

**The proof of the insolubility in natural numbers for  $n > 2$  , the Fermat's Last Theorem and Beal's conjecture for coprime integers arranged in a pair  $A, B, D$  in the equations  
 $A^n + B^n = D^n$  and  $A^n + B^y = D^z$ .  
 (elementary aspect)**

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**Abstract.** We give the corresponding identities for different solutions of the equations:

$$. aA^x + bB^x = cD^x \text{ [1] and}$$

$$aA^x + bB^y = cD^z \text{ [2]:}$$

As for coprime integers  $a, b, c, A, B, D$  and arbitrary positive integers  $x, y, z$  further, for not coprime integers, if

$$A_0^{x_0} + B_0^{x_0} = D_0^{x_0} \text{ [3] and}$$

$$A_0^{x_0} + B_0^{y_0} = D_0^{z_0} \text{ [4] , where}$$

$x_0, y_0, z_0, A_0, B_0, D_0$  - are any solutions in positive integers.

**Theorem.**

1. "If [3] and [4] have any solutions  $x_0, y_0, z_0, A_0, B_0, D_0$  - in positive integers, then from [1] and [2] we obtain an infinite number of solutions for arbitrary  $x, y, z$ ".

**Proof.**

1.1. Obviously, with respect to

$$a = kD_0^{x_0} + b; c = kA_0^{x_0} + b \text{ and}$$

$$a = kD_0^{z_0} + b; c = kA_0^{x_0} + b, \text{ where}$$

" $k$ " and " $b$ " – are arbitrary positive integers, then

$$(kD_0^{x_0} + b)A_0^{x_0} + bB_0^{x_0} \equiv (kA_0^{x_0} + b)D_0^{x_0} \text{ [5],}$$

$$(kD_0^{z_0} + b)A_0^{x_0} + bB_0^{y_0} \equiv (kA_0^{x_0} + b)D_0^{z_0} \text{ [6]}$$

1.2. For [3] there exists  $k = k'A_0^x B_0^x D_0^x$  and  $b = b'A_0^x B_0^x D_0^x$ , where  $x; (k', b') = 1$  -are arbitrary positive integers, since [5] we have

$$\begin{aligned} B_0^x D_0^x (k' D_0^{x_0} + b') A_0^{x+x_0} + A_0^x D_0^x B_0^{x+x_0} &\equiv \\ &\equiv A_0^x B_0^x (k' A_0^{x_0} + b') D_0^{x+x_0} \text{ [7].} \end{aligned}$$

1.3. Similarly, 1.2. from [6] it follows that

$$B_0^y D_0^z (k' D_0^{z_0} + b') A_0^{x+x_0} + A_0^x D_0^z B_0^{x+y_0} \equiv A_0^x B_0^y (k' A_0^{x_0} + b') D_0^{x+z_0} \text{ [8].}$$

1.4. If  $A_0, B_0, D_0$  -are for coprime integers arranged in a pair, then equations [5], [6], [7], [8] have an infinite set of solutions for each  $n = x + x_0$ , this means, that using Faltings (4) the equations [3] and [4] for each  $n = x + x_0 > 2$  does not have solutions in natural numbers. Then,  $x = 0$  and  $x_0 \leq 2$ . This means that Fermat's Last Theorem and Beal's conjecture based on previous articles completely resolved. The proof is completed. In the same way [7], [8] instead of  $x, x_0$  holds  $y, y_0$  or  $z, z_0$

**Examples:**

1.4.1.

$$13^2 \times 3^4 + 11^2 \times 2^4 = 5^2 \times 5^4$$

$$3^4 \times 2^4 \neq 5^4, \text{ but}$$

$$39^2 \times 3^2 + 11^2 \times 4^2 = 5^4 \times 5^2$$

$$39^2 = 56 \times 5^2 + 11^2$$

$$5^4 = 56 \times 3^2 + 11^2$$

$$7^2 + 2^5 = 3^4$$

1.4.2. Let

$$A^2 + B^2 = D^2$$

then, if  $A$  and  $B$  are not coprime, but

$$3^2 + 4^2 = 5^2,$$

therefore from [7]

$$\begin{aligned} 4^x \times 5^x (k'5^2 + 3^x b') 3^{x+2} + 3^x 5^x 4^{x+2} &\equiv \\ \equiv 3^x \times 4^x \times (k' \times 3^x + 5^x \times b') \times 5^{x+2} &\text{ etc.} \end{aligned}$$

1.4.3. If

$$A_0^n + B_0^n = D_0^n, \text{ then}$$

$$a^2 A_0^n + b^2 B_0^n \equiv c^2 D_0^n \text{ and } a = p^2 D_0^n - q^2 B_0^n,$$

$$b = p^2 D_0^n - 2pqD_0^n + q^2 B_0^n, \text{ where}$$

$p, q$  – are arbitrary natural positive integers.

If  $n = 2, p = 2, q = 1; D_0 = 3; B_0 = 4, D_0 = 5$ , then

$$7^2 \times 3^4 + 4^2 \times 2^4 = 13^2 \times 5^2,$$

$$21^2 \times 3^2 + 4^2 \times 4^2 = 13^2 \times 5^2,$$

$$209^2 \times 3^2 + 91^2 \times 4^2 = 145^2 \times 5^2 (p = 3; q = 1) \text{ etc.}$$

1.4.4. If

$$a + b = c,$$

then

$$a(p^2 c - q^2 b)^2 + b(p^2 c - 2pqc + q^2 b)^2 \equiv$$

$$\equiv c(p^2 c - 2pqb + q^2 b)^2, \text{ where}$$

$p, q$  – are arbitrary positive integers.

In addition, suppose that:

$$a' = a^n; b' = b^n; c' = c^n;$$

$$a' = a^n; b' = b^y; c' = c^z.$$

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**References:**

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