=p_2$ are primes $y = r$ is the primal radius. As there is no condition, there is an infinity of p_1, p_2.

If $(x_1-x_2)(x_1+x_3)=0 \Rightarrow (x_4+x_2)(x_4-x_3) \neq 0$

Let $\frac{x_4+x_2}{p_1+p_2} = \frac{p_1+p_2-4b}{p_1+p_2} = 1 - \frac{4b}{p_1+p_2}$

$$\frac{p_1+p_2}{x_4+x_2} = \frac{x_4+x_2+4b}{x_4+x_2} = 1 + \frac{4b}{x_4+x_2}$$

$$k = -\frac{2b}{p_1+p_2}, \quad k' = \frac{2b}{x_4+x_2}$$

$$\frac{x_4-x_3}{p_1-p_2} = \frac{p_1-p_2-4b}{p_1-p_2} = 1 - \frac{4b}{p_1-p_2}$$

$$\frac{p_1-p_2}{x_4-x_3} = \frac{x_4-x_3+4b}{x_4-x_3} = 1 + \frac{4b}{x_4-x_3}$$

$$m = -\frac{2b}{p_1-p_2}, \quad m' = \frac{2b}{x_4-x_3}$$

With the same calculus and reasoning, it implies that b = 0 . But b can not be equal to zero in all cases, it means there is an impossibility related to the fact that the conjecture is indecidable and if it is so, it is true ! Because, we would find in the case it is indecidable and false with the computer the 2x difference of all sums of primes and it is contradictory !

For $2 = p_1 - p_2$ a difference of an infinity of couples of primes. There is an infinity of consecutive primes. A[illegible] for all [illegible] > 2104 exists $p_1 = p_2 + 2$, p_2 primes for which $2x = p_1 + p_2$

Conclusion

The notion of the primal radius as defined in this study allows to confirm that for all integer x ≥ 3 exists a number r > 0 for which x + r and x − r are primes and that for all integer x ≥ 0 exists a number r > 0 for which x + r and r − x are primes and that exists an infinty of such primes. r is called the primal radius The corrolary is the proof of the Goldbach conjecture and de Polignac conjecture which stipulate, the first that an even number is always the sum of two prime numbers, the second that an even number is always the difference between two primes and that there is an infinity of such couples of primes. Another corollary is the proof of the twin primes conjecture which stipulates that there is an infinity of consecutive primes.

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