

## The second proof of the Fermat's Last theorem (elementary aspect)

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**Abstract.** We give a second proof of Fermat's Last theorem without reference to the closure of numerical systems with respect to the operation of superposition.

### Theorem.

We must to prove that the equation

$$x^n + y^n = z^n$$

has no solutions in natural numbers (positive integer)  $x, y, z, n$  for  $n > 2$ .

*Second proof.*

### 1. The statement:

Equation

$$x^n + y^n = z^n$$

is solvable in natural numbers, if the system of equations

$$\begin{cases} 2^\mu n^\vartheta - 2^{\mu'} (n-1)^{\vartheta'} = 1 & [\mathbf{A}]; \\ 2^{\mu'} (n-1)^{\vartheta'} = 1 \end{cases}$$

is consistent. It is consistent only for

$$\mu' = \vartheta' = 0.$$

It follows, that the system has only two solutions

$$\begin{aligned} 2^0 1^1 - 2^0 0^0 &= 1 \text{ and } n = 1; \\ 2^0 2^1 - 2^0 1^0 &= 1 \text{ and } n = 2. \end{aligned}$$

If  $\mu'$  and  $\vartheta'$  not simultaneously equal to zero, then  $[\mathbf{A}]$  has a number of solutions for each positive integer  $n > 2$ , in particular

$$\begin{aligned} 2^0 3^1 - 2^0 2^1 &= 1; \\ 2^0 4^1 - 2^0 3^1 &= 1 \end{aligned}$$

and etc.

**1.1. Second proof (p.1).**

**1.1.** Let  $x > 0; y > 0; z > x, y; n > k \geq 0$  – are an arbitrary natural numbers. Then, the equality

$$x^n(yz)^k + y^n(xz)^k = z^n(xy)^k \quad [1]$$

will be proved , if

$$x^{n-k} + y^{n-k} = z^{n-k} [2].$$

**1.2.** It also includes the case where

$$x^n + y^n = z^n \quad [3],$$

**1.2.1.** that from [1] and [2] for  $n > 2$  is possible only where  $k = 0$ .

**1.2.2.** And only for  $n = 2$  we obtain

$$\begin{aligned} x^2(yz)^k + y^2(xz)^k &= z^2(xy)^k, \\ x^{2-k} + y^{2-k} &= z^{2-k} \end{aligned}$$

and  $k = 0$  (for all  $n > 2$ ) . And  $k = 1$  ,since

$$x + y = z$$

for arbitrary  $x$  and  $y$ .

**1.3.**For  $n = k$   $1 + 1 \neq 1$  .

**1.4.** From [3]

$$z = (x^n + y^n)^{\frac{1}{n}} \quad [4].$$

Follows from [2] and [4]

**1.5.**  $n = (n - k)S$  ( $S$ -is a natural number) and

$$\begin{aligned} n &= k \frac{S}{S-1} \quad [5]; \\ S &= 2; k = 1; n = 2. \end{aligned}$$

**1.6.** From [5]

$$\begin{aligned} k &= n \frac{S-1}{S} \quad [6]; \\ S &= 1; k = 0 \text{ and from [6] } n = 2. \end{aligned}$$

**1.7.** Let's consider other variants.

Since

$$\frac{1}{2^\mu 5^\vartheta} ,$$

where  $\mu \geq 0; \vartheta \geq 0$  –are an arbitrary natural numbers,- a unique representation of fractions be given by finite decimal fractions, then

$$k \frac{S}{S-1} \text{ and } n \frac{S-1}{S}$$

can be both positive integers only if

1.7.1. from [5]

$$k = S' - 1 = 2^{\mu'} 5^{\theta'}$$

and

1.7.2.

$$n = S' = 2^{\mu'} 5^{\theta'} + 1;$$

1.7.3. from [6]

$$S = n = 2^{\mu} 5^{\theta},$$

1.7.4. then,

$$2^{\mu'} 5^{\theta'} + 1 = 2^{\mu} 5^{\theta} \quad [7]$$

1.7.5. and

$$2^{\mu} 5^{\theta} - 2^{\mu'} 5^{\theta'} = 1 \quad [7'].$$

1.8. Equation [7'] has only two solutions in positive integers

1.8.1.

$$2^0 5^1 - 2^2 5^0 = 1; n = 5; k = 4 > 0;$$

therefore,  $n \neq 5$  (p 1.2.1.).

1.8.2. It remains only to

$$2^1 5^0 - 2^0 5^0 = 1 \text{ and } n = 2; k = 1.$$

1.8.2.1. Further, we consider the equations

$$x^n + y^n = z^n$$

for  $n > 2$ , using the following system of equations:

$$\begin{cases} x^n (yz)^{n-1} + y^n (xz)^{n-1} = z^n (xy)^{n-1} & [8], \text{ if} \\ x + y = z & [9]; \end{cases}$$

$$\begin{cases} x^n (yz)^{n-2} + y^n (xz)^{n-2} = z^n (xy)^{n-2} & [10], \text{ if} \\ x^2 + y^2 = z^2 & [11]; \end{cases}$$

for arbitrary natural  $n \geq 1$  and  $n \geq 2$ , and within each pair [8] and [9], [10] and [11] accordingly  $x, y, z$  are equal in magnitude.

Examples:

$$\begin{cases} 3^2(4 \times 7)^1 + 4^2(3 \times 7)^1 = 7^2(3 \times 4)^1 & , \text{ as} \\ 3 + 4 = 7, \text{ but } 3^2 + 4^2 \neq 7^2. \end{cases}$$

$$\begin{cases} 3^3(4 \times 5)^1 + 4^3(3 \times 5)^1 = 5^3(3 \times 4)^1 & , \text{ as} \\ 3^2 + 4^2 = 5^2, \text{ but } 3^3 + 4^3 \neq 5^3. \end{cases}$$

$$\begin{cases} 3^3(4 \times 5)^2 + 4^3(3 \times 5)^2 \neq 5^3(3 \times 4)^2 & , \text{ as} \\ 3 + 4 \neq 5 \end{cases}$$

$$\begin{cases} 3^4(4 \times 5)^2 + 4^4(3 \times 5)^2 \neq 5^4(3 \times 4)^2 & , \text{ as} \\ 3^2 + 4^2 = 5^2, \text{ but } 3^4 + 4^4 \neq 5^4. \end{cases}$$

$$\begin{cases} 3^4(4 \times 5)^3 + 4^4(3 \times 5)^3 \neq 5^4(3 \times 4)^3 & , \text{ as} \\ 3 + 4 \neq 5. \end{cases}$$

and so on consecutively ascending up to the  $n < \infty$  (infinity).

**1.9.** Confirmation of the correctness of (p.1.8.1.) using the following variant of the proof:

**1.9.1.** If

$$x^5 + y^5 = z^5$$

would be a solution in positive integers, then the equality

$$x^5(yz)^1 + y^5(xz)^1 = z^5(xy)^1$$

would be satisfied, if

$$x^{5-1} + y^{5-1} = z^{5-1}$$

and

$$x^4 + y^4 = z^4 \quad [12].$$

But as yet proved Fermat, [12] has no solutions in positive integers.

**1.10.** This completes the proof of Fermat's Last theorem without reference to the closure of numerical systems with respect to the operation of superposition, that is

$$x^n + y^n = z^n$$

has no solutions in positive integers for  $n > 2$ .

**1.11.** Now, given that  $2^{\mu'} 5^{\theta'}$  may be both even and odd [7] if and only if

$$(2^{\mu'} 5^{\theta'})^0 = [2^{\mu'} (n - 1)^{\theta'}]^0 = 2^0 (n - 1)^0 = 1,$$

where  $n$  is an arbitrary natural number, perhaps an approval of (p1).

1.12. Combining [1] and [2], we get

$$(n - k)s = k, n = k \frac{s+1}{s}, s=1, k=1, n=2$$

and

$$k = n \frac{s}{s+1}, s = 1, n = 2, k = 1.$$

**References:**

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