The proof of the insolubility in natural numbers for $n > 2$, the Fermat's Last Theorem and Beal's conjecture for coprime integers arranged in a pair $A, B, D$ in the equations

$$A^n + B^n = D^n$$

and

$$A^y + B^y = D^z.$$ (elementary aspect)

PROF. DR. K. RAJA RAMA GANDHI$^1$ AND REUVEN TINT$^2$

Resource person in Math for Oxford University Press and Professor at BITS-Vizag$^1$
Number Theorist, Israel$^2$
Email: editor126@gmail.com, reuven.tint@gmail.com

Abstract. We give the corresponding identities for different solutions of the equations:

$$aA^x + bB^x = cD^x [1]$$

and

$$aA^y + bB^y = cD^z [2].$$

As for coprime integers $a, b, c, A, B, D$ and arbitrary positive integers $x, y, z$ further, for not coprime integers, if

$$A_0^{x_0} + B_0^{x_0} = D_0^{x_0} [3]$$

and

$$A_0^{y_0} + B_0^{y_0} = D_0^{z_0} [4],$$

where $x_0, y_0, z_0, A_0, B_0, D_0$ - are any solutions in positive integers.

Theorem.

1. “If [3] and [4] have any solutions $x_0, y_0, z_0, A_0, B_0, D_0$ - in positive integers, then from [1] and [2] we obtain an infinite number of solutions for arbitrary $x, y, z$.”

Proof.

1.1. Obviously, with respect to

$$a = kD_0^{x_0} + b; \quad c = kA_0^{x_0} + b$$

and

$$a = kD_0^{z_0} + b; \quad c = kA_0^{x_0} + b,$$

"$k$" and "$b$" – are arbitrary positive integers, then

$$(kD_0^{x_0} + b)A_0^{x_0} + bB_0^{x_0} \equiv (kA_0^{x_0} + b)D_0^{x_0} [5],$$
\[(kD_0^{x_0} + b)A_0^{x_0} + bB_0^{y_0} \equiv (kA_0^{x_0} + b)D_0^{z_0} \quad [6]\]

1.2. For [3] there exists \(k = k'A_0^xD_0^y\) and \(b = b'A_0^xD_0^y\), where \(x; (k', b') = 1\) – are arbitrary positive integers, since [5] we have

\[B_0^xD_0^y(k'D_0^{x_0} + b')A_0^{x+x_0} + A_0^xD_0^yB_0^{x+x_0} \equiv \]

\[\equiv A_0^xB_0^y(k'A_0^{x_0} + b')D_0^{x+x_0} \quad [7].\]

1.3. Similarly, 1.2. from [6] it follows that

\[B_0^yD_0^x(k'D_0^{z_0} + b')A_0^{x_0} + A_0^yD_0^xB_0^{x+y_0} \equiv A_0^yB_0^x(k'A_0^{x_0} + b')D_0^{x+y_0} \quad [8].\]

1.4. If \(A_0, B_0, D_0\) - are for coprime integers arranged in a pair, then equations [5], [6], [7], [8] have an infinite set of solutions for each

\(n = x + x_0\), this means, that using Faltings (4) the equations [3] and [4] for each

\(n = x + x_0 > 2\) does not have solutions in natural numbers.

Then, \(x = 0\) and \(x_0 \leq 2\). This means that Fermat's Last Theorem and Beal's conjecture based on previous articles completely resolved. The proof is completed. In the same way [7], [8] instead of \(x, x_0\) holds \(y, y_0\) or \(z, z_0\)

**Examples:**

1.4.1.

\[13^2 \times 3^4 + 11^2 \times 2^4 = 5^2 \times 5^4\]

\[3^4 \times 2^4 \neq 5^4, \text{ but}\]

\[39^2 \times 3^2 + 11^2 \times 4^2 = 5^4 \times 5^2\]

\[39^2 = 56 \times 5^2 + 11^2\]

\[5^4 = 56 \times 3^2 + 11^2\]

\[7^2 + 2^5 = 3^4\]
1.4.2. Let
\[ A^2 + B^2 = D^2 \]
then, if \( A \) and \( B \) are not coprime, but
\[ 3^2 + 4^2 = 5^2, \]
therefore from [7]
\[ 4^x \times 5^x (k' 5^2 + 3^x b') 3^x + 2 + 3^x 5^x 4^x + 2 \equiv \]
\[ \equiv 3^x \times 4^x \times (k' \times 3^x + 5^x \times b') \times 5^x + 2 \, \text{etc.} \]

1.4.3. If
\[ A_0^n + B_0^n = D_0^n, \text{ then} \]
\[ a^2 A_0^n + b^2 B_0^n \equiv c^2 D^n \text{ and } a = p^2 D_0^n - q^2 B_0^n, \]
\[ b = p^2 D_0^n - 2pq D_0^n + q^2 B_0^n, \text{ where} \]
p, q \text{ — are arbitrary natural positive integers.}
If \( n = 2, p = 2, q = 1; D_0 = 3; B_0 = 4, D_0 = 5 \), then
\[ 7^2 \times 3^4 + 4^2 \times 2^4 = 13^2 \times 5^2, \]
\[ 21^2 \times 3^2 + 4^2 \times 4^2 = 13^2 \times 5^2, \]
\[ 209^2 \times 3^2 + 91^2 \times 4^2 = 145^2 \times 5^2 (p = 3; q = 1) \, \text{etc.} \]

1.4.4. If
\[ a + b = c, \]
then
\[ a(p^2 c - q^2 b)^2 + b(p^2 c - 2pq + q^2 b)^2 \equiv \]
\[ \equiv c(p^2 c - 2pq b + q^2 b)^2, \text{ where} \]
p, q \text{ — are arbitrary positive integers.}
In addition, suppose that:
\[ a' = a^n; b' = b^n; c' = c^n; \]
\[ a' = a^n; b' = b^y; c' = c^z. \]
References:


