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The reproductive solution for Fermat's Last theorem (elementary aspect)

PROF. DR. K. RAJA RAMA GANDHI[1] AND REUVEN TINT[2]

Resource person in Math for Oxford University Press and Professor at BITS-Vizag[1]

Number Theorist, Israel[2]

Email: editor126@gmail.com, reuven.tint@gmail.com

Abstract. We give a proof of the solvability in a natural numbers for Fermat's Last theorem and the equations

$$a^x + b^y = c^z$$

and

$$X_1^{n_1} + X_2^{n_2} + \dots + X_{k-1}^{n_{k-1}} = X_k^{n_k}$$

has not been found earlier, significantly different from known, and allow us to obtain infinite set of solutions in natural numbers, and examples.

Theorem 1.

We must to prove that the equation

$$x^n + y^n = z^n$$

has no solutions in natural numbers (positive integers) for $n > 2$.

Proof:

1.1. Suppose

$$x' + y' = z',$$

where x', y' - are arbitrary natural numbers, as

$$x^{\alpha a - pbc > 0} + y^{\beta b - qac > 0} = z^{\gamma c - mab > 0} \qquad (1).$$

Then,

$$\alpha a > pbc,\ \beta b > qac,\ \gamma c > mab.$$

1.1.1. Whenever

$$p = \alpha - 1, q = \beta - 1, m = \gamma - 1;$$

then

$$\alpha < \frac{1}{1-\frac{a}{bc}}\ , \beta < \frac{1}{1-\frac{b}{ac}}, \gamma < \frac{1}{1-\frac{c}{ab}}\ .$$

For

$$a = b = c = n$$

$$\alpha = \beta = \gamma < \frac{1}{1-\frac{n}{n^2}} = \frac{n}{n-1} = \frac{2}{2-1} = 2 = 1 + \frac{1}{n-1} = 1 + \frac{1}{2-1}.$$

Therefore, $n = 2$ and $\alpha = \beta = \gamma = 1$, since the inequality is valid only for the case when comparing the elements of the same numerical system - the natural numbers. It follows that, ultimately, to the equation
(for $\alpha = \beta = \gamma = 1$ and $a = b = c = n$):

$$x^{1\times n-(1-1)\times n^2 > 0} + y^{1\times n-(1-1)\times n^2 > 0} = z^{1\times n-(1-1)\times n^2 > 0},$$

and since "n" is not greater than two, that lead to equations

$$x^2 + y^2 = z^2 \text{ and } x + y = z.$$

1.1.2. If

$$\alpha a > (\alpha - k)bc\ ,$$

then

$$\alpha(bc - a) < kbc$$

and

$$\alpha < k \times \frac{bc}{bc - a} = k \times \frac{n}{n-1}\ (k > 1).$$

Multiply the two numbers "k" and $\frac{n}{n-1}$ in the system of natural numbers for$(n > 2)$ is incorrectly.

1.1.3. If

$$b = c = n,$$

then

$$\alpha a - pn^2 > 0$$

and

$$p < \frac{\alpha a}{n^2}.$$

For $a = n$ $\quad p < \alpha \times \frac{1}{n}$

Multiply the two numbers α and $\frac{1}{n}$ in the system of natural numbers $n > 1$ is incorrectly.
Sets of functions $\frac{1}{n}$ and $\frac{n}{n-1}$ are not closed under superposition.

1.2. It is easy to verify, that if

$$x + y = z,$$

then

$$\alpha a - pbc = 1, \beta b - qac = 1, \gamma c - mab = 1, \qquad (2)$$

where

$$(\alpha a, pbc) = 1, (\beta b, qac) = 1, (\gamma c, mab) = 1, (\alpha, p) = 1, (\beta, q) = 1, (\gamma, m) = 1$$

- are relatively prime integers, a, b, c - are pairwise relatively prime integers(having no common factor other than one).

1.3.

$$\alpha a - pbc = \beta b - qac = \gamma c - mab = 0$$

is impossible since

$$1 + 1 \neq 1.$$

1.4. If

$$\alpha = \beta = \gamma = p = q = m = 1$$

$$a > bc, b > ac, c > ab \text{ and } 1 > a^2, 1 > b^2, 1 > c^2,$$

then it is impossible.

1.5. Thus, consider all possible variants for our case the solutions of equation

$$x + y = z.$$

1.6. Therefore, the equation

$$x^n + y^n = z^n$$

is not solvable in natural numbers for $n > 2$ as required. **This completes the proof.**
This is an elementary proof, with a high degree of probability could be obtained by Fermat.

2.1. Multiplying **[1]** by

$$t = x^{pbc} \times y^{qac} \times z^{mab}.$$

Given

$$(x^{\alpha} \times y^{qc} \times z^{mb})^a + (x^{pc} \times y^{\beta} \times z^{ma})^b = (x^{pb} \times y^{qa} \times z^{\gamma})^c.$$

2. 2.

$$a = 4, b = 5, c = 7,$$

$$\alpha \times 4 - p \times 5 \times 7 = 1, p = 1, \alpha = 9,$$

$$\beta \times 5 - q \times 4 \times 7 = 1 \ , q = 3; \beta = 17,$$

$$\gamma \times 7 - m \times 4 \times 5 = 1, m = 1, \gamma = 3,$$

$$(x^9 \times y^{21} \times z^5)^4 + (x^7 \times y^{17} \times z^4)^5 = (x^5 \times y^{12} \times z^3)^7,$$

as $x + y = z.$

2.3. If

$$3^2 + 4^2 = 5^2,$$

then

$$[3 \times (3^{35} \times 4^{84} \times 5^{20})]^2 + [4 \times (3^{35} \times 4^{84} \times 5^{20})]^2 = [5 \times (3^{35} \times 4^{84} \times 5^{20})]^2$$

and

$$[(3^9 \times 4^{21} \times 5^5)^2]^4 + [(3^7 \times 4^{17} \times 5^4)^2]^5 = [(3^5 \times 4^{12} \times 5^3)^2]^7$$

- In fact, there is a slightly modified solution of L. Yushmanovich –(6, page 74).

2.4. Using the equation [**2**], if

$$\alpha_0, \beta_0, \gamma_0, \rho_0, q_0, m_0$$

are any (or minimal solution) related above equations in natural numbers for fixed values a, b, c ,then

$$\alpha = \alpha_0 + bc\theta_1 \quad , \rho = \rho_0 + a\theta_1,$$

$$\beta = \beta_0 + ac\theta_2 \quad , q = q_0 + b\theta_2,$$

$$\gamma = \gamma_0 + ab\theta_3 \quad , m = m_0 + c\theta_3,$$

$\theta_1, \theta_2, \theta_3$- are arbitrary natural (whole) numbers or zero, and

$$(x^{\alpha_0 + bc\theta_1} \times y^{q_0 c + bc\theta_2} \times z^{m_0 b + bc\theta_3})^a +$$

$$+ (x^{p_0 c + ac\theta_1} \times y^{\beta_0 + ac\theta_2} \times z^{m_0 a + ac\theta_3})^b =$$

$$= (x^{p_0 b + ab\theta_1} \times y^{q_0 a + ab\theta_2} \times z^{\gamma_0 + ab\theta_3})^c.$$

2.5.

$$a = 4; b = 5; c = 7$$

$$\alpha = 9 + 5 \times 7 = 44; \rho = 1 + 4 = 5$$

$$\beta = 17 + 4 \times 7 = 45;\ q = 3 + 5 = 8$$

$$\gamma = 3 + 4 \times 5 = 23;\ m = 1 + 7 = 8$$

$$(x^{44} \times y^{56} \times z^{40})^4 + (x^{35} \times y^{45} \times z^{32})^5 =$$

$$=(x^{25} \times y^{32} \times z^{23})^7$$

2.6. If

$$x_1 + x_2 + \ldots.. + x_{k-1} = x_k,$$

then

$$\sum_{i=1}^{k-1}\left[\prod_{i=1}^{k-1}\left(x_i^{\alpha_i} \times \prod_{k\neq j=1}^{k} x_j^{\prod_{i\neq j=1}^{k} p_j \times \prod_{i,j\neq\vartheta=1}^{k} n_\vartheta}\right)\right]^{n_i} =$$

$$= \left(x_k^{\alpha_k} \times \prod_{k\neq j=1}^{k-1} x_j^{\prod_{k\neq j=1}^{k-1} p_j \times \prod_{k,k-1\neq\vartheta=1}^{k-2} n_\vartheta}\right)^{n_k},$$

where

$x_1, x_2, \ldots.., x_{k-1}$ – are arbitrary natural numbers;

$x_k = \sum_{i=1}^{k-1} x_i$;

$n_1, n_2, \ldots, n_{k-1}$ – are pairwise relatively prime arbitrary natural numbers,

$k \geq 3$;

$i = 1,2, \ldots., k$ – are numbers of brackets;

$j = 1,2, \ldots., k$ - are numbers inside brackets;

$\sum$, $\prod$ - are signs of sum and products;

2.7. If given n_i and n_k of "k" values, then α_i and p_j determined by the "k" in the following equations:

$$\alpha_i n_i - \rho_j \prod_{i\neq j=1}^{k} n_j = 1.$$

2.8. For example, for $k = 4$

$$(x_1^{\alpha_1} \times x_2^{p_2 n_3 n_4} \times x_3^{p_3 n_2 n_3} \times x_4^{p_4 n_2 n_3})^{n_1} +$$

$$+ (x_2^{\alpha_2} \times x_1^{p_1 n_3 n_4} \times x_3^{p_3 n_1 n_4} \times x_4^{p_4 n_1 n_3})^{n_2} +$$

$$+ (x_3^{\alpha_3} \times x_1^{p_1 n_2 n_4} \times x_2^{p_2 n_1 n_4} \times x_4^{p_4 n_1 n_2})^{n_3} =$$

$$= (x_4^{\alpha_4} \times x_1^{p_1 n_2 n_3} \times x_2^{p_2 n_1 n_3} \times x_3^{p_3 n_1 n_2})^{n_4}$$

, and if

$$n_1 = 3, n_2 = 4, n_3 = 5, n_4 = 7,$$

$$47 \times 3 - 1 \times 4 \times 5 \times 7 = 1$$

$$79 \times 4 - 3 \times 3 \times 5 \times 7 = 1$$

$$17 \times 5 - 1 \times 3 \times 4 \times 7 = 1$$

$$43 \times 7 - 5 \times 3 \times 4 \times 5 = 1.$$

$$\alpha_1 = 47, \alpha_2 = 79, \alpha_3 = 17, \alpha_4 = 43,$$

$$p_1 = 1, p_2 = 3, p_3 = 1, p_4 = 5.$$

$$(x_1^{47} \times x_2^{105} \times x_3^{28} \times x_4^{100})^3 + (x_2^{79} \times x_1^{35} \times x_3^{21} \times x_4^{75})^4 +$$

$$+(x_3^{17} \times x_1^{28} \times x_2^{63} \times x_4^{60})^5 = (x_4^{43} \times x_1^{20} \times x_2^{45} \times x_3^{12})^7.$$

2.9. If

$$27^5 + 84^5 + 110^5 + 133^5 = 144^5$$

- is well-known solution, then

$$(27^{1502} \times 84^{5005} \times 110^{5148} \times 133^{819} \times 144^{2541})^{5\times 2} +$$

$$+ (27^{1001} \times 84^{3337} \times 110^{3432} \times 133^{546} \times 144^{1694})^{5\times 3} +$$

$$+ (27^{429} \times 84^{1430} \times 110^{1471} \times 133^{234} \times 144^{726})^{5\times 7} +$$

$$+ (27^{273} \times 84^{910} \times 110^{936} \times 133^{149} \times 144^{462})^{5\times 11} =$$

$$= (27^{231} \times 84^{770} \times 110^{792} \times 133^{126} \times 144^{391})^{5\times 13}.$$

- In fact, there is a slightly modified solution of L. Yushmanovich
for arbitrary natural numbers "n" and arbitrary dimensions ($k \geq 3$) –(6), стр.74.

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