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ABOUT SOME TRANSCENDENTAL NUMBERS

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Abstract. Mathematicians of the XIX century have proved that π, e and the Champernowne number are transcendentals, but what about *π+e* or *πe*? The Riemann hypothesis can it be generalized? In this paper, we demonstrate a method in order to know if it is possible to solve these two problems.

The approach of the first problem

A number is transcendental if it is not the root of a polynomial equation : $a_n x^n + a_{n-1}x^{n-1} + \ldots + a_0 = 0$ where the coefficients are rationals and different of zero. Otherwise it would be algebraic. We know since the XIX century that π, a and the Champernowne number are transcendentals but we do not know anything about their sum, their difference not their product.Effectively if A is algebraic and B is transcendental, A+B and AB are transcendentals, but if A is also transcendental we still do not the nature of AB, A-B or A+B. Let us try to solve this problem.

Definition

A real number is composound if it can be written as $\prod_j p_j^{n_j}$ where p_j are integer primes and n_j are rationals. This decomposition in prime factors is unique. A prime real number can be written only as p=p.1. Thus we define other real prime nubers like π, e, ln(2) . Thus $\sqrt[q]{p} = p^{\frac{1}{q}}$ is compound. Also $\sqrt[q]{p} + 1 = p^{\frac{1}{q}} + 1$ prime and we have $\sqrt[2^l]{p} - 1 = (p-1)(\sqrt[2^l]{p}+1)^{-1}(\sqrt[2^{l-1}]{p}+1)^{-1}\ldots(\sqrt{p}+1)^{-1}$ compound, for example.

Theorem

If T and T' are transcendental prime numbers then T+T', T-T' and TT' are transcendentals.

Proof of the theorem

Let C and C' two compound transcendental numbers. We have 4 possibilities.
1) CC'^n, $C+mC'$ are algebraics.
2) CC'^n is algebraic and $C+mC'$ is transcendental.
3) CC'^n is transcendental and $C+mC'$ is algebraic.
4) $CC'^n, C+mC'$ are transcendentals.

Thus
1) $CC'^n + mC'^{n+1} = (C+mC')C'^n = A + mC'^{n+1} = A'C'^n$ where A and A' are algebraics and C' is solution of an algebraic equation, it is impossible !
2) $CC'^n, CC'^{n'}$ are algebraics and $CC'^n(CC'^{n'})^{-1} = C'^{n-n'}$ is algebraic if n=n'. There is only one n=M for which CC'^n is algebraic, all the others are transcendentals. If M is not unique, there are three possibilities : $CC''(C'C''')^n$ is transcendental for all n or $CC''^{-1}(C'C'''^{-1})^n$ is transcendental for all n or there exists L, L' for which
$CC'^M = A, C''C'''^{M'} = A', CC''(C'C''')^L = A'', CC''^{-1}(C'C'''^{-1})^{L'} = A'''$ are algebraics.

$$A^2 C'^{L+L'-2M} C'''^{L-L'} \quad A''A'''$$

$$A'^2 C'^{L-L'} C'''^{L+L'-2M'} \quad A''A'''^{-1}$$

$$C'^X \quad BC'''^Y$$

$$C'^{-Y} \quad DC'''^Z$$

$$X = L' + L - 2M$$

$$Y = L' - L$$

$$Z = -(L + L' - 2M')$$

$$B \quad A''A'''A^{-2}$$

$$D \quad A'^2 A''^{-1} A'''$$

And

$$AA'C'^{L-M} C'''^{L-M'} \quad A''$$

$$AA'^{-1} C'^{L'-M} C'''^{M'-L'} \quad A'''$$

$$C'^U \quad EC'''^V$$

$$C'^{U'} \quad FC'''^{V'}$$

$$U = L' - M$$

$$V = M' - L$$

$$U' = L' - M$$

$$V' = L' - M'$$

$$E \quad A''A^{-1}A'^{-1}$$

$$F \quad A'''A^{-1}A'$$

$$C'^{-XY} = B^{-Y}C'''^{-Y^2} = D^X C'''^{XZ} \Rightarrow -Y^2 = -(L-L')^2 = XZ = -(L+L'-2M)(L+L'-2M')$$

$$C'^{UV'} = E^{V'}C'''^{VV'} = F^{-V}E^{V'}C'^{U'V} \Rightarrow U'V = UV' = (L-M)(M'-L) = (L'-M)(L'-M')$$

$$C'^{YU} = E^Y C'''^{VY} = E^Y B^{-V} C'^{XV} \Rightarrow YU = XV = (L-M)(L'-L) = (M'-L)(L'+L-2M)$$

$$C'^{ZU} = E^Z C'''^{ZV} = E^Z D^{-V} C'^{XV} \Rightarrow ZU = -VY = -(L+L'-2M')(L'-M) = -(L'-L)(L'-M')$$

$$\Rightarrow L^2 + L'^2 - 2LL' = L^2 + L'^2 + 2LL' - 2(M'+M)(L+L') + 4MM'$$

$$\Rightarrow M^2 + M'^2 - 2MM' = M^2 + M'^2 + 2MM' - 2(M'+M)(L+L') + 4LL'$$

$$\Rightarrow (M-M')^2 = (M+M'-2L)(M+M'-2L')$$

Thus

$$(L-M)(L'-M') = (M'-L)(L'+L-2M) + (M-L')(L'-M')$$

$$(L'-M)(L'-M') = (M'-L)(L'-M) + (M'-L)(L-M)$$

$$(L'-M)(L+L'-M-M') = (M'-L)(L-M) = (L'-M)(L'-M')$$

$$\Rightarrow (L'-M)(L-M) = 0 \Rightarrow (M'-L)(M'-L') = 0$$

We deduce

M=M'=L=L' is unique !

3) $C + mC' - (C + m'C') = (m-m')C' \Rightarrow m = m'$ because this number is algebraic at this condition. There is only one m=N for which C+mC' is algebraic, all the others are transcendentals. If N is not unique there are three possibilities :

C+C''+m(C'+C''') is transcendental for all m, C-C''+m(C'-C''') is transcendental for all m or there exists L, L' for which C+NC'=A, C''+N'C'''=A', C+C''+L(C'+C''')=A'', C-C''+L'(C'-C''')=A''' are algebraics, thus

A''+A'''=2A+(L+L'-2N)C'+(L-L')C'''

A’’-A’’’=2A’+(L-L’)C’+(L+L’-2N’)C’’’
A’’=A+A’+(L-N)C’+(L-N’)C’’
A’’’=A-A’+(L’-N)C’+(N’-L’)C’’

Thus $(L - L')^2 = (L + L' - 2N)(L + L' - 2N')$ and $(L - N)(N' - L) = (L' - N)(L' - N')$ and $(L - N)(L' - L) = (N' - L)(L' + L - 2N)$ and $-(L + L' - 2N')(L' - N) = -(L' - L)(L' - N')$ by the same calculus than higher N=N’=L=L’ is unique !

Thus, there are finaly two possibilities

I) There are three subpossibilities : $CC''(C'C''')^n$ is transcendental for all n, for all T, T’ prime transcendental numbers, there exist $C = TC''^{-1}, C' = T'C'''^{-1} \Rightarrow TT'^n$ is transcendental for all n, particularly n=1 and TT’ and T+T’ are transcendentals. Or $CC''^{-1}(C'C'''^{-1})^n$ is transcendental for all n, for all T, T’ prime transcendental numbers, there exist $C = TC''^{-1}, C'^{-1} = T'C'''^{-1} \Rightarrow TT'^n$ is transcendental for all n, particularly n=1 and TT4 and T+T’ are transcendentals.

Third subpossibility : CC'^n for all $n \neq M$ for all T, T’ prime transcendental numbers there exist 1 1 $C = T^{1/2}, C' = T'^{1/n} \Rightarrow TT'$ transcendental and T+T’ transcendental.

II) There are three subpossibilities C+C’’$^{+m}$(C’+C’’’) is transcendental for all n, for all T, T’ prime transcendental numbers, there exist C=T-C’’, C’=T’-C’’’ and T+mT’ is transcendental for all m, particularly m=1 and T+T’ and TT’ are transcendentals. Or C’’+ m(C’-C’’’) is transcendental for all m, for all T, T’ prime transcendental numbers, there exist C=T+C’’, C’=T’+C’’’ and T+mT’ is transcendental for all m, particularly m=1 and T+T’ and TT’ are transcendentals. Third subpossibility : C+mC’ is transcendental for all $m \neq N$. For all T, T’ prime transcendental numbers, there exist 2C=T, mC’=T’ and T+T’ and TT4 are transcendentals.

The theorem application

$T = \pi, T' = e$ implie that the sum and the product of π and e are transcendentals.

The approach of Riemann hypothesis

The Rieman hypothesis states that the non trivial zeros of the Riemann zeta function $\zeta(z) = \sum_{t=1}^{\infty} \frac{1}{t^z}$ lie on the critical line $\frac{1}{2} + iy$. We have

For t integer, Euler has proved that $\zeta(z) = \sum_{t=1}^{\infty} \frac{1}{t^z} = \prod_{primes} \frac{1}{1 - p^{-z}}$, it is the Euler identity. For t real, it is still tru =e and becomes

$\prod_{primes} \frac{1}{1 - p^{-z}} = \int_1^{\infty} \frac{dt}{t^z} = \frac{1}{1-z}\left[t^{1-z}\right]_1^{\infty}$ But there are the tivial zeros : we have $\zeta(-2k) = 0, \forall k \in N$ and $\left[t^{1+2k}\right]_1^{\infty} = 0$ but if $\left[t^z\right]^{\infty}$ but if $\left[t^z\right]^{\infty}$ is the limit in the infinity, $\left[t^{1+2k}\right]^{\infty} = 1, \forall k \in N$ and

$$\left[t^{1-\frac{1}{2}-iy}\right]^{\infty} = \left[t^{\frac{1}{2}-iy}\right]^{\infty} = \left[t^{\frac{1}{2}+iy}\right]^{\infty} = \frac{1}{2}\left(\left[t^{\frac{1}{2}-iy}\right]^{\infty} + \left[t^{\frac{1}{2}+iy}\right]^{\infty}\right) = a$$

$$\Rightarrow \left[t^{2(\frac{1}{2}+iy)} - 2at^{\frac{1}{2}+iy} + t\right]^{\infty} = \left[t^{2(\frac{1}{2}+iy)} - 2at^{\frac{1}{2}+iy} + 1\right]^{\infty} = 0 \Rightarrow \left[t^{\frac{1}{2}+iy}\right]^{\infty} = a = a + \sqrt{a^2 - 1} = 1$$

it means that

$$\zeta(\frac{1}{2}+iy)=\left[\frac{1}{1-\frac{1}{2}-iy}t^{1-\frac{1}{2}-iy}\right]_1^{\infty}=0$$

Let now

$$\zeta(x+iy)=0=\left[\frac{t^{1-x-iy}}{1-x-iy}\right]_1^{\infty}=\left[\frac{t^{\frac{1}{2}-iy}t^{\frac{1}{2}-x}}{1-x-iy}\right]_1^{\infty}=\left[\frac{t^{\frac{1}{2}-x}}{1-x-iy}\right]_1^{\infty}=0\Rightarrow x=\frac{1}{2}$$

We have proved that the non trivial zeros of the Riemann function for the reals lie in the critical line ! So the hypothesis for the real numbers is proved. The Riemann hypothesis is important because it gives information about the zeros of the Riemann function and the distribution of those zeros are related to real primes !

Conclusion

Through this exposé, we have given a method to find the nature of several numbers, we have shown the nature of some of them. We have also generalized the Riemann hypothesis to the real numbers and proved it !

References

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