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The proof of the insolubility in natural numbers for $\mathbf{n > 2}$, the Fermat's Last Theorem and Beal's conjecture for coprime integers arranged in a pair $\mathbf{A, B, D}$ in the equations $\mathbf{A^n + B^n = D^n}$ and $\mathbf{A^n + B^y = D^z}$. (elementary aspect)

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Abstract. We give the corresponding identities for different solutions of the equations:

$$. \mathrm{aA^x + bB^x = cD^x}\ [\mathbf{1}] \text{ and}$$

$$\mathrm{aA^x + bB^y = cD^z}\ [\mathbf{2}]:$$

As for coprime integers $\mathrm{a, b, c, A, B, D}$ and arbitrary positive integers $\mathrm{x, y, z}$
further,for not coprime integers , if

$$\mathrm{A_0^{x_0} + B_0^{x_0} = D_0^{x_0}}\ [\mathbf{3}] \text{ and}$$

$$\mathrm{A_0^{x_0} + B_0^{y_0} = D_0^{z_0}}\ [\mathbf{4}] \text{ ,where}$$

$\mathrm{x_0}, y_0, z_0, A_0, B_0, D_0$ - are any solutions in positive integers.

Theorem.

1. "If $[\mathbf{3}]$ and $[\mathbf{4}]$ have any solutions $\mathrm{x_0}, y_0, z_0, A_0, B_0, D_0$ - in positive integers, then from $[\mathbf{1}]$ and $[\mathbf{2}]$ we obtain an infinite number of solutions for arbitrary x, y, z".

Proof.

1.1. Obviously, with respect to

$$a = kD_0^{x_0} + b;\ c = kA_0^{x_0} + b \text{ and}$$

$$a = kD_0^{z_0} + b;\ c = kA_0^{x_0} + b, \text{ where}$$

"k" and "b" – are arbitrary positive integers, then

$$\left(kD_0^{x_0} + b\right)A_0^{x_0} + bB_0^{x_0} \equiv \left(kA_0^{x_0} + b\right)D_0^{x_0}\ [\mathbf{5}],$$

$$\left(kD_0^{z_0}+b\right)A_0^{x_0}+bB_0^{y_0}\equiv\left(kA_0^{x_0}+b\right)D_0^{z_0}\ [\mathbf{6}]$$

1.2. For $[\mathbf{3}]$ there exists $k=k'A_0^xB_0^xD_0^x$ and $b=b'A_0^xB_0^xD_0^x$, where $x;(k',b')=1$ –are arbitrary positive integers, since $[\mathbf{5}]$ we have

$$B_0^xD_0^x\left(k'D_0^{x_0}+b'\right)A_0^{x+x_0}+A_0^xD_0^xB_0^{x+x_0}\equiv$$

$$\equiv A_0^xB_0^x\left(k'A_0^{x_0}+b'\right)D_0^{x+x_0}[\mathbf{7}].$$

1.3. Similarly, **1.2.** from $[\mathbf{6}]$ it follows that

$$B_0^yD_0^z\left(k'D_0^{z_0}+b'\right)A_0^{x+x_0}+A_0^xD_0^zB_0^{x+y_0}\equiv A_0^xB_0^y\left(k'A_0^{x_0}+b'\right)D_0^{x+z_0}[\mathbf{8}].$$

1.4. If A_0,B_0,D_0 -are for coprime integers arranged in a pair, then equations $[\mathbf{5}],[\mathbf{6}],[\mathbf{7}],[\mathbf{8}]$ have an infinite set of solutions for each $n=x+x_0$, this means, that using Faltings (4) the equations $[\mathbf{3}]$ and $[\mathbf{4}]$ for each $n=x+x_0>2$ does not have solutions in natural numbers. Then, $x=0$ and $x_0\leq 2.$ This means that Fermat's Last Theorem and Beal's conjecture based on previous articles completely resolved. The proof is completed. In the same way $[\mathbf{7}],[\mathbf{8}]$ instead of x,x_0 holds y,y_0 or z,z_0

Examples:

1.4.1.

$$13^2\times 3^4+11^2\times 2^4=5^2\times 5^4$$

$$3^4\times 2^4\neq 5^4, \text{ but}$$

$$39^2\times 3^2+11^2\times 4^2=5^4\times 5^2$$

$$39^2=56\times 5^2+11^2$$

$$5^4=56\times 3^2+11^2$$

$$7^2+2^5=3^4$$

1.4.2. Let

$$A^2 + B^2 = D^2$$

then, if A and B are not coprime, but

$$3^2 + 4^2 = 5^2,$$

therefore from [7]

$$4^x \times 5^x(k'5^2 + 3^x b')3^{x+2} + 3^x 5^x 4^{x+2} \equiv$$

$$\equiv 3^x \times 4^x \times (k' \times 3^x + 5^x \times b') \times 5^{x+2} \text{ etc.}$$

1.4.3. If

$$\mathrm{A}_0^n + \mathrm{B}_0^n = \mathrm{D}_0^n \text{ , then}$$

$$a^2 A_0^n + b^2 B_0^n \equiv c^2 D^n \text{ and } a = p^2 D_0^n - q^2 B_0^n \text{ ,}$$

$$b = p^2 D_0^n - 2pqD_0^n + q^2 B_0^n \text{ , where}$$

$p, q -$ are arbitrary natural positive integers.
If $n = 2, p = 2, q = 1; D_0 = 3;\ B_0 = 4, D_0 = 5$,then

$$7^2 \times 3^4 + 4^2 \times 2^4 = 13^2 \times 5^2,$$

$$21^2 \times 3^2 + 4^2 \times 4^2 = 13^2 \times 5^2,$$

$$209^2 \times 3^2 + 91^2 \times 4^2 = 145^2 \times 5^2\ (p = 3; q = 1) \text{ etc.}$$

1.4.4. If

$$a + b = c,$$

then

$$a(p^2c - q^2b)^2 + b(p^2c - 2pqc + q^2b)^2 \equiv$$

$$\equiv c(p^2c - 2pqb + q^2b)^2, \text{ where}$$

$p, q -$ are arbitrary positive integers.
In addition, suppose that:

$$a' = a^n; b' = b^n; c' = c^n;$$

$$a' = a^n; b' = b^y; c' = c^z.$$

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